CRITERIA FOR USING A ONE-DIMENSIONAL HYDRODYNAMIC MODEL FOR TWO BARRAGES SITUATED IN MEDITERRANEAN REGION

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Abstract:
In order to characterise the barrage regime is usually very important to check the validity of assumptions on which its application is based. Accordingly, in this study we have estimated a set of numbers comprising the Lake number, the Wedderburn and Burger numbers, and inflow and outflow Froude numbers. Therefore we have used two years worth of time series data (2000-2001) consisting of temperature profiles, morphometry data, meteorological data, and inflow and withdrawal data of two reservoir located in the Mediterranean region.

Key Words: Hydrodynamic., Reservoir regime

Introduction
The understanding of lake hydrodynamics has made much progress in the last twenty years. However, it is still difficult for the general limnological practitioner to gain a quantitative description of the hydrodynamical regimes in a particular lake at a particular time (Imberger, J. (1994))

In general hydraulics characterisation of flow is based essentially on the Reynolds and Froude numbers. So flows are subdivided into laminar or turbulent and super or sub-critical. Therefore, an analogy methodology may be established to permit the limnologist to classify hydrodynamic regime in lakes. Such hydrodynamical regime is important because the mixing and transport process operating in a lake determine, to a large degree, the ecological response of the lake to meteorological forcing, inflows and outflows. Thus, one-dimensional model like the DYRESM is applied to a Mediterranean reservoir such as the Sau or Boadella, it is important to check the validity of assumptions on which its application is based. Accordingly, in this study we have estimated a set of numbers comprising the Lake number, the Wedderburn and Burger numbers, and inflow and outflow Froude numbers. For both reservoirs, we have used two years worth of time series data (2000-2001) consisting of temperature profiles, morphometry data, meteorological data, and inflow and withdrawal data. The temperature profiles have been converted to density profiles using the UNESCO (1981) state formula equation. Later, we have used these density profiles to calculate the non-dimensional numbers.

Lake Number
The Lake number $L_N$ is a dimensionless parameter defined as the ratio of the moments of the stabilizing force of gravity. This, $L_N$ describes the water upwelling from the hypolimnion to the surface layer, expressed by the equation:

$$L_N = \frac{\left(Z_g - Z_o\right) M g \left(1 - \frac{Z_f}{Z_H}\right)}{A^{1/2} \left(1 - \frac{Z_g}{Z_H}\right) \rho_H u_s^2} \tag{1.1}$$

where $Z_g$ is the centre of gravity of the water mass and $Z_g$ is the centre of volume for the entire lake body; $Z_f$ is the height to the centre of the metalimnion; $Z_H$ is the depth from the bottom.
of the reservoir, $M$ is the total mass of water [Kg]; $g$ is acceleration due to gravity [$m.s^{-2}$]; $\rho_0$ is the average water density [$kg.m^{-3}$]; $A$ is the surface area of the reservoir $A(Z)$; and $u_s^*$ is the water friction velocity [$m.s^{-1}$].

$Z_0$ and $Z_g$ are defined respectively as follows:

$$Z_0 = \frac{\sum_{i=0}^{m_c} \rho(Z_i)Z_i A(Z_i)}{\sum_i \rho(Z_i)A(Z_i)}$$ (12)

$$Z_g = \frac{\sum_i \rho(Z_i)A(Z_i)}{\sum_i A(Z_i)}$$ (1.3)

$$u_{s^*}^2 = \frac{\rho_s}{\rho_0} C_D U_{10}^2$$ (1.4)

where $U_{10}$ is the wind velocity at 10 m above the water surface [$m.s^{-1}$]; $C_D$ is the drag coefficient $=1.3.10^{-3}$ [dimensionless]; and $\frac{\rho_s}{\rho_0}$ is the ratio between air and water densities $= 1.2.10^{-3}$ [dimensionless].

Under these circumstances, the isopycnals are expected to be primarily horizontal and little seiching of the seasonal thermocline or turbulent mixing in the hypolimnion are expected. For $L_N << 1$, stratification is weak with respect to wind stress. Under these circumstances, the seasonal thermocline is expected to experience strong seiching and the hypolimnion is expected to experience extensive turbulent mixing due to internal shear (Imberger 1989). Thus the hypolimnion water, very rich in nutrients, will reach the surface layer during the wind episode (Imberger 2001).

**Wedderburn Number**

The Wedderburn number represents the ratio of the baroclinic restoring force to the wind disturbance force, or the ratio of the restoring moment about the centre of the volume of the lake to the disturbance moment. $W$ describes the upwelling of water from the metalimnion into the water surface, expressed by:

where $g_{h}$ is the modified acceleration due to gravity across the uppermost thermocline.

$$W = \frac{g' h^2}{u_{s^*}^2 L}$$ (1.5)

This is represented by $g' = \frac{\Delta \rho}{\rho_s} g$, where $\Delta \rho$ is the density difference between the surface layer and the mean water density, and $h$ is the depth of the diurnal thermocline; $u_s^*$ is the water shear velocity due to wind stress and approximated by the bulk aerodynamic formula as previously defined in the Lake number calculation; and $L$ is the length of the lake.

c. For $W >> 1$, tilting of the isotherms due to applied wind stress will be small and horizontal variations negligible. This coincides with strong stratification, light winds, and slow deepening of the mixed layer. For $W << 1$, deepening is dominated by internal shear production and will occur over a much shorter time scale than horizontal convection in the surface layer (Imberger & Patterson 1990). Where $W$ is small and $L_N$ large, only the upper region of the thermocline will respond to wind forcing. Where $W$ and $L_N$ are small, the lake as a whole should respond, and vertical mixing should occur throughout it (Imberger and Patterson 1990).
Burger Number

The Burger number $S_i$ is an indicator of the influence of the earth’s rotation on water motion in reservoirs; influence on the water internal waves. $S_i$ is expressed by:

$$S_i = \frac{c_i}{L_w f}$$

(1.6)

where $L_w$ is the width of the reservoir and $c_i$ is the Rossby radius; $c_i$ is the wave velocity expressed by:

$$c_i = \sqrt{g' H}$$

(1.7)

$g' = g \frac{\Delta \rho}{\rho_0}$ is the reduced gravity; $\Delta \rho$ is the difference between surface water and mean $H$ is the mean reservoir water depth, which depends on the inflow entering the reservoir and water withdrawal from the reservoir. $f$ is the Coriolis parameter equal to the double rate of rotation of the earth at the latitude of the lake, $S_i = 1$ is the critical value indicating that the rotation is of the same magnitude as gravity. When $S_i >> 1$, the internal oscillations increasingly take on the characteristics of simple gravitational seiches (Antenucci & Imberger 2001).

When $S_i << 1$, the waves have characteristics similar to those of an inertial oscillation, with the majority of the energy in the wave being in the form of kinetic energy.

Inflow Froude number

The regime behaviour of the river inflow entering the reservoir is described by the inflow Froude number, $F_{ri}$:

$$h_i \left[ \frac{Q_i}{g^{1/2} H^{3/2} L_w} \right]^{2/3} = F_{ri}^{2/3}$$

(1.8)

where $Q_i$ is the peak inflow discharge; $H$ is the total depth of the reservoir; $g' = \frac{\Delta \rho}{\rho_0}$ is the reduced gravity; $\Delta \rho$ is the difference between inflowing water density and the mean reservoir water density; $\rho_0$ is the mean water density; and $F_{ri} = 1$ is the critical value of the plunge or rise. Where $F_{ri} >> 1$, the inflow is too large to separate as an underflow or an overflow. When $F_{ri} << 1$, the inflow separates as an underflow or an overflow. Once it has been established that the river water underflows ($\Delta \rho_i > 1$ and $h_i/H < 1$), it is necessary to carry out a more detailed analysis to estimate its entrainment into the downflow and thus the depth of the inflow intrusion.

Outflow Froude number

The outflow Froude number characterises the type of water withdrawal, expressed by the equation:

$$F_{ro} = \left[ \frac{Q_o}{g^{3/2} H^{5/2}} \right]$$

(1.9)

where $Q_o$ is the outflow discharge; $g' = \frac{\Delta \rho}{\rho_0}$ is the reduced gravity; $\Delta \rho$ is the difference between outflow water density and the mean reservoir water density; $\rho_0$ is the mean water density; $H$ is the total depth of the reservoir. Where $F_{ro} << 1$, then the outflow is selective from a depth corresponding to the outflow level.

Methods

Lake numbers were computed for each recorded temperature profile using equation 1.1. We used twenty measured over two years (2000-2001) for the Sau Reservoir and fifteen for the Boadella
Reservoir. As water depth decreases and increases with water inflow and outflow, the water surface and the reservoir volume change. To calculate both, interpolation from the bathymetric data was necessary. To determine the daily velocity friction we used equation 2.4, with a drag coefficient of 1.3.10^{-3}, and a ratio between air and water densities of 1.2.10^{-3}. Wind velocity is given in meteorological data in Fig. 2.8 for the Sau Reservoir and in Fig. 2.21 Chapter 2 for the Boadella Reservoir. The mean water density was taken as 1000 kg m^{-3}. The depth of the reservoir Z_H and its area were estimated using morphometry data (Sections 2.4.1 and 2.5.1, Chapter 2). The estimated lake numbers are 712 in Sau reservoir and 636 in Boadella Reservoir.

As described above for lake number, Wedderburn number values were estimated by linear interpolation between existing profiles. For each temperature profile using equation 1.5 daily friction velocity u_\alpha had been previously defined in the calculation of the Lake number.

Modified acceleration due to gravity g' is the ratio of the difference in diurnal thermocline density from the mean water density, divided by the mean water density multiplied by gravity g. L is the average length of the reservoir: L = 3000 m for Sau and L = 1500 m for Boadella.

The Burger number was obtained using equation 1.6 in which the Rossby radius was calculated as the ratio between longwave phase velocity and inertial frequency.

Longwave phase speed is a function of reduced gravity g', which depends on the difference between surface water density and mean water density divided by the mean reservoir water density. Inertial frequency depends on the latitude of a reservoir; the Sau and Boadella reservoirs are located in the same region and have approximately the same altitude.

The inflow Froude number was calculated using equation 1.8. The discharge or inflow rate Q_i is the peak inflow discharge entering the reservoir and was estimated from the inflow file: approximately 217 m^3 s^{-1} for the Sau Reservoir and 111 m^3 s^{-1} for Boadella. The reduced gravity g_i is the difference between inflow density and mean reservoir density divided by the mean density. The average width of the reservoir is B = 700 m for Sau and 600 m for Boadella. The total depth of the reservoir H is variable, depending on the volume entering and leaving it.

The outflow Froude number was computed using equation 1.9. The outflow Q_o was deduced from the withdrawal data. The reduced gravity for outflow is the ratio of the outflow density minus the mean water density to the mean water density. H is the reservoir water depth.

Results and discussions

Lake number

The L_N values obtained for the Sau Reservoir corresponding to lowest wind velocity 0.8 m/s and the minimum is around 2 approximately in the middle of December 2001. Thus, L_N > 1 indicating that stratification is the dominant force when compared to wind stress. Minimum and maximum wind velocities are 0.8 and 2.9 (m/s) respectively. There is low turbulence and no mixing in deeper water, due to the isolation of the bottom waters from the surface. Consequently, However at the end of 2001 the Lake number was equal 2, indicating that the hypolimnion was relatively mixed.

The L_N of the Boadella Reservoir fluctuated from a minimum value of 5 in the beginning of June 2000 corresponding relatively to high wind velocity 5.6 m/s (see section 2.5.2.4 Chapter 2) to a maximum of 2226 in the end of Mars 2000 corresponding to the lowest wind velocity 0.4 m/s. Compared to Sau the stratification is strong in the first year (2000) and during the second (2001). This is linked to low wind velocity, for which minimum value and maximum values are 0.8 and 2.9 for Sau and 0.7 and 2.1 for Boadella (see Figs. 2.8 and 2.21, Chapter 2.) It should also be noted that morphometry plays an important role in the determination of Lake number, and that determination of the thermocline is difficult. In both reservoirs, water upwelling from the hypolimnetic occurs at the winter end autumn in 2000 and 2001. Fig 1.1 shows the differences in Sau and Boadella’s Lake numbers. In this figure we see that lake number diminished corresponding to the peak inflow (see Fig. 2.10B, Chapter 2) in the end of the year 2000 means that may inflow also contribute in hypolimnion mixing.
However, in Boadella Reservoir the lack of profiles from the end of August until the end of April influence the estimation of lake number for comparison purposes versus inflow. Also, withdrawal being selective may enhance stratification. For both reservoirs through the period 2000 to 2001 Lake numbers are bigger than the critical value. This indicates that stratification is relatively strong and that the deepest part of the hypolimnion remains unmixed if not influenced by inflow/outflow and cooling forces. Unfortunately, there are fewer profiles for Boadella than there are for Sau. However, the variation in its Lake number is small, as is the case for Sau. Also, it should be noted that the wind velocity in Boadella, on the days when profiles were taken, was relatively low. We can therefore conclude that both reservoirs were not completely mixed, at least at the time the profiles were taken. It has should also be noted, however, that the profiles were taken during the day. In winter, during the day, and the mean velocity of the wind is in general not high enough to overcome stratification. For example, on 09/02/2000:

\[ Z_g - Z_0 = 1.6 \times 10^{-3} \text{ m}, \quad u^* = 0.0017 \text{ m.s}^{-1} \quad \text{and} \quad L_N = 60. \]

However, it is very likely that at night time \( Z_g - Z_0 \) was close to 0 and \( L_N \) went to 0. In these circumstances hypolimnetic mixing could be expected.

Fig 1.1 The differences in the Sau and Boadella Lake numbers

**Wedderburn number**

The Wedderburn numbers in the Sau Reservoir do change substantially, tending to oscillate over the two years between 0 and 750. Only one value is below 1 corresponding to the value in the beginning of November 2000 corresponding to the high wind velocity 5.6 m/s. 0, indicating that wind stress on the surface of the lake is able to overcome the stratification in the water column and metalimnetic water can be expected to be vented into the surface layer if we neglect other disturbing forces such as inflow/outflow and cooling.

Wedderburn numbers in the Boadella Reservoir range from approximately 0 to 454 at the end of May and are smaller than those of Sau, See Fig. 1.2 for Sau and Boadella Wedderburn number differences.
Burger number

The Sau Reservoir experiences substantial fluctuation in Si values throughout the two years, with all the values above 1. Burger numbers range from 1.20 to 2.3, with a mean value of approximately 1.75. Both constant. The Burger number is high than the critical value 1.0 and therefore the rotational effects are insignificant. However, given that Si is close to 1, rotation would be discarded. The Sau and Boadella Burger numbers are shown in Fig.1.3.
**Inflow Froude number**

The average Sau inflow Froude number is 0.01. It is important to note that it would be necessary to carry out a more detailed analysis to estimate the entrainment into the inflow and the depth of the downflow insertion.

The inflow Froude number for Boadella is smaller for Sau. Arnera rivers which flow into Boadella. Fig.1.4 shows the inflow Froude numbers for Sau and Boadella.

![Figure 1.4 Sau and Boadella inflow Froude numbers](image)

**Outflow Froude number**

Sau’s average outflow Froude number is about 0.01, which is lower than the critical value of 1.0. Boadella’s average outflow Froude number is 0.001, ten times smaller than Sau’s one. This indicates that in both reservoirs the outflow is selective. Fig. 1.5 shows the Sau and Boadella outflow Froude numbers.

![Figure 1.5 Sau and Boadella outflow Froude numbers](image)

**Conclusions**

The Lake number $L_N$ while the Wedderburn number $W$ is an indicator of surface layer mixing. In both reservoirs, $L_N$ is large and $W$ is small for the profiles under study. This means that
the one-dimensional DYRESM model can be used. (Imbeger, J. 2001). Where \( L_N < 1 \). It should be pointed out, however, that to calculate the \( L_N \) we have used daily mean wind taken during the daytime. It is likely that there are times during the night and/or when there are very high winds that \( L_N < 0 \), Wedderburn number are not enough because it leads to an overestimation of these numbers especially when the days when profiles were taken are calm therefore daily temperature profiles and daily inflow are needed to judge perfectly whether the one dimensional hydrodynamic may be applied or not. For this reason, daily lake numbers, which were obtained by interpolation of daily temperature from monthly observed water temperature profiles, density structure is approximately horizontal and the one dimensional assumption is valid. Also we have estimate the daily wedderburn number as we did for lake numbers. Wedderburn numbers, are mostly greater than one indicating that tilting of the isotherms is small; this is due to the weakness of the wind stress.

The Burger number \( S_i \) is slightly larger than one, indicating that rotation might be discarded.

\[ F_r << 1 \text{ and } F_s << 1 \] indicate that in both reservoirs separation occurs

References:


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