INTERVAL DATA ENVELOPMENT ANALYSIS AND AN APPLICATION IN TURKISH BANKING SECTOR

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Abstract

Interval Data Envelopment Analysis (Interval DEA) which is the approach that could make efficiency measurement with interval or fuzzy input and output data of Data Envelopment Analysis (DEA) which is used to measure the relative efficiencies of Decision-Making Units (DMU) in multi-input and multi-output processes was examined in this study. This model which was based on arithmetic operations with interval numbers, provides, unlike other DEA models, the relative efficiency measurements of DMU with inputs and outputs which had interval data, by using fixed and combined production limit without the need of extra variable or scale transformation. Interval efficiency of 21 commercial banks in Turkish banking sector in 2011 was calculated to show how related method was implemented in this study.

Keywords: Data Envelopment Analysis, Interval Data Envelopment Analysis, Interval efficiency, Fuzzy data, Turkish banking sector

Introduction

DEA of efficiency analysis methods, is quite successful method for measuring the relative effect of DMU whose input and output values completely and correctly known in multi-input and multi-output processes. But, DEA may give wrong results due to the data of complex input and output variables of real world problems. Many studies was done for more certain results of DEA in such circumstances by using fuzzy theory.

Fuzzy DEA was developed in order to measure efficiency in the condition of the disuse of classical DEA models in unknown data. Interval DEA of Fuzzy DEA that the lower and upper bounds of data were known, was examined in this study.

Data Envelopment Analysis

DEA, is non-parametric method based on linear programming which aimed to measure the relative efficiency of DMU in such conditions that multi input and multi output variables, which were measured by various measures, had complicated the comparison.

DEA is first based on the study of Farrell in 1957 (Farrell, 1957). Although Farrell used multi input and one output in its study, its linear equation system of efficiency measurement was an infrastructure for measurement of efficiency in conditions with multi input and multi output.

Charnes, Cooper and Rhodes (CCR) presented the first model of DEA named as CCR model which was non-parametric and based on linear programming in 1978. This model which was known as fractional model today is as follows (Charnes *et al.* 1978).

$$E_k = \max\left(\sum_{r=1}^{p} u_r Y_{rk}\right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ik}\right) \qquad (1)$$

$$\left(\sum_{r=1}^{p} u_r Y_{rj}\right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ij}\right) \leq 1$$

$$u_r \geq \varepsilon,\ v_i \geq \varepsilon$$

In equations; E_k: efficiency value of DMU k, u_r: weight given to output r, v_i: weight given to input i, Y_{rk}: output r produced by DMU k, X_{ik}: input i utilized by DMU k, Y_{rj}: output r produced by DMU j, X_{ij}: input i utilized by DMU j, ε: sufficiently small positive number (for example 0.00001), i = 1 to m (number of input), r = 1 to p (number of output) and j = 1 to n (number of DMU). The DMU as E_k=1 is mentioned as efficient in the result of equation.

After the emergence of CCR model with constant returns to scale, BCC model with variable returns to scale was generated by the study of Banker, Charnes and Cooper (BCC) in 1984 (Banker *et al.* 1984). After the generation of CCR and BCC models many studies were done to contribute the theoretical development of DEA (Cook and Seiford, 2009 and Hatami *et al.* 2011).

Interval Data Envelopment Analysis

DEA which was quite useful method for performance measurement can be resulted false due to wrong data or uncertainty data because of its sensitivity to data. An extreme point on data highly affects the efficiency measurement of most of DMU (Kao and Liu, 2000).

One of the most important difficulties of classical DEA model is the need of certain values on input and output values (Kao and Liu, 2000). For example sensitive measurements and certain data sometimes are impossible, because production or service processes generally have complex input and output. Therefore probability distribution and membership functions of fuzzy set theory can be used to concretize the uncertain data. Organizing the uncertain data with probability distributions needs whether predictable regularity data or frequency data. Generally membership function of fuzzy set theory is used to organize the uncertain data because of this condition complicated the use of probability distributions method (Liu and Chuang, 2009). Fuzzy DEA was generated by using fuzzy set theory to measure the efficiency with uncertain data.

Kao and Liu generated a method that could provide fuzzy efficiency measurement which applied α-cuts and Zadeh's extension principle to turn the fuzzy DEA to final DEA for DMUs which had fuzzy observations. Fuzzy DEA model of Kao and Liu which was transformed into linear model family by using α-cuts approach is mentioned as (Kao and Liu, 2000);

$$\tilde{E}_k = \max \left(\sum_{r=1}^{p} u_r \tilde{Y}_{rk} \right) \Big/ \left(\sum_{i=1}^{m} v_i \tilde{X}_{ik} \right) \tag{2}$$

$$\left(\sum_{r=1}^{p} u_r \tilde{Y}_{rj} \right) \Big/ \left(\sum_{i=1}^{m} v_i \tilde{X}_{ij} \right) \le 1$$

$$u_r, v_i \ge \varepsilon \ge 0$$

Whether inputs and outputs in equation are known and respectively can be showed with fuzzy sets of $\mu_{\tilde{X}_{ij}}$ and $\mu_{\tilde{Y}_{rj}}$ membership functions. When $S(\tilde{X}_{ij})$ and $S(\tilde{Y}_{ij})$, show the support of $\tilde{X}_{ij}$ and $\tilde{Y}_{ij}$, the α-cuts of $\tilde{X}_{ij}$ and $\tilde{Y}_{ij}$ can be defined as (Kao and Liu, 2003);

$$(X_{ij})_\alpha = \left\{ x_{ij} \in S(\tilde{X}_{ij}) \setminus \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha \right\}, \quad \forall i, j \tag{3.a}$$

$$(Y_{rj})_\alpha = \left\{ y_{rj} \in S(\tilde{Y}_{rj}) \setminus \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha \right\}, \quad \forall r, j \tag{3.b}$$

or in other words,

$$\begin{aligned} (X_{ij})_\alpha &= \left[(X_{ij})^L{}_\alpha , (X_{ij})^U{}_\alpha \right] \\ &= \left[\min_{x_{ij}} \left\{ x_{ij} \in X_{ij} \setminus \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha \right\}, \max_{x_{ij}} \left\{ x_{ij} \in X_{ij} \setminus \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha \right\} \right] \end{aligned} \tag{4.a}$$

$$\begin{aligned} (Y_{rj})_\alpha &= \left[(Y_{rj})^L{}_\alpha , (Y_{rj})^U{}_\alpha \right] \\ &= \left[\min_{y_{rj}} \left\{ y_{rj} \in Y_{rj} \setminus \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha \right\}, \max_{y_{rj}} \left\{ y_{rj} \in Y_{rj} \setminus \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha \right\} \right] \end{aligned} \tag{4.b}$$

Membership function of DMU k can be defined according to Zadeh's extension principle (Yager, 1986; Zadeh, 1978; Zimmermann, 1996) as,

$$\mu_{\tilde{E}_k}(z) = \sup_{x,y} \min \left\{ \mu_{\tilde{X}_{ij}}(x_{ij}), \mu_{\tilde{Y}_{rj}}(y_{rj}),\ \forall i, r, j \setminus z = E_k(x,y) \right\} \tag{5}$$

Suggested approach for establishment of membership function is to remove the α-cuts of $\mu_{\tilde{E}_k}$. In other words, lower and upper bounds in various α levels must obtain for establishment of $\mu_{\tilde{E}_k}$ membership function. According to equality (5); $\mu_{\tilde{E}_k}$ is minimum of $\mu_{\tilde{X}_{ij}}(x_{ij})$ and $\mu_{\tilde{Y}_{rj}}(y_{rj})$, for $\forall i, r, j$. $\mu_{\tilde{X}_{ij}}(x_{ij}) \geq \alpha$, $\mu_{\tilde{Y}_{rj}}(y_{rj}) \geq \alpha$ and at least one of $\mu_{\tilde{X}_{ij}}(x_{ij})$ or $\mu_{\tilde{Y}_{rj}}(y_{rj})$ must be equal to α for crosscheck of $z = E_k(x,y)$ and $\mu_{\tilde{E}_k}(z) = \alpha$ for $\forall i, r, j$. Also, all α-cuts constitute a nested structure according to α (Kao and Liu, 2003). For example, for $0 < \alpha_2 < \alpha_1 \leq 1$ it must be like $\left[(X_{ij})^L{}_{\alpha_1}, (X_{ij})^U{}_{\alpha_1} \right] \subseteq \left[(X_{ij})^L{}_{\alpha_2}, (X_{ij})^U{}_{\alpha_2} \right]$ and $\left[(Y_{rj})^L{}_{\alpha_1}, (Y_{rj})^U{}_{\alpha_1} \right] \subseteq \left[(Y_{rj})^L{}_{\alpha_2}, (Y_{rj})^U{}_{\alpha_2} \right]$ (Zimmermann, 1996).

According to formulation (5),

$$(E_k)^L_\alpha = \max_{\substack{(X_{ij})^L_\alpha \leq x_{ij} \leq (X_{ij})^U_\alpha \\ (Y_{rj})^L_\alpha \leq y_{rj} \leq (Y_{rj})^U_\alpha \\ \forall i, r, j}} \left\{ \begin{aligned} & E_k = \max \left(\sum_{r=1}^{p} u_r Y_{rk} \right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ik} \right) \\ & \left(\sum_{r=1}^{p} u_r Y_{rj} \right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ij} \right) \leq 1, \quad j = 1, \ldots, n \\ & u_r, v_i \geq \varepsilon \geq 0 \end{aligned} \right. \tag{6.a}$$

$$(E_k)^U_\alpha = \max_{\substack{(X_{ij})^L_\alpha \leq x_{ij} \leq (X_{ij})^U_\alpha \\ (Y_{rj})^L_\alpha \leq y_{rj} \leq (Y_{rj})^U_\alpha \\ \forall i, r, j}} \left\{ \begin{aligned} & E_k = \max \left(\sum_{r=1}^{p} u_r Y_{rk} \right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ik} \right) \\ & \left(\sum_{r=1}^{p} u_r Y_{rj} \right) \Big/ \left(\sum_{i=1}^{m} v_i X_{ij} \right) \leq 1, \quad j = 1, \ldots, n \\ & u_r, v_i \geq \varepsilon \geq 0 \end{aligned} \right. \tag{6.b}$$

can be solved as above (Kao and Liu, 2003).

In the relative efficiency measurement of DMU k; the lowest value of DMU k is obtained to take possible lowest value of input levels of all other DMUs with output level of this DMU and to take possible highest value of output levels of all other DMU with input level of this DMU. Contrarily, to take possible highest value of input levels of all other DMUs with output level of this DMU and to take possible lowest value of output levels of all other DMUs with input level of this DMU to obtain the highest relative efficiency of DMU. Therefore the two level mathematical model in (6),

$$(E_k)_\alpha^L = \max\left(\sum_{r=1}^{p} u_r (Y_{rk})_\alpha^L\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ik})_\alpha^U\right)$$
$$\left(\sum_{r=1}^{p} u_r (Y_{rk})_\alpha^L\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ik})_\alpha^U\right) \le 1$$
$$\left(\sum_{r=1}^{p} u_r (Y_{rj})_\alpha^U\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ij})_\alpha^L\right) \le 1, \quad j=1,\dots,n,\ i \ne j \tag{7.a}$$
$$u_r, v_i \ge \varepsilon \ge 0$$

$$(E_k)_\alpha^U = \max\left(\sum_{r=1}^{p} u_r (Y_{rk})_\alpha^U\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ik})_\alpha^L\right)$$
$$\left(\sum_{r=1}^{p} u_r (Y_{rk})_\alpha^U\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ik})_\alpha^L\right) \le 1$$
$$\left(\sum_{r=1}^{p} u_r (Y_{rj})_\alpha^L\right)\Big/\left(\sum_{i=1}^{m} v_i (X_{ij})_\alpha^U\right) \le 1, \quad j=1,\dots,n,\ i \ne j \tag{7.b}$$
$$u_r, v_i \ge \varepsilon \ge 0$$

is simplified as traditional single level model. Model (7), is a classical DEA model which could be solved by transforming linear programming model (Kao and Liu, 2003).

Following model was generated to create lower and upper bounds of interval efficiency of each DMU in order to mention the uncertainty state in the condition that was known to locate between lower and upper bounds such as $\left[x_{ij}^L,\ x_{ij}^U\right]$ and $\left[y_{rj}^L,\ y_{rj}^U\right]$ (where $x_{ij}^L > 0$ and $y_{rj}^L > 0$) (Despotis and Smirlis, 2002 and Wang *et al.* 2005).

$$(E_k)_\alpha^L = \max \sum_{r=1}^{p} u_r (Y_{rk})_\alpha^L$$
$$\sum_{i=1}^{m} v_i (X_{ik})_\alpha^U = 1$$
$$\sum_{r=1}^{p} u_r (Y_{rk})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ik})_\alpha^U \le 0 \tag{8.a}$$
$$\sum_{r=1}^{p} u_r (Y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^L \le 0, \quad j=1,\dots,n,\ i \ne j$$
$$u_r, v_i \ge \varepsilon \ge 0$$

$$(E_k)^U_\alpha = \max \sum_{r=1}^{p} u_r (Y_{rk})^U_\alpha$$
$$\sum_{i=1}^{m} v_i (X_{ik})^L_\alpha = 1$$
$$\sum_{r=1}^{p} u_r (Y_{rk})^U_\alpha - \sum_{i=1}^{m} v_i (X_{ik})^L_\alpha \le 0 \qquad (8.b)$$
$$\sum_{r=1}^{p} u_r (Y_{rj})^L_\alpha - \sum_{i=1}^{m} v_i (X_{ij})^U_\alpha \le 0, \quad j = 1,...,n, \ i \ne j$$
$$u_r, v_i \ge \varepsilon \ge 0$$

Zhu (2003) showed to simplify Interval DEA model of Cooper *et al.* (1999) to the upper bound DEA model in model (8.b) in the condition of interval data. Entani *et al*. (2002) also used upper bound DEA model in order to measure possible relative efficiency of all DMU.

Every bound set used to measure the efficiency of DMU seems different for each DMU when the lower and upper bound DEA models in Model (8.a) and (8.b). Therefore bound sets which were used to measure upper and lower efficiency of the same DMU are different. For example, although the bound set used to measure upper bound efficiency of DMU *k* occurred from $\left\{(x^L_{ik}, y^U_{rk}), (x^U_{ij}, y^L_{rj}) \ (j = 1,...,n; j \ne k; i = 1,...,m; r = 1,..., p)\right\}$ dataset, the bound set used to measure lower bound efficiency of DMU *k* occurred from $\left\{(x^U_{ik}, y^L_{rk}), (x^L_{ij}, y^U_{rj}) \ (j = 1,...,n; j \ne k; i = 1,...,m; r = 1,..., p)\right\}$ dataset. It is obvious that these two data sets are different *(Wang et al. 2005)*.

Various bound sets in the measurement of efficiency of DMU do not allow the comparison among efficiency because of different product bounds were embraced in efficiency measurement process.

In Interval DEA model pair based on interval arithmetic which used same bound set every time for either all DMU or all of the lower-upper bound efficiency was generated in order to avoid the use of different production bounds for measuring efficiency of different DMU (Wang *et al.* 2005) .

Let be efficiency of DMU *j* is $\theta_j = \dfrac{\sum_{r=1}^{p} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$, $j = 1,...,n$. According to the operation rules of interval data process

$\theta_j = \frac{\sum_{r=1}^{p} u_r [y_{rj}^L, y_{rj}^U]}{\sum_{i=1}^{m} v_i [x_{ij}^L, x_{ij}^U]} = \frac{\left[\sum_{r=1}^{p} u_r y_{rj}^L, \sum_{r=1}^{p} u_r y_{rj}^U,\right]}{\left[\sum_{i=1}^{m} v_i x_{ij}^L, \sum_{i=1}^{m} v_i x_{ij}^U\right]} = \left[\frac{\sum_{r=1}^{p} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U}, \frac{\sum_{r=1}^{p} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L}\right], j = 1,...,n$ is obtained. θ_j, which seemed as interval number can be displayed as $[\theta_j^L, \theta_j^U] \subseteq (0,1]$, $j = 1,...,n$. Thus, can be written as (Wang *et al.* 2005) ;

$$\theta_j^L = \frac{\sum_{r=1}^{p} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U} > 0 \ , \ j = 1,...,n \tag{9.a}$$

and

$$\theta_j^U = \frac{\sum_{r=1}^{p} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1 \ , \ j = 1,...,n \tag{9.b}$$

Therefore, to measure lower and upper bounds efficiency of DMU *k*,

$$\max \theta_k^L = \frac{\sum_{r=1}^{p} u_r y_{rk}^L}{\sum_{i=1}^{m} v_i x_{ik}^U}$$

$$\frac{\sum_{r=1}^{p} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1 \ , \ j = 1,...,n \tag{10.a}$$

$$u_r, v_i \geq \varepsilon \ , \ \forall r, i$$

$$\max \theta_k^U = \frac{\sum_{r=1}^{p} u_r y_{rk}^U}{\sum_{i=1}^{m} v_i x_{ik}^L}$$

$$\frac{\sum_{r=1}^{p} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1 \ , \ j = 1,...,n \tag{10.b}$$

$$u_r, v_i \geq \varepsilon \ , \ \forall r, i$$

Fractional programming model pair is established as above (Wang *et al.* 2005*)*. Linear programming models such as

$$\begin{aligned} \max \theta_k^L &= \sum_{r=1}^{p} u_r y_{rk}^L \\ & \sum_{i=1}^{m} v_i x_{ik}^U = 1 \\ & \sum_{r=1}^{p} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 \ , \ j = 1,\ldots,n \\ & u_r, v_i \geq \varepsilon \ , \ \forall r,i \end{aligned} \tag{11.a}$$

$$\begin{aligned} \max \theta_k^U &= \sum_{r=1}^{p} u_r y_{rk}^U \\ & \sum_{i=1}^{m} v_i x_{ik}^L = 1 \\ & \sum_{r=1}^{p} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 \ , \ j = 1,\ldots,n \\ & u_r, v_i \geq \varepsilon \ , \ \forall r,i \end{aligned} \tag{11.b}$$

are generated by applying Charnes-Cooper transformation to this fractional programming model pair (Wang *et al.* 2005).

Model (11.a) determines the production bound of all DMUs. Model (11.b) uses this production bound as a reference in order to measure lower bound efficiency of each DMU. θ_k^L displays the best possible relative lower bound efficiency and θ_k^U the best possible relative efficiency was achieved by DMU *k,* when all DMU were in the best production bound. These create the best possible relative efficiency $[\theta_k^L, \theta_k^U]$ (Despotis and Smirlis, 2002 and Wang *et al* 2005).

These Interval DEA models which were generated finally can be solved such as certain data classical DEA (Despotis and Smirlis, 2002). It is mentioned that the DMU which had the best possible upper bound efficiency as $\theta_k^U = 1$ after solving of model, was efficient (Wang *et al.* 2005).

Application

To calculate of relative efficiency values of commercial banks working in Turkey in 2011 by using classical and interval DEA was aimed. Input-oriented CCR model was used to calculate efficiency values of the banks and analyses were performed by excel based EMS 1.3 (Efficiency Measurement System) package program.

The select of DMUs and variables which would be used in DEA is rather important because of its high effect on efficiency values. For the assumption to be homogeneous of units that were measured efficiency, investment bank, development bank, participation bank and branches of the foreign banks in Turkey were excluded from the study 21 commercial banks selected for analysis. 2 input and 3 output variables which usually used in similar studies were selected and 2011 end-of-year data of aforementioned variables were gathered from the official website of Banks Association of Turkey.

In order to apply interval DEA, the certain data in Appendix 1 were transformed to fuzzy data, then lower and upper bounds were obtained by α-cuts method according to these data.

To express of certain data as fuzzy data, standard deviation values were used and fuzzy data in Appendix 2 and Appendix 3 to blur it 1 percent of standard deviation with the formulas in (12.a), (12.b) and (12.c).

*Lower Bound Data (L) = Actual Data - Standard Deviation*0,01* (12.a)

Midpoint (M) = Actual Data (12.b)

*Upper Bound Data (U) = Actual Data + Standard Deviation*0,01* (12.c)

Lower and upper bound data in $\mu \geq \alpha$ are determined by generating set of α-cuts for fuzzy input and output data. Therefore set of α-cuts are such as in (13.a) and (13.b) for fuzzy input mentioned as $\tilde{X}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U)$ and fuzzy output data mentioned as $\tilde{Y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U)$ (Saati *et al.* 2002).

$$\mu_{\tilde{X}_{ij}}(x_{ij}) \geq \alpha = \begin{cases} \dfrac{x_{ij} - x_{ij}^L}{x_{ij}^M - x_{ij}^L} \geq \alpha \\ \dfrac{x_{ij}^U - x_{ij}}{x_{ij}^U - x_{ij}^M} \geq \alpha \end{cases} \Rightarrow \alpha x_{ij}^M + (1-\alpha) x_{ij}^L \leq x_{ij} \leq \alpha x_{ij}^M + (1-\alpha) x_{ij}^U \qquad (13.a)$$

$$\mu_{\tilde{Y}_{rj}}(y_{rj}) \geq \alpha = \begin{cases} \dfrac{y_{rj} - y_{rj}^L}{y_{rj}^M - y_{rj}^L} \geq \alpha \\ \dfrac{y_{rj}^U - y_{rj}}{y_{rj}^U - y_{rj}^M} \geq \alpha \end{cases} \Rightarrow \alpha y_{rj}^M + (1-\alpha) y_{rj}^L \leq y_{rj} \leq \alpha y_{rj}^M + (1-\alpha) y_{rj}^U \qquad (13.b)$$

Lower and upper bounds of $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$ are expressed such as in (14.a) and (14.b) with α-cuts method (Saati *et al.* 2002).

$$\tilde{X}_{ij} = (\alpha x_{ij}^M + (1-\alpha) x_{ij}^L \ , \ \alpha x_{ij}^M + (1-\alpha) x_{ij}^U) \qquad (14.a)$$

$$\tilde{Y}_{rj} = (\alpha y_{rj}^M + (1-\alpha) y_{rj}^L \ , \ \alpha y_{rj}^M + (1-\alpha) y_{rj}^U) \qquad (14.b)$$

Set of α-cuts lower and upper bounds of input and output variables for all DMUs, were obtained such as Appendix 4 and Appendix 5 by the application of formulation in (14.a) and (14.b) to the fuzzy data in Appendix 2 and Appendix 3. When the tables in Appendix 4 and Appendix 5 is examined, lower and upper bounds of input and output variables seems to be connected with α. This condition means various lower and upper bounds could be obtained with various α values and therefore various lower and upper bound efficiency value could be calculated by various α values.

Efficiency values of classical DEA model which was calculated by input-oriented CCR method and interval DEA model which was obtained by α = 0.25 are placed on Table 1.

Table 1: Calculated efficiency values

DMUs	Efficiency Values		
	Classical DEA	Interval DEA	
		Lower Bound	Upper Bound
Ziraat Bank	0.5863	0.5867	0.5682
Is Bank	0.8267	0.8356	0.7928
Akbank	0.9191	0.9183	0.9055
Garanti Bank	1.0000	1.0000	1.0000
Yapı Kredi Bank	0.9060	0.9050	0.8780
Halkbank	0.8235	0.8216	0.8242
Vakıfbank	0.9397	0.9373	0.8770
Denizbank	0.4956	0.4912	0.5001
Finansbank	0.6502	0.6510	0.6494
TEB	0.5471	0.5440	0.4985
HSBC Bank	0.5343	0.5256	0.5430
ING Bank	0.5858	0.5740	0.5447
Şekerbank	0.5250	0.5071	0.5433
Anadolubank	0.4585	0.4198	0.4828
ABank	0.7210	0.6621	0.7101
Eurobank Tekfen	0.4824	0.4102	0.5550
Tekstil Bank	0.5772	0.5183	0.5714
Citibank	1.0000	1.0000	1.0000
Turkland Bank	0.5853	0.4406	0.7081
Fibabank	1.0000	0.8434	1.0000
Turkish Bank	0.2485	0.0766	0.5068

Garanti Bank, Citibank and Fibabank whose upper bound efficiency value equal to 1 was accepted as efficient as a result of analysis.

Conclusion

One of the important characteristic of DEA is to be too much sensitive for data. This characteristic cause wrong results on the situation of uncertain data. In this study, the importance of this characteristic was mentioned and moreover Interval DEA model, developed to measure the efficiency with fuzzy data, was examined.

If the data of input and output variables is fuzzy, the objective function and constraints of DEA model becomes fuzzy. Thus the model cannot be solved by the classical DEA method. On the situation of uncertain data, Interval DEA method transforms this data to interval numbers and suggests establishing a new pair of model that could be solved by classical DEA. Interval DEA stands out with easier to implement method than other fuzzy DEA methods because of has advantages of classical DEA method, does not require any scale transformation and does not need a new constraint.

Two different models are generated to ensure the measurement of lower and upper bounds efficiency by Interval DEA. Thus, a lower and upper bound is obtained for efficiency value of each DMUs and efficiency value of DMU is stated as a interval numbers. Wang *et al*. (2005) mentioned that the upper bound efficiency value which was displayed the best possible efficiency value, equals to 1 was enough for efficient DMU *(Wang et al. 2005)*. Also both upper and lower efficiency value of DMU equal to 1 can be accepted as efficient for providing more selective an analysis because of the thought that the possibility of increasing the efficient DMU with this condition.

An application was performed for better understanding of the operation of this method by referring Interval DEA method theory, in this study. The banking system which is one of the prominent field of DEA because of has the upper level of competition and has efficiency conception, was handled for application of this study.

2011 end-of-year data of number of branch, number of staff, total credit, total income and net profit variables were used to measure the efficiency value of banks and results were displayed in Table 1. Classical DEA results were obtained by exact data and Interval DEA results were obtained interval data were found by blurring of exact data, in the Table. Generally both results seem to be consistent. Although a very low rate blur to data, the lower bound efficiency value of Fibabank who was found efficient by classical DEA, was not equal to 1 in interval DEA method. Thus, Interval DEA method provides more selective results rather than classical DEA, can be said.

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Appendices:

Appendix 1: Exact input and output data of DMUs

DMUs	Inputs		Outputs		
	Number of branch	Number of staff	Total credit	Total income	Net profit
Ziraat Bank	1458	24374	71173.26	6209.94	2100.67
İş Bank	1201	24887	91620.64	8163.27	2667.49
Akbank	927	15339	70213.22	6083.46	2394.53
Garanti Bank	914	16773	83532.93	7936.87	3070.58
Yapı Kredi Bank	907	14859	67044.87	5821.77	1857.49
Halkbank	771	13643	55949.45	5051.93	2045.13
Vakıfbank	680	12222	57200.61	4422.16	1226.79
Denizbank	588	9772	22196.37	2291.80	873.97
Finansbank	522	10837	29867.20	3000.00	848.11
TEB	507	9356	25443.64	1793.32	206.68
HSBC Bank	330	6155	13662.37	1534.52	240.79
ING Bank	322	5232	15264.74	1100.21	79.01
Şekerbank	272	3530	8282.22	876.92	118.04
Anadolubank	88	1911	3715.39	335.91	85.23
ABank	63	1185	4220.59	288.19	28.26
Eurobank Tekfen	59	954	2291.94	209.61	37.37
Tekstil Bankası	44	880	2461.06	147.24	22.02
Citibank	37	2233	2652.96	421.70	5.51
Turkland Bank	21	438	2095.04	91.00	5.19
Fibabank	20	284	256.57	33.39	0.69

Appendix 2: Fuzzy input data of DMUs

DMUs	Number of branch			Number of staff		
	L	M	U	L	M	U
Ziraat Bank	1453.65	1458	1462.35	24296.14	24374	24451.86
İş Bank	1196.65	1201	1205.35	24809.14	24887	24964.86
Akbank	922.65	927	931.35	15261.14	15339	15416.86
Garanti Bank	909.65	914	918.35	16695.14	16773	16850.86
Yapı Kredi Bank	902.65	907	911.35	14781.14	14859	14936.86
Halkbank	766.65	771	775.35	13565.14	13643	13720.86
Vakıfbank	675.65	680	684.35	12144.14	12222	12299.86
Denizbank	583.65	588	592.35	9694.14	9772	9849.86
Finansbank	517.65	522	526.35	10759.14	10837	10914.86
TEB	502.65	507	511.35	9278.14	9356	9433.86
HSBC Bank	325.65	330	334.35	6077.14	6155	6232.86
ING Bank	317.65	322	326.35	5154.14	5232	5309.86
Şekerbank	267.65	272	276.35	3452.14	3530	3607.86
Anadolubank	83.65	88	92.35	1833.14	1911	1988.86
ABank	58.65	63	67.35	1107.14	1185	1262.86
Eurobank Tekfen	54.65	59	63.35	876.14	954	1031.86
Tekstil Bank	39.65	44	48.35	802.14	880	957.86
Citibank	32.65	37	41.35	2155.14	2233	2310.86
Turkland Bank	22.65	27	31.35	418.14	496	573.86
Fibabank	16.65	21	25.35	360.14	438	515.86
Turkish Bank	15.65	20	24.35	206.14	284	361.86

Appendix 3: Fuzzy output data of DMUs

DMUs	Total credit			Total income			Net profit		
	L	M	U	L	M	U	L	M	U
Ziraat Bank	70857.77	71173.26	71488.75	6181.87	6209.94	6238.01	2090.15	2100.67	2111.20
İş Bank	91305.15	91620.64	91936.12	8135.21	8163.27	8191.34	2656.96	2667.49	2678.01
Akbank	69897.74	70213.22	70528.71	6055.39	6083.46	6111.52	2384.00	2394.53	2405.05
Garanti Bank	83217.44	83532.93	83848.42	7908.81	7936.87	7964.94	3060.05	3070.58	3081.10
Yapı Kredi Bank	66729.38	67044.87	67360.36	5793.71	5821.77	5849.84	1846.96	1857.49	1868.01
Halkbank	55633.96	55949.45	56264.93	5023.87	5051.93	5080.00	2034.61	2045.13	2055.66
Vakıfbank	56885.12	57200.61	57516.10	4394.09	4422.16	4450.22	1216.26	1226.79	1237.31
Denizbank	21880.89	22196.37	22511.86	2263.74	2291.80	2319.87	863.45	873.97	884.50
Finansbank	29551.71	29867.20	30182.68	2971.93	3000.00	3028.07	837.59	848.11	858.64
TEB	25128.16	25443.64	25759.13	1765.25	1793.32	1821.39	196.15	206.68	217.20
HSBC Bank	13346.89	13662.37	13977.86	1506.45	1534.52	1562.58	230.26	240.79	251.31
ING Bank	14949.25	15264.74	15580.22	1072.14	1100.21	1128.27	68.49	79.01	89.54
Şekerbank	7966.74	8282.22	8597.71	848.85	876.92	904.98	107.52	118.04	128.57
Anadolubank	3399.91	3715.39	4030.88	307.85	335.91	363.98	74.70	85.23	95.76
ABank	3905.10	4220.59	4536.07	260.13	288.19	316.26	17.74	28.26	38.79
Eurobank Tekfen	1976.46	2291.94	2607.43	181.54	209.61	237.67	26.84	37.37	47.89
Tekstil Bank	2145.57	2461.06	2776.54	119.18	147.24	175.31	11.49	22.02	32.54
Citibank	2337.48	2652.96	2968.45	393.64	421.70	449.77	-5.02	5.51	16.03
Turkland Bank	1129.89	1445.38	1760.86	77.76	105.83	133.90	-6.96	3.56	14.09
Fibabank	1779.55	2095.04	2410.52	62.93	91.00	119.06	-5.34	5.19	15.71
Turkish Bank	-58.91	256.57	572.06	5.32	33.39	61.46	-9.83	0.69	11.22

Appendix 4: α-cuts bounds of input variables

DMUs	Number of branch		Number of staff	
	Lower bound	Upper bound	Lower bound	Upper bound
Ziraat Bank	α1458+(1-α)1453.65	α1458+(1-α)1462.35	α24374+(1-α)24296.14	α24374+(1-α)24451.86
İş Bank	α1201+(1-α)1196.65	α1201+(1-α)1205.35	α24887+(1-α)24809.14	α24887+(1-α)24964.86
Akbank	α927+(1-α)922.65	α927+(1-α)931.35	α15339+(1-α)15261.14	α15339+(1-α)15416.86
Garanti Bank	α914+(1-α)909.65	α914+(1-α)918.35	α16773+(1-α)16695.14	α16773+(1-α)16850.86
Yapı Kredi Bank	α907+(1-α)902.65	α907+(1-α)911.35	α14859+(1-α)14781.14	α14859+(1-α)14936.86
Halkbank	α771+(1-α)766.65	α771+(1-α)775.35	α13643+(1-α)13565.14	α13643+(1-α)13720.86
Vakıfbank	α680+(1-α)675.65	α680+(1-α)684.35	α12222+(1-α)12144.14	α12222+(1-α)12299.86
Denizbank	α588+(1-α)583.65	α588+(1-α)592.35	α9772+(1-α)9694.14	α9772+(1-α)9849.86
Finansbank	α522+(1-α)517.65	α522+(1-α)526.35	α10837+(1-α)10759.14	α10837+(1-α)10914.86
TEB	α507+(1-α)502.65	α507+(1-α)511.35	α9356+(1-α)9278.14	α9356+(1-α)9433.86
HSBC Bank	α330+(1-α)325.65	α330+(1-α)334.35	α6155+(1-α)6077.14	α6155+(1-α)6232.86
ING Bank	α322+(1-α)317.65	α322+(1-α)326.35	α5232+(1-α)5154.14	α5232+(1-α)5309.86
Şekerbank	α272+(1-α)267.65	α272+(1-α)276.35	α3530+(1-α)3452.14	α3530+(1-α)3607.86
Anadolubank	α88+(1-α)83.65	α88+(1-α)92.35	α1911+(1-α)1833.14	α1911+(1-α)1988.86
ABank	α63+(1-α)58.65	α63+(1-α)67.35	α1185+(1-α)1107.14	α1185+(1-α)1262.86
Eurobank Tekfen	α59+(1-α)54.65	α59+(1-α)63.35	α954+(1-α)876.14	α954+(1-α)1031.86
Tekstil Bank	α44+(1-α)39.65	α44+(1-α)48.35	α880+(1-α)802.14	α880+(1-α)957.86

Citibank	α37+(1-α)32.65	α37+(1-α)41.35	α2233+(1-α)2155.14	α2233+(1-α)2310.86
Turkland Bank	α27+(1-α)22.65	α27+(1-α)31.35	α496+(1-α)418.14	α496+(1-α)573.86
Fibabank	α21+(1-α)16.65	α21+(1-α)25.35	α438+(1-α)360.14	α438+(1-α)515.86
Turkish Bank	α20+(1-α)15.65	α20+(1-α)24.35	α284+(1-α)206.14	α284+(1-α)361.86

Appendix 5: α-cuts bounds of output variables

DMUs	Total credit		Total income		Net profit	
	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Ziraat Bank	α71173.26+(1-α)70857.77	α71173.26+(1-α)71488.75	α6209.94+(1-α)6181.87	α6209.94+(1-α)6238.01	α2100.67+(1-α)2090.15	α2100.67+(1-α)2111.2
İş Bank	α91620.64+(1-α)91305.15	α91620.64+(1-α)91936.12	α8163.27+(1-α)8135.21	α8163.27+(1-α)8191.34	α2667.49+(1-α)2656.96	α2667.49+(1-α)2678.01
Akbank	α70213.22+(1-α)69897.74	α70213.22+(1-α)70528.71	α6083.46+(1-α)6055.39	α6083.46+(1-α)6111.52	α2394.53+(1-α)2384	α2394.53+(1-α)2405.05
Garanti Bank	α83532.93+(1-α)83217.44	α83532.93+(1-α)83848.42	α7936.87+(1-α)7908.81	α7936.87+(1-α)7964.94	α3070.58+(1-α)3060.05	α3070.58+(1-α)3081.1
Yapı Kredi Bank	α67044.87+(1-α)66729.38	α67044.87+(1-α)67360.36	α5821.77+(1-α)5793.71	α5821.77+(1-α)5849.84	α1857.49+(1-α)1846.96	α1857.49+(1-α)1868.01
Halkbank	α55949.45+(1-α)55633.96	α55949.45+(1-α)56264.93	α5051.93+(1-α)5023.87	α5051.93+(1-α)5080	α2045.13+(1-α)2034.61	α2045.13+(1-α)2055.66
Vakıfbank	α57200.61+(1-α)56885.12	α57200.61+(1-α)57516.1	α4422.16+(1-α)4394.09	α4422.16+(1-α)4450.22	α1226.79+(1-α)1216.26	α1226.79+(1-α)1237.31
Denizbank	α22196.37+(1-α)21880.89	α22196.37+(1-α)22511.86	α2291.8+(1-α)2263.74	α2291.8+(1-α)2319.87	α873.97+(1-α)863.45	α873.97+(1-α)884.5
Finansbank	α29867.2+(1-α)29551.71	α29867.2+(1-α)30182.68	α3000+(1-α)2971.93	α3000+(1-α)3028.07	α848.11+(1-α)837.59	α848.11+(1-α)858.64
TEB	α25443.64+(1-α)25128.16	α25443.64+(1-α)25759.13	α1793.32+(1-α)1765.25	α1793.32+(1-α)1821.39	α206.68+(1-α)196.15	α206.68+(1-α)217.2
HSBC Bank	α13662.37+(1-α)13346.89	α13662.37+(1-α)13977.86	α1534.52+(1-α)1506.45	α1534.52+(1-α)1562.58	α240.79+(1-α)230.26	α240.79+(1-α)251.31
ING Bank	α15264.74+(1-α)14949.25	α15264.74+(1-α)15580.22	α1100.21+(1-α)1072.14	α1100.21+(1-α)1128.27	α79.01+(1-α)68.49	α79.01+(1-α)89.54
Şekerbank	α8282.22+(1-α)7966.74	α8282.22+(1-α)8597.71	α876.92+(1-α)848.85	α876.92+(1-α)904.98	α118.04+(1-α)107.52	α118.04+(1-α)128.57
Anadolubank	α3715.39+(1-α)3399.91	α3715.39+(1-α)4030.88	α335.91+(1-α)307.85	α335.91+(1-α)363.98	α85.23+(1-α)74.7	α85.23+(1-α)95.76
ABank	α4220.59+(1-α)3905.1	α4220.59+(1-α)4536.07	α288.19+(1-α)260.13	α288.19+(1-α)316.26	α28.26+(1-α)17.74	α28.26+(1-α)38.79
Eurobank Tekfen	α2291.94+(1-α)1976.46	α2291.94+(1-α)2607.43	α209.61+(1-α)181.54	α209.61+(1-α)237.67	α37.37+(1-α)26.84	α37.37+(1-α)47.89
Tekstil Bank	α2461.06+(1-α)2145.57	α2461.06+(1-α)2776.54	α147.24+(1-α)119.18	α147.24+(1-α)175.31	α22.02+(1-α)11.49	α22.02+(1-α)32.54
Citibank	α2652.96+(1-α)2337.48	α2652.96+(1-α)2968.45	α421.7+(1-α)393.64	α421.7+(1-α)449.77	α5.51+(1-α)-5.02	α5.51+(1-α)16.03
Turkland Bank	α1445.38+(1-α)1129.89	α1445.38+(1-α)1760.86	α105.83+(1-α)77.76	α105.83+(1-α)133.9	α3.56+(1-α)-6.96	α3.56+(1-α)14.09
Fibabank	α2095.04+(1-α)1779.55	α2095.04+(1-α)2410.52	α91+(1-α)62.93	α91+(1-α)119.06	α5.19+(1-α)-5.34	α5.19+(1-α)15.71
Turkish Bank	α256.57+(1-α)-58.91	α256.57+(1-α)572.06	α33.39+(1-α)5.32	α33.39+(1-α)61.46	α0.69+(1-α)-9.83	α0.69+(1-α)11.22