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The Use of STEM Approaches To Improve Formula Derivation Steps in Material Science and Engineering Programmes at Higher Education Institutions

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Abstract

Many complex formula derivation steps found within material science and engineering programmes are essential skill-developing activities that enhance students' learning. Most students lack the required mathematical knowledge to fully comprehend some of those derivation steps. This work involves the development of a framework for clarifying some of the original derivations by adding further mathematical steps that support students' constructive and cognitive learning. Modifications were made on the theoretical tensile strength derivations, the Maxwell's and the Voigt-Kelvin's models. The aim is to facilitate students' learning and help them acquire transferable skills when solving other material science and engineering problems. Students benefited from the activities in two folds; firstly, they understood the mathematical reasons behind each derivation step and secondly, their study periods were reduced. These activities provide a platform for widening STEM activities at higher education institutions. The ongoing work will look at other important formula derivation steps within material science and engineering.

Keywords: Mathematics, Engineering, Theoretical formula, Formula derivations, STEM

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Introduction

Most material science and engineering courses at higher education institutions involve complex formula derivation steps aimed to enhance students' analytical skills (Jaworski, 2008). Regardless of the instructional approach, good mathematical skills in engineering programs are very essential (Cardella, 2008; Ní Fhloinn et al., 2014). In engineering education, lecturers have been urged to formulate clear instructional materials that support students' cognitive and creative thinking, construct deep conceptual knowledge and improve their problem-solving abilities (Felder, et al., 2000; Litzinger et al., 2011; Daly et al., 2014). The main problem is the level of facilitation needed to guide students in achieving the desired learning objectives without influencing their self-learning efficacies. Mathematical thinking among engineering students involves the concepts, metacognitive processes, beliefs and effects during problem-solving practices (Schoenfeld, 1992). Schoenfeld (1992) also noted that problem solvers have limited cognitive resources during the metacognitive process and their working memory can hold approximately seven pieces of information at a time. Hence, the working memory of students must be managed for effective problem-solving processes.

The learning of mathematics is indispensable within the engineering community and the best `practical' mathematical approach is to understand it as a language for describing physical and chemical laws (Sazhin, 1998). This is because students learn physical concepts much easier than mathematical concepts. Thus, the understanding of engineering problems means the conversion of problems into physical and/or chemical problems that can be formulated in terms of mathematical equations.

Purposes of this study

This work seeks to motivate and encourage students' conceptual thinking by adding further mathematical derivation steps to original equations found in the literature. This will help deepen students' understanding and support them to acquire the necessary transferable mathematical problem-solving skills that can be applied in other engineering programmes.

The Helping Engineers Learn Mathematics (HELM) projects confirmed the decline in basic mathematics at the institutions and other subject review reports also noted the lack of basic mathematical skills among students (Davis et al., 2005). Many students will continue their education in engineering with mathematical difficulties since engineering institutions cannot always recruit students with higher qualifications in mathematics (Jaworski, 2008). It is therefore essential to link mathematics in Science, Engineering and Technology to promote the acquisition of transferable skills.

Widening Science, Technology, Engineering & Mathematics (STEM)

STEM education refers to the teaching and learning in the fields of science, technology, engineering, and mathematics at all educational levels (Gonzalez & Kuenzi, 2012). Currently, the focus of STEM activities has been shifted towards the lower education sectors ((Bybee, 2010; Corlu et al., 2014; Moore et al., 2014; Chiu et al., 2015; Khanlari & Mansourkiaie, 2015; English, 2016; Ros et al., 2016) but that must be spread across all higher education institutions as this will allow the exploration of fundamental principles in engineering and mathematics. STEM education must increase students' understanding of how things work in different technological processes and must improve their problem-solving and innovation mindset (Bybee, 2010).

At higher education, there have been difficulties in the efforts to introduce STEM activities because of several associated barriers such as the difficulties in providing mathematical education, the lack of students' diverse knowledge, as well as the lack of quantitative skills among engineering students (Booth, 2004; Czaplinski et al., 2019; Lee, et al., 2019). Booth (2004) suggested that mathematics and engineering educators must collaborate to provide mathematics instruction to engineers while Czaplinski and colleagues (2019) emphasized the need of assisting students in developing their diverse knowledge to enable them to construct transdisciplinary solutions for complex problems. Similar to Reid and Wilkes (2016), this work will support STEM approaches in higher education and provide platforms to enhance students' quantitative skills.

Teaching and students' learning

Feedforward teaching styles require lecturers to give illustrative instructions that guide the learner towards the desired results (Hershberger, 1990; Basso & Belardinelli, 2006). In those approaches, too many scaffolding practices might impact students' abilities negatively even though some students may improve their academic achievements (Murtagh and Webster, 2010). Moreover, the constructivism learning theory suggests knowledge to be acquired through student experiences (Bada and Olusegun, 2015) but cognitively, it is difficult for the brain to process information among students with weaker working memory (Cowan, 2014). For those students with weaker working memory, scaffolding and feedforward approaches might be the best strategy to enhance their learning and that may boost their self-efficacies (Van Dinther et al., 2011).

In all, students must actively engage and find solutions to problems during their learning without passively receiving it from the lecturer (Liu and Chen, 2010). To improve students' engagement, Green et al. (2017) noted that flipping the lecture by providing instructional information before lectures promotes deeper thinking and creativity among students and allow them to

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develop problem-solving skills. This work facilitates the students' learning by guiding them towards the required learning objectives without influencing their constructive engagements. This work guided the students by adding further additional derivation steps to enhance their learning.

Methodology

In this work, modifications were made to derivation steps found in modern literature. Further mathematical steps were added to the derivation of the theoretical tensile strength formula, Maxwell's and Voigt-Kelvin's models. These equations are essential models within the material science programme because they describe the strength and dynamic mechanical behaviours within materials. While the theoretical tensile strength gives insight into the classical crack strength estimation, the Maxwell and the Voigt-Kelvin models explain the deformation mechanisms of materials having both viscous and elastic properties. For clarity, all added steps were *relabelled* to avoid confusion. The modified derivations workouts were incorporated into two of the materials science and engineering units and were used in all lessons for two year period. Feedback from hundred & fifty (150) students was recorded after usage.

In the following sections, the original formula derivations (as appeared in the literature) were reported followed by the modifications written in **bold text**. Furthermore, extensive explanations were provided to improve students' understanding.

Theoretical tensile strength

Theoretical tensile strength from literature (Young & Lovell, 2011)

The expected dependence of stress σ upon the displacement x during cleavage is as shown below.

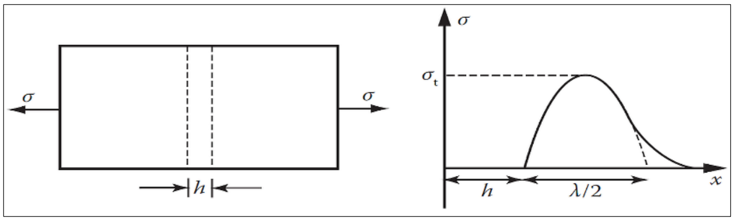

Figure 1. The model used to calculate the theoretical cleavage stress of a polymer crystal (Young & Lovell, 2011, p. 558)

It is envisaged that this can be approximated to a sine function of wavelength λ which takes the form;

$$\sigma = \sigma_T sin\left(\frac{2\pi x}{\lambda}\right) \tag{1}$$

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where σ_t is the maximum tensile stress, λ = wavelength, h the equilibrium separation of the atomic planes in the crystal, perpendicular to the tensile axis. This can be written for low strains as;

$$\sigma = \sigma_T \left(\frac{2\pi x}{\lambda}\right) \quad (2)$$

For perfectly elastic behaviour, the textbook quoted Hooke's law as

$$\sigma = \frac{Ex}{h} \quad (3)$$

Comparing the two equations results in the following equation.

$$\sigma = \frac{\lambda E}{2\pi h} \quad (4)$$

After assuming a perfectly elastic situation and adding some integration steps, the theoretical strength was given to be

$$\sigma_T = \sqrt{\frac{E\gamma}{h}} \quad (5)$$

This final equation is the known formula for the estimation of the tensile strength of polymer materials.

Feedback from students indicated that some integration steps within the derivations were missing and some students struggled to follow the steps..

Modification of theoretical tensile strength formula's derivation

From

$$\sigma_T = \frac{\lambda E}{2\pi h} \quad (6)$$

By rearranging, the surface energy per unit area (γ) is given by;

$$= \frac{2\pi h}{E} \sigma_T \quad (7)$$

A sinusoidal graph was drawn to explain the procedures involved and the γ applied to the area under the curve (***New***).

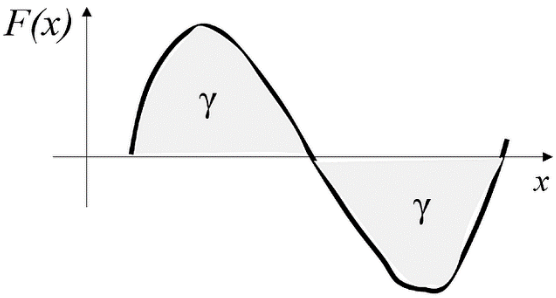

Figure 2. A schematic sinusoidal graph showing surface energy per unit area required during material cleavage

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The total energy required to create one new surface is given by

$$U_{total} = \gamma = \int_0^{\lambda/2} \sigma_T sin\frac{2\pi x}{\lambda} \quad (8)$$

The result from integration gives

$$\int_0^{\lambda/2} \sigma_T sin\frac{2\pi x}{\lambda} = \sigma_T \left[-\frac{\lambda}{2\pi} cos\frac{2\pi x}{\lambda}\right]_0^{\lambda/2} \quad (9)$$

When the definite integral limits are applied, it becomes

$$\sigma_T \left[-\frac{\lambda}{2\pi} cos\frac{2\pi}{\lambda}\frac{\lambda}{2}\right] - \sigma_T \left[-\frac{\lambda}{2\pi} cos\frac{2\pi(0)}{\lambda}\right] \quad (10)$$

Since cos(π) = -1 and cos(0) = 1, the equation becomes

$$\sigma_T \left[-\frac{\lambda}{2\pi} \times -1\right] - \sigma_T \left[-\frac{\lambda}{2\pi} \times 1\right] \quad (11)$$

$$\sigma_T \left[\frac{\lambda}{2\pi}\right] + \sigma_T \left[\frac{\lambda}{2\pi}\right] \quad (12)$$

Hence, the total energy required to create one new surface is given by

$$\gamma = 2\sigma_T \left[\frac{\lambda}{2\pi}\right] = \sigma_T \frac{\lambda}{\pi} \quad (13)$$

The total energy required to create ***two*** new surfaces as described in figure 2 is given by

$$2\gamma = \sigma_T \frac{\lambda}{\pi} \quad (14)$$

By rearranging

$$\lambda = \frac{2\gamma\pi}{\sigma_T} \quad (15)$$

Equating (7) and (15) gives;

$$\lambda = \frac{2\pi h}{E}\sigma_T = \frac{2\gamma\pi}{\sigma_T} \quad (16)$$

After rearranging and cancellations,

$$\sigma_T = \sqrt{\frac{E\gamma}{h}} \quad (17)$$

There was positive feedback from the students and they expressed their satisfaction of acquiring the necessary understanding. The added mathematical steps appeared to be basic differential equations but that motivated the students in applying and linking mathematics to engineering problems.

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Maxwell's model for describing stress relaxations of viscoelastic materials

Derivation of Maxwell's model from the literature (Young & Lovell, 2011, p. 489)

Below are the derivation steps for Maxwell's model (i.e. from literature) using a spring and dashpot in series, as displayed in figure 3. As mentioned earlier, Maxwell's model is used to describe the viscoelastic material's stress relaxation behaviour. Hence, while the spring represents the elastic part of the material, the dashpot is used for the viscous deformation part.

Figure 3. A schematic diagram of Maxwell's model built up by a spring and a dashpot in series (source: Young & Lovell, 2011, p. 489)

Since the elements are in series, the stress will be identical in each one and so

$$\sigma = \sigma_1 = \sigma_2 \quad (18)$$

For the spring and dashpot, respectively, Hook's law gives

$$\sigma = E\varepsilon_1 \quad (19)$$

Or

$$\frac{d\sigma}{dt} = E\frac{d\varepsilon_1}{dt} \quad (20)$$

Newton's law is used to describe the linear viscous behaviour through the equation

$$\frac{d\sigma}{dt} = \eta\frac{d\varepsilon_2}{dt} \quad (21)$$

Where η is the viscosity characteristic parameter for the description of the viscous medium, E is the elastic modulus and ε is the strain within the model.

The strain characterizing the total deformation is given by the sum

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad (22)$$

and the strain rate when differentiated with respect to time gives

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} \quad (24)$$

Inserting the derivations from equations (20) and (21) into (23) yields

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\eta}\sigma \quad (24)$$

Since

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$$\frac{d\varepsilon}{dt} = 0 \qquad (25)$$

Hence,

$$0 = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\eta}\sigma \qquad (26)$$

Rearranging result in the following equation.

$$\frac{1}{\sigma}d\sigma = -\frac{E}{\eta}\mathrm{dt} \qquad (27)$$

This can be readily integrated when $t = 0$ and $\sigma = \sigma_0$ to give

$$\sigma = \sigma_0\, exp\left(-\frac{E}{\eta}t\right) \qquad (28)$$

Looking at the above derivation steps, it was evident that some mathematical integration steps were not displayed before reaching the final equation and this affected the learning of the weaker students. It is normally believed that this teaching style of eliminating some mathematical steps promote constructivist learning among students and the students might develop their own knowledge through research and self-studies (Bada and Olusegun, 2015). However, some of the students failed to find the required connection to the final answer and that disrupts their problem-solving and transferable skills.

Modification of Maxwell's model's formula derivation

Using the same model described above, some modifications were made as follows.

From Hook's law,

$$\sigma_1 = E\varepsilon_1 \qquad (29)$$

When there is a change in the stress and strain with time, it becomes

$$\frac{d\sigma_1}{dt} = E\frac{d\varepsilon_1}{dt} \qquad (30)$$

Rearranging of the above equation becomes

$$\frac{d\varepsilon_1}{dt} = \frac{1}{E}\frac{d\sigma_1}{dt} \qquad (31)$$

The same stress σ for viscous deformation is expressed as

$$\sigma = \eta\frac{d\varepsilon_2}{dt} \qquad (32)$$

Also, rearranging the above equation becomes

$$\frac{d\varepsilon_2}{dt} = \frac{1}{\eta}\sigma \qquad (33)$$

Where η is the viscosity characteristic parameter for the description of the viscous medium, E is the elastic modulus and ε is the strain within the model.

In an equilibrium of forces and assuming constant area, the stress becomes constant (i.e. $\sigma = \sigma_1 = \sigma_2$) but strain varies.

The strain (ε) characterizing the total deformation is given by the sum

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{34}$$

and the total strain rate *with respect to* time yields

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} \tag{35}$$

Inserting the derivations from equations (31) and (33) into (35) yields

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\eta}\sigma \tag{36}$$

But for **stress relaxation** (i.e. the reason for Maxwell's model), the strain is held constant ($\varepsilon = \varepsilon_0$). This means

$$\frac{d\varepsilon}{dt} = 0 \tag{37}$$

Inserting this into equation (36) becomes

$$0 = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\eta}\sigma \tag{38}$$

And rearranging the above equation becomes

$$\frac{d\sigma}{\sigma} = -\frac{E}{\eta}dt \tag{39}$$

Using differential equations, the above equation can be written as

$$\int_{\sigma_0}^{\sigma}\frac{d\sigma}{\sigma} = \int_0^t -\frac{E}{\eta}dt \tag{40}$$

Applying integration principles to the above equation and solving the differential equation becomes

$$\ln(\sigma) - \ln(\sigma_0) = -\frac{E}{\eta}(t - 0) \tag{41}$$

And the above equation can be rearranged as

$$\ln\left(\frac{\sigma}{\sigma_0}\right) = -\frac{E}{\eta}(t) \tag{42}$$

Using Logarithms laws, it can be expressed as

$$\frac{\sigma}{\sigma_0} = \exp\left(-\frac{E}{\eta}t\right) \tag{43}$$

The final answer can then be expressed as

$$\sigma = \sigma_0\, exp\left(-\frac{E}{\eta}t\right) \tag{44}$$

After the above modifications, most students who had previous difficulties overcame their barriers and assimilated the whole derivation steps. Students were also able to explain the final mathematical equation graphically as shown in figure 4. In figure 4, it is evident that the stress relaxes exponentially with time while the strain remains constant.

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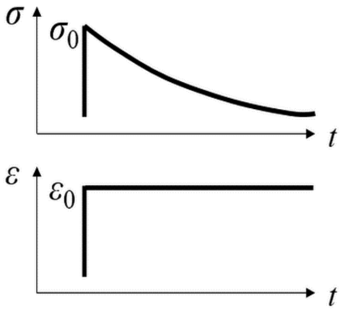

Figure 4. A schematic diagram showing how the equation derived from the Maxwell model predicts an exponential decay of stress expected from a viscoelastic polymer material

Further investigations were also made to answer the question of the reasons why Maxwell's model, designed normally for the description of stress relaxation behaviour, cannot be employed in explaining the strain retardation behaviour of the materials. Additional mathematical manipulations were made during the investigations below.

Why Maxwell's model cannot predict the strain retardation behaviour of viscoelastic materials

It is worth mentioning that the strain retardation or creeping behaviour of viscoelastic material occurs when the strain within the material increases exponentially with time (Arrospide et al., J. 2017). Meaning, the equation fails if the strain increases linearly.

From the previous derivations, the following manipulations can be made.

For creeping to occur, the stress must be held constant ($\sigma = \sigma_0$). This means

$$\frac{d\sigma}{dt} = 0 \tag{45}$$

Using the equation below;

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{1}{\eta}\sigma \tag{46}$$

If the stress is held constant and dσ/dt = 0, the equation will become

$$\frac{d\varepsilon}{dt} = \frac{1}{\eta}\sigma_0 \tag{47}$$

Using differential equations on the above equation becomes

$$\int_{\varepsilon_0}^{\varepsilon} d\varepsilon = \int_0^t \frac{\sigma_0}{\eta} dt \tag{48}$$

Solving the equation above gives

$$\varepsilon - \varepsilon_0 = \frac{\sigma_0}{\eta}(t - 0) \tag{49}$$

And the above equation can be rearranged as

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$$\boldsymbol{\varepsilon = \frac{\sigma_0}{\eta} t + \varepsilon_0} \tag{50}$$

but since

$$\boldsymbol{\varepsilon_0 = \frac{\sigma_0}{E}} \tag{51}$$

The final answer can then be expressed as

$$\boldsymbol{\varepsilon = \frac{\sigma_0}{\eta} t + \frac{\sigma_0}{E}} \tag{52}$$

The above equation predicts the strain to increase linearly with time (i.e. not exponential) and this is as illustrated in figure 5 below. This equation predicts Newtonian flow and not viscoelastic material behaviour. Hence, Maxwell's model cannot predict the strain retardation behaviour of viscoelastic materials.

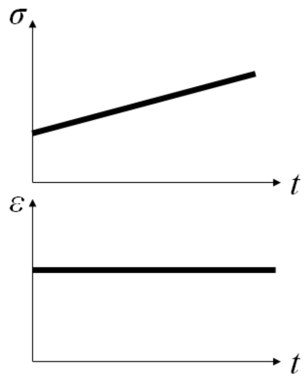

Figure 5. A schematic diagram showing why Maxwell's model cannot be used to describe the exponential strain increment

Voigt-Kelvin model for retardation behaviour

Derivation of Voigt-Kelvin's model from the literature (Young & Lovell, 2011)

From the literature, the Voigt-Kelvin model was derived using a parallel arrangement of spring, that represents the elastic part of viscoelastic material, and dashpot for the viscous part (fig. 6). The Voigt-Kelvin model is used for the description of the material's retardation behaviour. The following derivation steps were obtained from the literature (Young & Lovell, 2011).

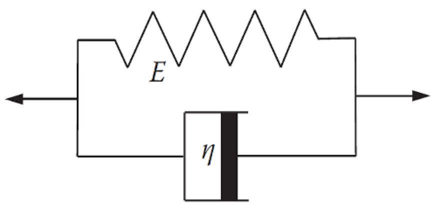

Figure 6. Voigt mechanical model used to represent the viscoelastic behaviour of polymers (Young & Lovell, 2011, p. 489)

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The parallel arrangement of the spring and the dashpot means that the strains are uniform, i.e.

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \tag{53}$$

And the stress in each component will add up to

$$\sigma = \sigma_1 + \sigma_2 \tag{54}$$

But the stress for the spring and the viscous deformation are expressed as

$$\sigma_1 = E\varepsilon \tag{55}$$

and

$$\sigma_2 = \eta \frac{d\varepsilon}{dt} \tag{56}$$

Inserting equations (55) and (56) into (54) becomes

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt} \tag{57}$$

Rearranging the above equation becomes

$$\frac{\sigma}{\eta} = \frac{E\varepsilon}{\eta} + \frac{d\varepsilon}{dt} \tag{58}$$

And further rearranging becomes

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} - \frac{E\varepsilon}{\eta} \tag{59}$$

The Voigt or Kelvin model is particularly useful in describing the behaviour during creep where the stress is held constant at $\sigma = \sigma_0$. Hence,

$$\frac{d\varepsilon}{dt} = \frac{\sigma_0}{\eta} - \frac{E\varepsilon}{\eta} \tag{60}$$

This simple differential equation has the solution

$$\varepsilon = \frac{\sigma_0}{E}\left[1 - exp\left(-\frac{Et}{\eta}\right)\right] \tag{61}$$

The constant ratio η/E can again be replaced by τ_0 which is the relaxation time and so the variation of strain with time for a Voigt model undergoing creep loading is given by

$$\varepsilon = \frac{\sigma_0}{E}\left[1 - exp\left(\frac{-t}{\tau_0}\right)\right] \tag{62}$$

Similarly, the weaker students had difficulties in linking the final equation to the derivation steps. Additional derivation steps were added without affecting the final results in the section below.

Modification of Voigt-Kelvin model's formula derivation

Using the same model illustrated in figure 6, the following additional mathematical steps were added.

In an equilibrium of forces, the strain remains constant. This means,

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \tag{63}$$

The applied load is supported by the spring and the dashpot and can be written as

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$$\sigma = \sigma_1 + \sigma_2 \tag{64}$$

The stresses for the spring and viscous deformation are expressed as

$$\sigma_1 = E\varepsilon \tag{65}$$

and

$$\sigma_2 = \eta \frac{d\varepsilon}{dt} \tag{66}$$

Inserting equations (65) and (66) into (64) becomes

$$\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt} \tag{67}$$

Rearranging the above equation becomes,

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} - \frac{E\varepsilon}{\eta} \tag{68}$$

And can be written as,

$$\frac{d\varepsilon}{dt} = \frac{\sigma - E\varepsilon}{\eta} \tag{69}$$

Further rearrangement of the above equation becomes,

$$\frac{\eta \, d\varepsilon}{\sigma - E\varepsilon} = dt \tag{70}$$

Applying differential equations gives

$$\boldsymbol{\eta \int_0^{\varepsilon} \frac{1}{\sigma - E\varepsilon} d\varepsilon = \int dt} \tag{71}$$

The solution after integrating the above equation gives

$$\boldsymbol{\eta \left[-\frac{1}{E} \ln(\sigma - E\varepsilon) \right] = t} \tag{72}$$

Rearranging and applying the limits gives

$$\boldsymbol{-\frac{\eta}{E} [\ln(\sigma - E\varepsilon)]_0^{\varepsilon} = t} \tag{73}$$

Using the limits and solving the above equation gives

$$\boldsymbol{-\frac{\eta}{E} [\ln(\sigma - E\varepsilon) - \ln \sigma] = t} \tag{74}$$

Rearranging the above equation becomes

$$\boldsymbol{-\frac{\eta}{E} \left[\ln\left(\frac{\sigma - E\varepsilon}{\sigma}\right) \right] = t} \tag{75}$$

Further rearranging becomes

$$\boldsymbol{\ln\left(\frac{\sigma - E\varepsilon}{\sigma}\right) = -\frac{Et}{\eta}} \tag{76}$$

Using Laws of Logarithms, the above equation can then be expressed as

$$\boldsymbol{\frac{\sigma - E\varepsilon}{\sigma} = e^{-\frac{Et}{\eta}}} \tag{77}$$

Rearranging the above equation becomes

$$\boldsymbol{1 - \frac{E\varepsilon}{\sigma} = e^{-\frac{Et}{\eta}}} \tag{78}$$

Also, the above equation can be written as

$$\boldsymbol{1 - e^{-\frac{Et}{\eta}} = \frac{E\varepsilon}{\sigma}} \tag{79}$$

And further rearrangement becomes

$$\boldsymbol{\frac{\sigma}{E} \left[1 - e^{-\frac{Et}{\eta}} \right] = \varepsilon} \tag{80}$$

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Furthermore, since the stress is constant (σ_0), the final original equation at any time (t) is obtained as

$$\varepsilon(t) = \frac{\sigma_0}{E}\left[1 - e^{-\frac{Et}{\eta}}\right] \tag{81}$$

The above added mathematical steps made it possible for all students to follow the derivation steps and relate that to the final equation. The equation predicts how the strain increases exponentially with time (fig. 7) and students could explain the equation graphically. The main point was to allow the students to make connections between mathematical knowledge and material science and engineering concepts.

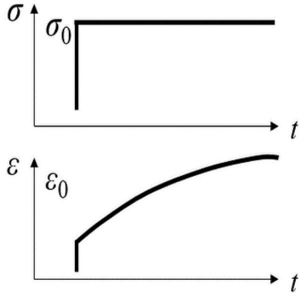

Figure 7. A schematic diagram showing how the equation obtained from the Voigt-Kelvin model predicts an exponential increment in the strain that is expected from a viscoelastic polymer material

Similar to the activities above, investigations were also made on why the Voigt-Kelvin model cannot be used to describe the stress relaxation behaviour of viscoelastic materials.

Why Voigt-Kelvin model cannot predict the stress relaxation behaviour of viscoelastic materials

As mentioned earlier, the equation must demonstrate stress that decays exponentially with time during stress-relaxation processes and that is the basis for this investigation. Using the same model shown in figure 6, the following mathematical manipulations were made.

From the equation,

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} - \frac{E\varepsilon}{\eta} \tag{82}$$

In a stress relaxation situation, the strain is held constant. It means

$$\boldsymbol{\varepsilon = \varepsilon_0} \tag{83}$$

and

$$\boldsymbol{\frac{d\varepsilon}{dt} = 0} \tag{84}$$

Inserting the above two equations (i.e. 83 and 84) into (82) becomes

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$$\mathbf{0} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\eta}} - \frac{\boldsymbol{E\varepsilon_0}}{\boldsymbol{\eta}} \quad (85)$$

Rearranging the equation gives

$$\boldsymbol{\sigma} = \boldsymbol{E\varepsilon_0} \quad (86)$$

Mathematically, the above equation predicts a linear elastic response such that the stress must be constant with time and this is as illustrated in figure 8. Hence, the parallel arrangments of the spring and the dashpot (i.e. Voig-Kelvin model) is not appropriate for the description of stress relaxation processes.

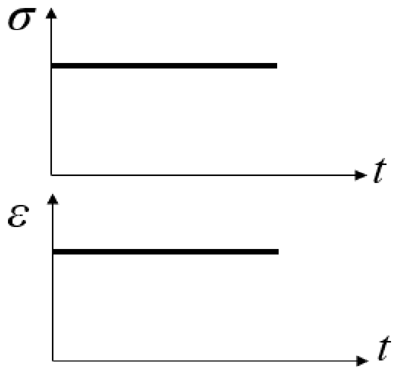

Figure 8. A schematic diagram showing why the Voigt-Kelvin model cannot be used to describe the stress relaxation behaviour of viscoelastic materials

Effects of the derivation modifications on students' learning

The essential characteristics of an effective engineering lecturer are the concern for students, stimulation of interest, and the ability to explain clearly irrespective of the teaching style (Feldman, 1976) but a good engineering lecturer must have the following characteristics (Davies et al., 2006);

- Enthusiastic,
- Gives clear, well-structured presentations,
- Uses real-world engineering examples backed up by industrial experience,
- Has a genuine interest in students as individuals and as members of an audience (i.e. friendly, approachable, patient and responds to feedback),
- encourages learning,
- Has depth of knowledge and command of the material,
- Uses visual material and demonstrate effectively,
- Gives good handouts,
- Makes classes enjoyable,
- Good at simplifying difficult concepts, and
- Well organised and reliable.

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Given the characteristics shown above, the modified equations supported most of the students who had difficulties in mathematics and stimulated their interest in the derivation activities. They assimilated the derivation steps with ease and that improved their learning. This approach can be linked to the feedforward teaching and learning approach (Hershberger, 1990; Basso & Belardinelli, 2006) and promotes STEM activities among material engineering departments.

The students were actively engaged and they could learn by themselves without relying too much on peer support. The additional mathematical steps did not affect students with strong mathematical backgrounds but instead fostered the development of connections between mathematical and engineering problems. Students with strong mathematical backgrounds had enough time to apply the information obtained in solving other complex problems in line with the constructivism learning theory (Bada and Olusegun, 2015). It means material science and engineering students can work on any complex problems with ease when lectures are flipped (Green et al., 2017).

Information gathered from the students after the two years are as follows;

"*The added mathematical steps enhanced my understanding*"

"*I struggled initially to make meaning of the derivation steps. The modified steps made it easier for me to understand*"

"*My learning hours for this unit was reduced throughout the semester because of the added mathematical steps*"

"*It was very useful to see how to apply mathematical knowledge in the Material Science program*".

The students' feedback can be summarized as follows;

- **Understanding**: Most of the students revealed that the added derivation steps helped clarify their misconceptions and made initially perceived complex derivation steps becoming less difficult (fig. 9*a*). Figure 9a is a schematic diagram that illustrates students' improved understanding after the modification. The concave-shaped curve depicts the areas where students benefited enormously using the modified derivations.
- **Study periods**: Students noted that adding additional mathematical steps to the original steps reduced their learning hours and this is as depicted in figure 9*b*. The added steps influenced their learning hours without affecting their learning styles.

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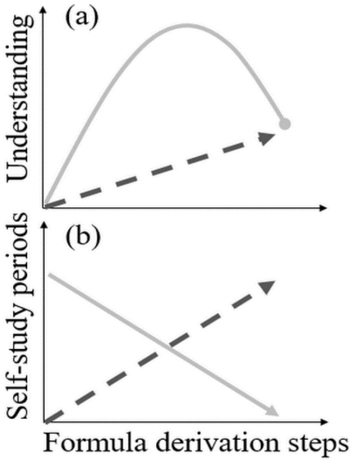

Figure 9. Schematic diagrams showing how students' (*a*) understanding and (*b*) learning hours were shaped by the added derivation steps. Broken deep lines indicate the original formula derivation step while grey lines represent scenarios after adding the derivation steps

Conclusion

The approach of adding further mathematical principles to some original derivation steps found in material science and engineering literature improve students' learning without influencing their learning outcomes. This is very important because it is impossible to recruit only students who have strong mathematical backgrounds for material science and engineering programmes and such activities are excellent ways to improve the mathematical knowledge of weaker students when admitted. From my experience, many weak students in mathematics tend to abandon material science and engineering programmes when they face complex mathematical problems.

Blending mathematical principles and engineering concepts provides a platform to develop STEM activities among engineering students and goes a long way to widen the STEM approach at higher education institutions. This strategy must be followed by other engineering education institutions. The general strategies when using modifying formula derivation steps by adding mathematical principles are;

1. Reading through the derivation steps and specifying the required mathematical principles.
2. Using external mathematical literature to support missing derivation steps.
3. Working out the original derivation steps from the literature to ascertain whether the individual steps are complete.
4. Documenting the benefits of adding further necessary mathematical steps.

5. Transfering acquired knowledge in any material science or engineering work.

The ongoing work will look at other important formula derivation steps within material science and engineering to promote deep students' learning.

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