THE ELECTROMAGNETIC FIELD OF THE POINT
HERTZIAN RADIATOR IN UNIAXIAL MAGNETIC
CRYSTAL

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Abstract
Earlier received new analytical solution of the equation of Maxwell are expressed by
several wave potentials, one of them is constructed by means of an integrated sine and a
cosine. In this work asymptotic solution from the analytical solution of the equations of
Maxwell for electromagnetic field strength of the emitter radiator of Hertz in uniaxial
magnetic crystal are received.

Keywords: Asymptotic solution, radiator of Hertz, crystal

Introduction
At the solution of many problems of radio engineering, telecommunication and other
scientific and technical branches it is much important to know structure of an electromagnetic
field in the considered part of space. These issues include, for example, the development of
radiating systems, ensuring electromagnetic compatibility of radio devices and
telecommunications systems, and etc. For calculation the electromagnetic field in each case it
is required to solve the corresponding electrodynamic problem.

In work [1] the method of calculation of an electromagnetic field of an electric dipole
in a homogeneous medium with uniaxial anisotropy of electric and magnetic properties with
use of vector potential was considered.

Now at a high level the mathematical apparatus of the theory of the generalized
functions is developed. The class of the generalized functions contains the juddering and
singular generalized functions, in particular, and a class of smooth functions. In work [2] for
the solution of such task the method of the generalized functions is used.

In the presented work electromagnetic field of a point radiator of Hertz in a far zone
for the uniaxial anisotropic medium, a tensor of magnetic permeability which has a diagonal
appearance with various diagonal components is investigated. Elements \( \hat{\mu} \) are selected in
the form of a diagonal matrix: \( \mu_{zz} = \mu_1, \mu_{xx} = \mu_{yy} = \mu \), which corresponds to the magnetic
uniaxial crystal. The axis of a crystal is directed along an axis \( z \).

Statement of the problem
In work [2] exact solutions of the equations of Maxwell for electric and magnetic
dipoles in uniaxial magnetic crystals are received. Exact solutions are received as the sum of
two independent solutions. The first composed one of decisions is defined by the wave
potential \( \Psi_0 \) and a vector of the moment of the electric dipole \( p_0^E \), operating along an axis of
a magnetic crystal:
\[
\begin{aligned}
E_1 &= -\frac{1}{\varepsilon_0} (\nabla \nabla + k_0^2) (\Psi_0 p_0^e), \\
H_1 &= i\omega \cdot [\nabla, (\Psi_0 p_0^e)],
\end{aligned}
\]

(1)

where \( \nabla \) – operator of Hamilton,

\[
\nabla = \frac{\partial}{\partial x} e_x + \frac{\partial}{\partial y} e_y + \frac{\partial}{\partial z} e_z, \quad \Psi_0 = -\exp(ik_0 r) \frac{\mu}{4\pi r},
\]

\[
r = \sqrt{x^2 + y^2 + z^2}, \quad k_0^2 = \omega^2 \varepsilon_0 \varepsilon_\mu \mu_0, \quad p_0^e = (0,0,0).
\]

The second composed analytical solutions are defined by wave potentials \( \Psi_0, \Psi_1^m, \Psi_2^m \) and a vector of the moment of the electric dipole \( p_\perp^e \), which located perpendicularly to an axis of a crystal of \( z \):

\[
\begin{aligned}
E_2 &= -\frac{1}{\varepsilon_0} (k_n^\mu (p_\perp^e \Psi_1^m) - k_0^2 \nabla \nabla (\nabla, (p_\perp^e \Psi_2^m)) + \nabla (\nabla, (p_\perp^e \Psi_0))), \\
H_2 &= -i\omega (\nabla \nabla e_z [\nabla, (p_\perp^e \Psi_2^m)] + e_z \nabla (\nabla, (p_\perp^e \Psi_0 - \Psi_1^m))) - [\nabla, (p_\perp^e \Psi_0)],
\end{aligned}
\]

(2)

here the wave potential \( \Psi_1^m \) and the radius vector \( r' \) for the magnetic anisotropic medium are determined by:

\[
\Psi_1^m = -\sqrt{\frac{\mu}{\mu_1}} \frac{\exp(i k_n^\mu r' \frac{\mu}{4\pi r')}}{\mu_1}, \quad r' = \sqrt{x^2 + y^2 + \frac{z}{\mu_1^2}}.
\]

(3)

The wave number \( k_n^\mu \) for uniaxial magnetic crystal:

\[
k_n^\mu = k_0 \sqrt{\frac{\mu}{\mu_1}}, \quad \nabla \nabla = \nabla - \frac{\partial}{\partial z} e_z, \quad p_\perp^e = (p_x^e, p_y^e, 0).
\]

The function \( \Psi_2^m \) is defined by the integral sine and cosine:

\[
\Psi_2^m = \frac{-i}{8\pi k_0} \left[ \exp(ik_0 z)(\text{Ci}(k_0 (r - z)) + i \cdot \text{si}(k_0 (r - z))) + \exp(-ik_0 z)(\text{Ci}(k_0 (r + z)) + \\
+ i \cdot \text{si}(k_0 (r + z))) - \exp(ik_0 z)(\text{Ci}(k_n^\mu r' - k_0 z) + i \cdot \text{si}(k_n^\mu r' - k_0 z)) - \\
- \exp(-ik_0 z)(\text{Ci}(k_n^\mu r' + k_0 z) + i \cdot \text{si}(k_n^\mu r' + k_0 z)) \right].
\]

(4)

The integrated sine and cosine:
\[
\text{Ci}(z) = \gamma + \ln(z) + \int_0^z \frac{\cos \frac{t}{2} - 1}{t} \, dt, \\
\text{si}(z) = \int_0^z \frac{\sin \frac{t}{2}}{t} \, dt - \frac{\pi}{2},
\]

(5)

\( \gamma = 0.5772 \) – Euler number.

In this part it is required to find the asymptotic solution of the exact solution of vectors of intensity of an electromagnetic field for an electric dipole in the far zone, which is located parallel to and perpendicular to an axis of uniaxial magnetic crystal.

**Solution of the problem**

To find the strength of electromagnetic field for the point electric dipole operating parallel to an axis of a magnetic crystal in a far zone, it is necessary to consider communication of the Cartesian system of coordinates with spherical system of coordinates [3]:

\[
x = r \cos \varphi \cdot \sin \theta, \quad y = r \sin \theta \cdot \sin \varphi, \quad z = r \cos \theta.
\]

(6)

Now I differentiate (1) and using (6), it is possible to obtain an asymptotic solution of intensity of electromagnetic field for a point electric dipole, acting parallel to the magnetic axis of the crystal in the far zone:

\[
E_r = E_\varphi = H_r = H_\theta = 0,
\]

(7)

\[
E_\theta = \frac{k_0^2 \rho_z^e \cos \varphi \cdot \cos \theta \cdot \exp(i(k_0 r - \omega \cdot t))}{4\pi \varepsilon_0 r},
\]

(8)

\[
H_\varphi = -\frac{\omega \cdot k_0 \rho_z^e \exp(i(k_0 r - \omega \cdot t)) \cos \theta \cdot \cos \varphi}{4\pi r}.
\]

(9)

The received solutions (7)-(9) correspond with solutions for a dipole in the isotropic medium.

Further asymptotic solutions of intensity of electromagnetic field for a point electric dipole in the far zone, which is located perpendicularly to an axis to uniaxial magnetic crystal (2) will be received.

Using (6) in (3), it is easy to convert the radius vector \( r' \) and the wave potential \( \Psi_1^m \) for uniaxial magnetic crystal of the Cartesian coordinate system in the spherical coordinate system:

\[
r' = r \sqrt{\frac{\mu}{\mu_1}} \cos^2 \theta + \sin^2 \theta,
\]

(10)
\( \Psi_1^m = -\frac{\exp(i(k_\mu^\mu r \sqrt{\frac{\mu}{\mu_1} \cos^2 \theta + \sin^2 \theta} - \omega \cdot t))}{4\pi r \sqrt{\frac{\mu}{\mu_1} \cos^2 \theta + \sin^2 \theta}}. \) \hspace{1cm} (11)

The asymptotic solution of the integral sine and cosine (5) at \( z \to \infty \):

\[
\sin(z) \approx \frac{1}{z} \left( \cos(z) + \frac{\sin(z)}{z} \right), \quad \text{Ci}(z) \approx \frac{1}{z} \left( \sin(z) - \frac{\cos(z)}{z} \right), \hspace{1cm} (12)
\]

By means of (12) it is possible to find the asymptotic solution of function \( \Psi_2^m \) (4) in a far zone (Fig.2):

\[
\Psi_2^m \approx \frac{\Psi_0 r^2 - \Psi_1^m \cdot r'^2}{k_0^2 (r^2 - z^2)}. \hspace{1cm} (13)
\]

Fig.2. Comparative graph of the exact and the asymptotic solution function \( \Psi_2^m \)

The components of the vectors of intensity of electromagnetic field (2) are:
\[
E_{2x} = -\frac{1}{\varepsilon_0\varepsilon}(k_n^2 \frac{\partial^2}{\partial x^2}(p_x^e \Psi_1^m) + \frac{\partial^2}{\partial x^2}(p_x^e \Psi_0^m) - k_0^2 \frac{\partial^2}{\partial x^2}(p_x^e \Psi_2^m)),
\]
\[
E_{2y} = -\frac{1}{\varepsilon_0\varepsilon}(\frac{\partial^2}{\partial x\partial y}(p_x^e \Psi_0^m) - k_0^2 \frac{\partial^2}{\partial x\partial y}(p_x^e \Psi_2^m)),
\]
\[
E_{2z} = -\frac{1}{\varepsilon_0\varepsilon \frac{\partial^2}{\partial x\partial z}(p_x^e \Psi_0^m),
\]
\[
H_{2x} = i\omega \frac{\partial^3}{\partial x\partial y\partial z}(p_x^e \Psi_2^m),
\]
\[
H_{2y} = i\omega \left(\frac{\partial^3}{\partial y^2\partial z}(p_x^e \Psi_2^m) + \frac{\partial}{\partial z}(p_x^e \Psi_0^m)\right),
\]
\[
H_{2z} = -i\omega \frac{\partial}{\partial y}(p_x^e \Psi_0^m),
\]

As a result of differentiation (14), (15) in the far zone will be the following:
\[
E_{2x} = -\frac{k_0^2 p_x^e x^2}{4\pi\varepsilon_0\varepsilon} \left(\exp(i(k_0 r - \omega \cdot t)) - \frac{1}{x^2 + y^2} \frac{\exp(i(k_0 r - \omega \cdot t))}{r} - \sqrt{\frac{\mu_1}{\mu}} \frac{\exp(i(k_0 r - \omega \cdot t))}{r'}\right)
\]
\[
E_{2y} = -\frac{p_x^e xy}{4\pi\varepsilon_0\varepsilon} \frac{k_0^2 \exp(i(k_0 r - \omega \cdot t))}{r^3} - \frac{1}{x^2 + y^2} \frac{\exp(i(k_0 r - \omega \cdot t))}{r} - \sqrt{\frac{\mu_1}{\mu}} \frac{\exp(i(k_0 r - \omega \cdot t))}{r'}\right),
\]
\[
E_{2z} = -\frac{p_x^e xz}{\varepsilon_0\varepsilon} \frac{k_0^2 \exp(i(k_0 r - \omega \cdot t))}{r^3},
\]
\[
H_{2x} = -\frac{\omega k_0 xz}{x^2 + y^2} \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2} - \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2},
\]
\[
H_{2y} = -\frac{\omega k_0 yz}{x^2 + y^2} \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2} - \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2} + \omega k_0 z \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2},
\]
\[
H_{2z} = -\omega \cdot k_0 z \frac{\exp(i(k_0 r - \omega \cdot t))}{4\pi r^2}.
\]
Further to proceed to the solution of the problem, we consider the components of an arbitrary vector moving from a Cartesian coordinate system in the spherical coordinate system [3]:

\[
\begin{align*}
A_r &= A_x \sin \theta \cos \varphi + A_y \sin \theta \sin \varphi + A_z \cos \theta, \\
A_\theta &= A_x \cos \theta \cos \varphi + A_y \cos \theta \sin \varphi - A_z \sin \theta, \\
A_\varphi &= -A_x \sin \varphi + A_y \cos \varphi.
\end{align*}
\]

(18)

Now taking into account (6) and using (18) it is possible to find of intensity of electromagnetic field for the point electric dipole, which is operating perpendicular to the axis of magnetic crystal in the far zone:

\[
E_r = H_r = 0,
\]

(19)

\[
E_\theta = \frac{k_0^2 p_\varphi^e \cos \theta \cos \varphi \cdot \exp(i(k_0 r - \omega t))}{4\pi \varepsilon_0 r},
\]

(20)

\[
E_\varphi = -\sqrt{\frac{\mu}{\mu_1}} p_\varphi \sin \varphi \cdot \exp(i(k_0 r - \omega t))
\]

(21)

\[
H_\theta = \frac{\omega \cdot k_0 p_\varphi \cdot \exp(i(k_0 r - \omega t)) \sin \varphi}{4\pi ((\mu/\mu_1) \cos^2 \theta + \sin^2 \theta)},
\]

(22)

\[
H_\varphi = \frac{\omega \cdot k_0 p_\varphi \cdot \exp(i(k_0 r - \omega t)) \cos \theta \cdot \cos \varphi}{4\pi \cdot r}.
\]

(23)

The received asymptotic solutions (19)-(23) when using limit transition $\mu_1 \to \mu$ correspond with solutions of other authors for the isotropic medium. In fig.2 sections of the directional pattern of a dipole of the Hertz, which is located perpendicular to an axis are shown to uniaxial magnetic crystal in a far zone.

\[
\text{a) } r = 1, \ \phi = \pi / 2 \quad \text{b) } r = 4, \ \theta = \pi / 2 \quad \text{v) } r = 8, \ \theta = \pi / 2
\]
$- \mu_1 = 2, \quad \ldots \quad \mu_1 = 5, \quad \cdots \mu_1 = 8$

Fig.2. Sections of the directional pattern of a dipole of the Hertz, which is located perpendicular to an axis to a magnetic crystal

**Conclusion**

Thus, in this work asymptotic decisions from the new analytical solution of Maxwell’s equations for intensity of an electromagnetic field of a dot radiator of Hertz in a uniaxial magnetic crystal are received.

From drawing it is visible that directional pattern forms at various values of radius of the sphere changes, i.e. the more sphere radius, it is more than local petals. And also the magnetic anisotropy is higher the intensity of radiation of electromagnetic waves on an axis of a crystal of z is maximum. In the direction of the emitter (axis x) doesn't occur the radiation of electromagnetic waves.

**References:**