THE EFFECTIVENESS OF THE RADON TRANSFORM AGAINST THE QUANTIZATION NOISE

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Abstract

The aim of image compression consists to reduce the number of bits required to represent an image. The Radon transform has become a very interesting tool in the field of image processing. Its Robustness against noises such as white noise has boosted the researchers to realize methods of detection of objects in noisy images. The Discrete Cosine Transform has shown its efficacy in the energy compaction of the image to be compressed into a smaller number of coefficients. It is part of many international standards including JPEG and MPEG. In this paper, we present an image compression method, which is the modification of the scheme presented by Predeep and Manavalan. The modification consists to use a high scale quantization, which is 20 in order to realize a heavy quantization for the DCT to achieve a high compression. A comparative study is performed to show the contribution of this modification.

Keywords: Image Compression; Discrete Cosine Transform (DCT); Radon Transform; Integration Process; Peak Signal to Noise Ratio (PSNR); JPEG2000.

1. Introduction

The image compression consists to reduce the amount of data needed to represent an image. The exploitation of the spatial and spectral redundancy and weaknesses of psychovisual system helped to develop methods that lead to the reduction of the amount of data required to represent an image. This leads to eliminate any problem of transmission or archiving.

We distinguish two main classes of compression: lossless compression and lossy compression. Lossless compression allows reconstruction after decompression, an identical image to the original but with a low compression ratio. Among the most widely used coders, we distinguish Huffman coding (Huffman 1952), the arithmetic coding, Golomb-Rice coding (Rice & Plaunt 1971), (Bacha & Jinaga 2013), the Tunstall coding (Tunstall 1967) and the RLC coding (Run Length Coding) (Devaki & Raghavendra 2012). We also distinguish the predictive coding like the Linear Predictive Coding (LPC) (Devi & Mini 2012). These encoders are distinguished from the lossy compression, which introduces an irreversible degradation of the original image but allows much greater compression than that obtained by lossless compression methods. These methods are generally based on the quantization of blocks of coefficients resulting from a transform. Among of these transforms : The Karhunen-Loeve Transform (KLT), the DCT Transform (Ahmed, Natarajan & Rao 1974), the Discrete Sine Transform (DST), the Radon transform (Radon 1917), (Pradeep & Manavalan 2013), the fractal Transform (Lahdir, Ameur & Adane 2007), the Walsh-Hadamard Transform (DWHT), the Discrete Wavelet Transform (DWT) which forms the basis of various techniques like EZW algorithm (Shapiro 1993), the SPIHT algorithm (Said & Pearlman 1996), JPEG and JPEG2000 standard (Charrier, Santa Cruz & Larsson 1999), (Medouakh & Baarir 2011), the Mojette Transform (Guédon, Barba & Burger 1995). We also distinguish the lossy predictive coding such as the Differential Pulse Code Modulation (DPCM) (Taygi & Sharma 2012).

We show in this paper the power of the Radon transform by means of the of image compression method based on the Radon transform and a high quantization of the DCT, which is a modification of the method developed by Pradeep & Manavalan. A comparative study with several methods is performed to show the effectiveness of this compression scheme.

The Radon transform has become a very important tool that has attracted many researchers in the field of image compression seen its robustness against different noises such as white noise and the quantization noise compared to other types of transformations as the Fourier transform and wavelet transform (Murphy 1986). It is particularly used for detecting objects in noisy images like in the case of ship wake detection in radar images (Courmontagne 2005). Ahmed et al. have created the DCT in 1974. The ability of the DCT compression is very close to that of the KLT. It is also nearly equal to the KLT in its ability of energy compaction. Unlike KLT, the DCT is independent of the image in question, that is to say, the core of its matrix is set at a given size. Therefore, any information about the blocks size at the receiver side is required for the reconstruction of the original image.

The rest of this paper is organized as follows: Section 2 is devoted to the theory of the image compression method used in this paper. Section 3 gives the different results of the evaluation criteria used in the comparison and their discussions. Finally, Section 4 summarizes the entire paper with a conclusion.

2. Image compression method based on the Radon transform and high quantization of DCT

This method is a modification of the method developed by Pradeep & Manavalan. This approach exploits the Radon transform and DCT with high scale of quantization, which is 20 for image compression. This method begins with the application of the Radon transform to the original image to obtain Radon points. The next step consists to encode the Radon points using the DCT. These steps in reverse order allow to reconstruct the original image.

The Radon transform is used to represent an image in the Radon field as a collection of projections along different directions for each given angle. In other words, the Radon transform maps a line to a point in the Radon domain; a particular line to a particular point. One of the most important properties of the Radon transform is that it is reversible hence the possibility of reconstructing the image from the knowledge of its integrations along hyperplanes of its space.

The Radon transform is applied to an input image $\boldsymbol{I}$ in the range $\theta \in [0, 2\pi]$ to collect all of its projection along a specific direction in the $\boldsymbol{x}$ and $\boldsymbol{y}$ axis domain. All projections of the image $\boldsymbol{I}$ is given by equation 1.

$$\hat{I}_{\theta}(p) = \int_{-\infty}^{+\infty} I(\vec{r})\,\delta(p - x\cos\theta - y\sin\theta)\,d\vec{r} \qquad (1)$$

With:

$\boldsymbol{I}$: Input image

$\vec{r}$: Vector of position of the components $(x_1, x_2, ..., x_n)$.

$p = \vec{\rho}\,\vec{r}$: A hyperplane, with $\vec{\rho}$ is a unit vector.

δ : Dirac function.

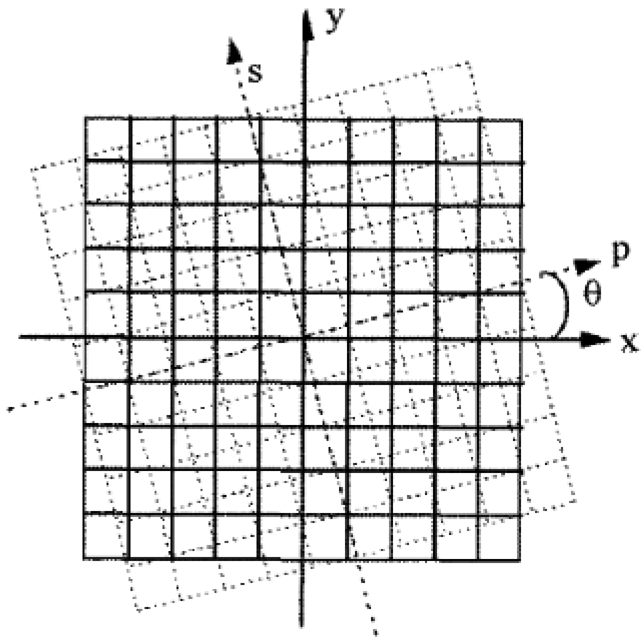

Figure 1: Relation between image field and Radon field.

The passage between the image field and the Radon field, as shown in Fig. 1 is provided by the following transition matrix:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p \\ s \end{pmatrix}$$

The graphical representation of the Radon transform gives a sinogram. Fig. 2 gives an example of the representation of the Radon transform of the Lena image.

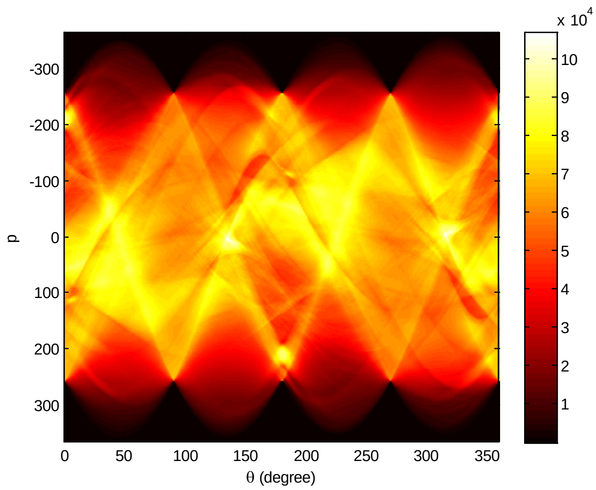

Figure 2: Sinogram of Lena image.

The Discrete Cosine Transform (DCT) takes its name from the fact that the rows of the matrix of the transform of size ***N* × *N*** are obtained by function of cosine as shown in the equation 2 (Sayood 2012):

$$C(i,j)=\begin{cases}\sqrt{\dfrac{1}{N}} & i=0, j=0,1,....,N-1\\ \sqrt{\dfrac{2}{N}}\cos\dfrac{(2j+1)i\pi}{2N} & i=0,1,....,N, j=0,1,....N-1\end{cases} \qquad (2)$$

With:

$\boldsymbol{C(i,j)}$: The DCT transform coefficient at the position ***(i, j).***

The quantization matrix that we use in our case is represented as follows:

$$\boldsymbol{Q(i,j)}=\begin{bmatrix} 320 & 220 & 200 & 320 & 480 & 800 & 1020 & 1220\\ 240 & 240 & 280 & 380 & 520 & 1160 & 1200 & 1100\\ 280 & 260 & 320 & 480 & 800 & 1140 & 1380 & 1120\\ 280 & 340 & 240 & 580 & 1020 & 1740 & 1600 & 1240\\ 360 & 440 & 740 & 1120 & 1360 & 2180 & 2060 & 1540\\ 480 & 700 & 1100 & 1280 & 1620 & 2080 & 2260 & 1840\\ 980 & 1280 & 1560 & 1740 & 2060 & 2420 & 2400 & 2020\\ 1440 & 1840 & 1900 & 1960 & 2240 & 2000 & 2060 & 1980 \end{bmatrix}$$

This matrix $\boldsymbol{S(i,j)}$ is obtained by multiplying the standard matrix used in the JPEG standard by 20. This matrix provides a quality level of 50 and renders both high compression and excellent decompression image quality. The standard quantization matrix is represented as follows:

$$\boldsymbol{S(i,j)}=\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61\\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55\\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56\\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62\\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77\\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92\\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101\\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

A coefficient is quantized by the following equation:

$$D(i, j) = \text{Round}\left(\frac{C(i,j)}{Q(i,j)}\right) \qquad (3)$$

With:

$\boldsymbol{C(i,j)}$: Is the DCT transform coefficient.

$\boldsymbol{Q(i,j)}$: The corresponding element in the quantization matrix.

The DCT coefficients are restored with the quantization error by the following equation:

$$C(i, j) = D(i, j) \times Q(i, j) \quad (4)$$

One of the drawbacks of image compression using the DCT is the pixilation effect. When a high quantization is applied, the block division becomes visible because an entire block is encoded with the same value (few non-zero coefficients to represent). We will use this compression technique in our comparison with a high level of quantization to quantify the difference between the two methods.

3. Results and discussions

The proposed method was applied to the standard 512×512 with 8 bpp, test Lena image represented in Fig. 3 (a). The reconstructed Lena image is represented in Fig 3 (b). The result was also compared with those obtained with the most common compression methods referred in this paper: DCT method with the same quantization scale, which is 20, Wavelet and fractals without iteration, the EZW and the JPEG2000 standard.

(a) (b)

Figure 3: Performance of the proposed method operating on Lena image.
(a) Original Lena image; (b) Reconstructed Lena image with ***PSNR*** = 34.07 dB/ ***Tc*** = 57.90 %.

We note according to Fig. 3 (b) that visually the reconstructed Lena image is of good quality. Therefore, it is accompanied by very minimal artifacts.

We used the ***PSNR*** and the compression ratio to evaluate the methods and quantify the difference between them. ***PSNR*** measures the distortion introduced by the compression operation. The ***PSNR*** is given by the following equation.

$$PSNR(I_0, I_c) = 20 \log_{10}\left(\frac{2^r - 1}{\sqrt{MSE(I_0, I_c)}}\right) \quad (5)$$

Where I_0 and I_c represents respectively the original image and the reconstructed image of size $M \times N$. and r represents the numerical resolution of the image. $r = 8$ in our case.

MSE is the Mean Square Error defines as:

$$MSE = \frac{1}{M\,N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(I_0(i,j) - I_c(i,j)\right)^2 \qquad (6)$$

The compression ratio allows evaluating the efficacy and potency of the image compression method. The compression ratio is given by the following equation.

$$CR = \frac{\text{Number of bits in the original image}}{\text{Number of bits in the compressed image}} \qquad (7)$$

The compression ratio in percentage is given by the following equation.

$$Tc = \left(1 - \frac{1}{CR}\right) \times 100 \qquad (8)$$

The different results of the evaluation criteria are listed in the Table I and represented as graphs in the Fig. 4 and 5.

TABLE I. PSNR AND COMPRESSION RATIO RESULTS

Method	**PSNR(dB)**	**Tc (%)**	**Reference**
The proposed method	34.07	57.90	-
DCT	24.02	98.78	-
Wavelet + Fractals	32.90	93.75	(Lahdir et al. 2007)
EZW	33.17	96.87	(Shapiro 1993)
JPEG2000	30.74	97.25	(Medouakh & Baarir 2011)

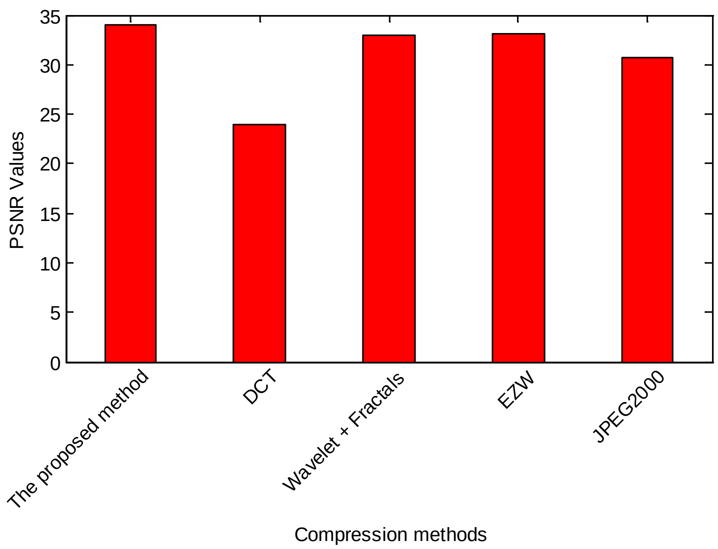

Figure 4: PSNR values vs. compression methods of Lena image.

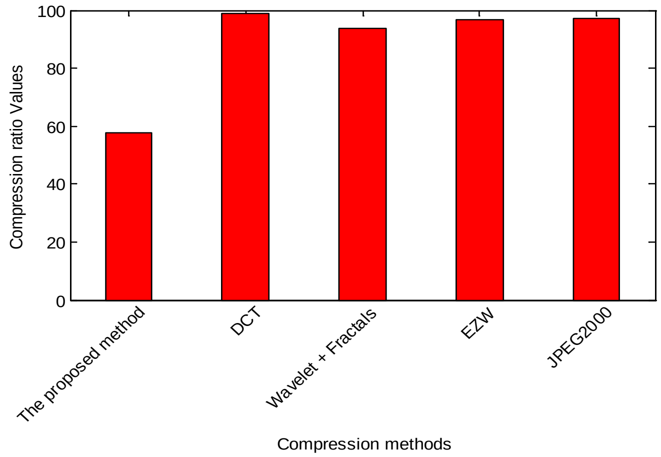

Figure 5: Compression ratio values vs. compression methods of Lena image.

We note from the Fig. 4 and 5 that the proposed method provides a compression ratio of 57.90 %, which is the lowest value. This value is less than 39.35 % of that of the JPEG2000 standard, 35.57 % of the wavelet with fractals method. This lower value is justified because the proposed method provides good images quality after their decompression as we will find out.

We also note from the Fig. 4 and 5 that the proposed method allows reconstruction of the Lena image with a higher ***PSNR*** than that obtained by the other methods. It is of 33.43 dB, higher than of the JPEG2000 standard, which is 30.74 dB and wavelet with fractals method, which is 32.90 dB. We also observe for the same quantization scale with the proposed method, which is 20 allows to achieve a ***PSNR*** of 24.02 dB for the DCT method. This difference shows the power of the Radon transform, which is the attenuation of the quantization noise by the integration process as shown in Fig. 6 and 7.

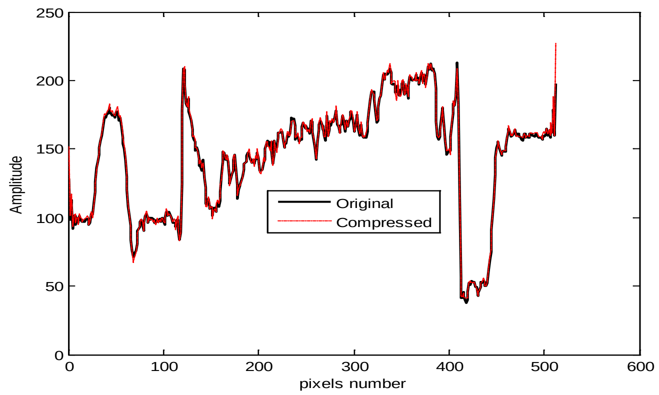

Figure 6: Intensity profile along line 164 of the Lena image reconstructed by the proposed method.

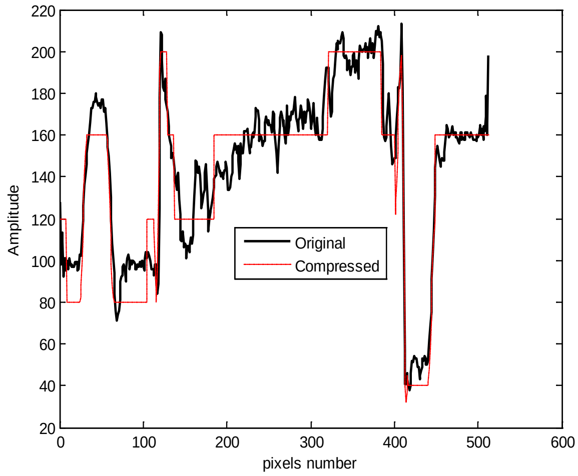

Figure 7: Intensity profile along line 164 of the Lena image reconstructed by the DCT method.

We observe from the Fig. 6 that the intensity profile of the original Lena image is almost the same with the reconstructed in the case of the proposed method. In the other hand, in the case of Fig. 7 the intensity fluctuations are remarkable, which justifies the pixilation effect caused by the DCT method.

Murphy showed that the Radon transform attenuates the intensity fluctuations by the integration process. This process comes from the fact that the inverse Radon transform is obtained using Filter-BackProjection algorithm (FBP). The FBP allows to filter each Radon point by the Ram-Lak filter (Ramachandran & Lakshminarayanan 1971). The filtered projection $\tilde{\hat{I}}_\theta(p)$ is given by the following integral:

$$\tilde{\hat{I}}_\theta(p) = \int_{-\infty}^{+\infty} F_\theta(v)|v| e^{i2\pi vp} dv \tag{9}$$

With $F_\theta(v)$ is the 1-D Fourier transform of the projection $\hat{I}_\theta(p)$.

The steps of the FPB are summarized as follows:

1. For each projection angle θ , calculate 1-D Fourier transform of the projection $\hat{I}_\theta(p)$ to obtain $F_\theta(v)$.
2. Multiply $F_\theta(v)$ by $|v|$ (Ram-Lak filter).
3. For each θ , calculate the 1-D inverse Fourier transform of $F_\theta(p)|v|$ to obtain the filtered projection $\tilde{\hat{I}}_\theta(p)$ by applying the Eq. 9.
4. Backproject the filtered sinogram $\tilde{\hat{I}}_\theta(p)$ to retrieve the image.

Ram-Lak filter is defined as:

$$H(\nu) = \begin{cases} |\nu| & \text{If } -\nu_{max} \leq \nu \leq \nu_{max} \\ 0 & \text{Otherwise} \end{cases} \quad (10)$$

The Ram-Lak filter is shown in Fig. 8.

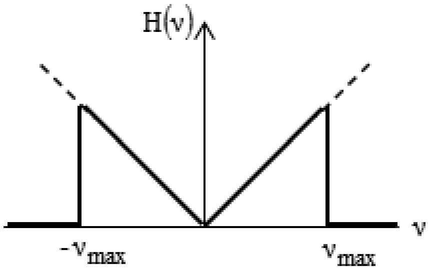

Figure 8: Ram-Lak filter.

The various excellent results obtained clearly shows that the proposed image compression method, which is the modification of the method proposed by Pradeep & Manavalan ensures an excellent rate-distortion compromise: high compression ratio while preserving the quality of the Lena image after its decompression with minimal traces of artifacts.

4. Conclusion

In this paper, an image compression method based on Radon transform and DCT with high quantization has been presented that provides excellent results. This method is a modification of the scheme of the method proposed by Pradeep & Manavalan. Furthermore, the performance of this method is competitive with many method like JPEG2000, EZW, DCT and Wavelet with fractals method. The application of the proposed method on Lena image provides an excellent rate-distorsion compromise: a high compression ratio, which is 57.90 % whith an excellent ***PSNR***, which is 33.43 dB. The results of the comparative study qualify the proposed method to be a good choice for archiving and transmission.

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