NEW EFFICIENT MODEL-BASED PID DESIGN METHOD

Farhan A. Salem, PhD

Mechatronics Sec. Dept. of Mechanical Engineering, Faculty of Engineering,

Taif University, Taif, Saudi Arabia

Abstract

Because of their simplicity, robustness and successful practical application, PID-controllers are the most popular and widely-used controllers in industry. Many PID design methods have been proposed, each has its advantages and limitations, However, finding appropriate parameters for the PID controller is still not easy task. This paper proposes a new simple and efficient model-based PID design method for achieving an important design compromise; acceptable stability, and medium fastness of response, the method is based on plant's parameters; the proposed PID design methodology was test, verified and compared using MATLAB/SIMULINK software.

Keywords: PID Controller, PID tuning methodology

Introduction

The PID- controllers are the most popular and widely-used controllers in industry, because of their simplicity, robustness and successful practical application that can provide excellent control performance despite the varied dynamic characteristics of plant. The PID algorithm consists of three basic control modes; Proportional, Integral and the Derivative modes**.** Before commencing tuning PID controller, it is important to know the configuration of the PID algorithm, there are three different types of PID algorithm; (1) Ideal (2) Series (also called "interacting" or "analog" or "classical") (3) Parallel (also called "non-interacting", "independent" and "gain independent), The difference between these algorithms is how the P, I and D gains affect each other.

The term control system design refers to the process of selecting feedback gains (poles and zeros) that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant (Ahmad A. Mahfouz, et al 2013). An important compromise for control system design is to result in acceptable stability, and medium fastness of response, one definition of acceptable stability is when the undershoot that follows the first overshoot of the response is small, or barely observable. Beside world wide known and applied PID design method including Ziegler–Nichols, Chiein-Hrones-Reswick (CHR), Wang–Juang–Chan, Cohen-Coon, many PID design methods have been proposed in different papers and texts including (Astrom K,J et al 1994)(Ashish Tewari, 2002)(Katsuhiko Ogata, 2010)(Norman S. Nise, 2011)(Gene F. Franklin, et al 2002)(Dale E. Seborg, et al, 2004)(Dingyu Xue et al, 2007)(Chen C.L et al, 1989)(R. Matousek, *2012*)(K. J. Astrom et al, 2001)(Susmita Das et al, 2012) (L. Ntogramatzidis, 2010)(M.Saranya et al, 2012), each method has its advantages, and limitations. (R. Matousek, *2012*) present multi-criterion optimization of PID controller by means of soft computing optimization method HC12. (K. J. Astrom et al, 2001) introduce an improved PID tuning approach using traditional Ziegler-Nichols tuning method with the help of simulation aspects and new built in function. (L. Ntogramatzidis et al, 2010) A unified approach has been presented that enable the parameters of PID, PI and PD controllers (with corresponding approximations of the derivative action when needed) to be computed in finite terms given appropriate specifications expressed in terms of steady-state performance, phase/gain margins and gain crossover frequency. (M.Saranya et al, 2012) proposed an Internal Model Control (IMC) tuned PID controller method for the DC motor for robust operation. (Fernando G. Martons, 2005) proposed a procedure for tuning PID controllers with simulink and MATLAB. (Saeed Tavakoli, 2003) presented Using dimensional analysis and numerical optimization techniques, an optimal method for tuning PID controllers for first order plus time delay systems.

This paper proposes a new simple and efficient model-based PID design method for achieving acceptable stability and medium fastness of response, the methodology is based on relating the plant's parameters to PID controller gains.

Modeling PID controller

The output of PID controller $u(t)$, is equal to the sum of three signals: The signal obtained by multiplying the error signal by a constant proportional gain K_P, plus the signal obtained by differentiating and multiplying the error signal by constant derivative gain K_D and the signal obtained by integrating and multiplying the error signal by constant internal gain K_I, . The output of PID controller is given by Eq.(1), taking Laplace transform, and solving for transfer function , gives *ideal* PID transfer function given by Eq.(2)

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int e(t)dt \quad \Leftrightarrow \quad U(s) = K_P E(s) + K_D E(s)s + K_I E(s)\frac{1}{s} \tag{1}$$

$$U(s) = E(s)\left[K_P + \frac{K_I}{s} + K_D s\right] \tag{2}$$

Equation (2) can be manipulated to result in the following form

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} \tag{3}$$

Equation (3) is second order system, with two zeros and one pole at origin, and can be expressed to have the following form:

$$G_{PID} = \frac{K_D\left(s + Z_{PI}\right)\left(s + Z_{PD}\right)}{s} = K_D\left(s + Z_{PI}\right)\frac{\left(s + Z_{PD}\right)}{s} = G_{PD}(s)G_{PI}(s) \tag{4}$$

Which indicates that PID transfer function is the product of transfer functions PI and PD , Implementing these two controllers jointly and independently will take care of both controller design requirements. The transfer function given by Eq.(34),can also be expressed to have the form:

$$G_{PID} = \frac{K_D\left(s + Z_{PI}\right)\left(s + Z_{PD}\right)}{s} = \frac{K_D s^2 + \left(Z_{PI} + Z_{PD}\right)K_D s + (Z_{PI}Z_{PD}K_D)}{s}$$

Rearranging, we have:

$$G_{PID} = \frac{K_D s^2}{s} + \frac{\left(Z_{PI} + Z_{PD}\right)K_D s}{s} + \frac{(Z_{PI}Z_{PD}K_D)}{s} = \left(Z_{PI} + Z_{PD}\right)K_D + \frac{(Z_{PI}Z_{PD}K_D)}{s} + K_D s$$

Substituting the following, $K_1 = \left(Z_{PI} + Z_{PD}\right)K_D$, $K_2 = (Z_{PI}Z_{PD}K_D)$, $K_3 = K_D$, gives:

$$G_{PID} = K_1 + \frac{K_2}{s} + K_3 s \tag{5}$$

Since PID transfer function is a second order system, it can be expressed in terms of damping ratio and undamped natural frequency to have the following form:

$$G_{PID}(s) = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} = \frac{K_D\left[s^2 + 2\xi\omega_n s + \omega_n^2\right]}{s} \tag{6}$$

Where: $\omega_n^2 = \frac{K_I}{K_D}$ and $2\xi\omega_n = \frac{K_P}{K_D}$

PID transfer function given by Eq.(3) can, also, be expressed in terms of derivative time and integral time to have the following form:

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) = K_P\frac{T_I T_D s^2 + T_I s + 1}{T_I s} \tag{7}$$

Where: The integral time, $T_I = K_P / K_I$, The derivative time, $T_D = K_D / K_P$

$K_I = K_P / T_I$, $K_D = K_P T_D$

Filtering PID controller

Two main approaches including are used to filter PID action ; (1) PID introduce a *zero* into the closed loop transfer function, the presence of zero may cause overshoot in the transient response for the closed loop system, to filter PID controller and eliminate the overshoot, a *prefilter* is used, (2) Since it not be desirable to implement the controller as given above , where the numerator has a higher degree than the denominator, the transfer function is not causal and can not be realized, also in practice, all signals will contain high frequency noise, and differentiating noise (by D-controller) will once again create signals with large magnitudes. To avoid this, the derivative term $K_D s$ is usually implemented in conjunction with a low-pass filter of the form: (1 /τs+1), (the addition of a lag to the derivative term) with small time constant e.g. shorter than *1/5* of derivative time T_D , for some small τ, this has the effect of attenuating the high frequency noise entering the D-controller, and produces the following controller proper transfer function:

$$G_{PID}(s) = K_P\left[1 + \frac{I}{T_I s} + \frac{T_D}{1 + \tau_D s}\right]$$

The transfer function of a PID controller with a filtered derivative is given by:

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D s}{N}}\right), \quad T_D/N \text{ - time constant of the added lag} \qquad (8)$$

N: determines the gain K_{HF} of the PID controller in the high frequency range, the gain K_{HF} must be limited because measurement noise signal often contains high frequency components and its amplification should be limited. Usually, the divisor N is chosen in the range 2 to 20. If no D-controller, then we have PI controller, given by Eq. (9), it is clear that, PI and PD controllers are special cases of the PID controller.

$$G_{PI} = K_P\left(1 + \frac{1}{T_I s}\right) = K_P\left(\frac{T_I s + 1}{T_I s}\right) \qquad (9)$$

The addition of the proportional and derivative components effectively predicts the error value at T_D seconds (or samples) in the future, assuming that the loop control remains unchanged. The integral component adjusts the error value to compensate for the sum of all past errors, with the intention of completely eliminating them in T_I seconds (or samples). The resulting compensated single error value is scaled by the single gain K_P

Dominant features of control systems

Most complex systems have dominant features that typically can be approximated by either a first or second order system response. Control system's response is largely dictated by those poles that are the closest to the imaginary axis, i.e. the poles that have the smallest real part magnitudes, such poles are called the dominant poles, many times, it is possible to identify a single pole, or a pair of poles, as the dominant poles. In such cases, a fair idea of the control system's performance can be obtained from the damping ratio and undamped natural frequency of the dominant poles (Farhan A. Salem, *2013*) .

The approximation conditions ;(1) for dominant one first order pole: *the pole closest to the imaginary axis* is the one that tend to dominate the response. (2) For higher-order than second system, if the *real* pole is *five* time-constants, 5T, *farther* to the left than the dominant poles, we assume that the system is represented by its dominant *second-order pair* of poles. Second order system e.g. given by Eq.(10), can be approximated as *one* order system, the condition for dominant one first order pole, is given by: *the pole closest to the imaginary axis* is the one that tend to dominate the response, e.g. the magnitude of β is very large , (typically if β/α > 5), this means *α* closest to *the imaginary axis*, and this second order system can be approximated as first order system with the following transfer function given by Eq.(11)

$$G(s) = K\,\frac{\alpha * \beta}{(s+\alpha)(s+\beta)} \tag{10}$$

$$G(s) = K\,\frac{\alpha}{(s+\alpha)} \tag{11}$$

Considering a third order system with *one real* root, and a *pair* of *complex conjugate roots* given by Eq.(12).

$$G(s) = \frac{K}{\left(s+\alpha\right)\left(s^2+2\xi\omega_n s+\omega_n^2\right)} \tag{12}$$

This system can be considered as consisting of two systems; first and second order systems; that it has three poles one real pole ,*at pole = α*, and two complex poles, the condition for dominant one first order pole , or two second order poles, is given below:

$$\frac{K/\omega_n^2}{\left(s+\alpha\right)} \qquad 10\alpha \ll \xi\omega_n \quad \textit{Approximated as first order system}$$

$$\frac{K/\alpha}{\left(s^2+2\xi\omega_n s+\omega_n^2\right)} \qquad \alpha \gg 10\xi\omega_n \quad \textit{Approximated as second order system}$$

Proposed PID design methodology

Proposed PID design methodology for second order systems

Based on Eqs.(6)(8), dominant features of control systems and internal characteristics of plants, a simple and easy to use PID design method is to be introduced, the proposed methodology is applied to all systems , including first and second order systems and system that can be approximated as first or second order system,

The general standard form of second order system, in terms of damping ratio ζ and undamped natural frequency ω_n is given by Eq.(13), Since PID transfer function is a second order system, it can be expressed in terms of damping ratio and undamped natural frequency as given by Eq.(6), the PID gains ; K_P, K_I, K_D, can be found in terms of plant's damping ratio and undamped natural frequency, as given by Eqs.(14)(15). Assigning proportional gain, the value of unity, K_P=1, and equating Eqs Eqs.(14)(15), to find K_I, result in Eqs.(16) that is used to find numerical values of PID gains based on plant parameters.

Testing these expressions, show that applying PID gaind calculated by Eqs.(16) *may* result in overshoot, and slow response, knowing that that performance of second order systems depends on damping ratio ζ and undamped natural frequency ω_n , where damping ratio determines how much the system oscillates as the response decays toward steady state and undamped natural frequency ω_n, determines how fast the system oscillates during any transient response (Farhan A. Salem, 2013), ω_n has a direct effect on the rise time, delay time, and settling time, therefore to speed up response and reduce (*remove*) overshot the main parameter that can be tuned is the Integral gain K_I, and seldom derivative gain K_D both given by Eqs.(17) , by multiplying each by softening factor a named ε and α.

Testing proposed PID design method, shows that the tuning range for multiplication factor ε, to result in smooth response without overshoot, is limited to ε = [0.1 : 2], it is noted that increasing the value of ε , will speed up response but will result in some oscillatory transient response without overshoot. Tuning K_D terms by multiplying it by factor α has minim effect on response curve. The proposed formulas for PID gains calculations and tuning range are given in Table 1

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \Leftrightarrow \quad G_{PID}(s) = \frac{K_D\left[s^2 + 2\xi\omega_n s + \omega_n^2\right]}{s} \qquad (13)$$

$$\omega_n^2 = \frac{K_I}{K_D} \Rightarrow K_D = \frac{K_I}{\omega_n^2} \quad (14)$$

$$2\xi\omega_n = \frac{K_P}{K_D} \Rightarrow K_D = \frac{K_P}{2\xi\omega_n} \quad (15)$$

$$\frac{K_P}{2\xi\omega_n} = \frac{K_I}{\omega_n^2} \Rightarrow \frac{1}{2\xi\omega_n} = \frac{K_I}{\omega_n^2} \Rightarrow K_I = \frac{\omega_n^2}{2\xi\omega_n} \Rightarrow K_I = \frac{\omega_n}{2\xi}$$
$$K_D = \frac{K_P}{2\xi\omega_n} \Rightarrow K_D = \frac{1}{2\xi\omega_n} \quad (16)$$
$$K_P = 1$$

$$K_P = 1 \quad , \quad K_D = \frac{1}{2\xi\omega_n} \quad , \; K_I = \frac{\omega_n}{2\xi} \quad (17)$$

Based on Eqs.(7)(8), the derived formulae for calculating PID controller gains in terms of derivate time T_D and integral time T_I , to be as given by Eq.(18), the divisor *N* is chosen in the range 2 to 20.

$$T_I = \frac{K_P}{K_I} = \frac{K_P}{\omega_n / 2\xi} = \frac{K_P 2\xi}{\omega_n} = \frac{2\xi}{\omega_n}$$

$$T_D = \frac{K_D}{K_P} = \frac{1/2\xi\omega_n}{K_P} = \frac{1}{2\xi\omega_n} = K_D$$

Table 1 Proposed formula for PID gains calculation, and softening ranges

Plant		**PID parameters**					
		K_P	**K_I**	**K_D**	**T_D**	**T_I**	**N**
ζ	**ω_n**	1	$\frac{\omega_n}{2\xi}$	$\frac{1}{2\xi\omega_n}$	$\frac{1}{2\xi\omega_n}$	$\frac{2\xi}{\omega_n}$	2 ÷ 20
Tuning limits		1	$\varepsilon\frac{\omega_n}{2\xi}, \; \varepsilon = 0.1 \div 2$	$\alpha\frac{1}{2\xi\omega_n}, \; \alpha = 0.58 \div 1.5$	$\frac{\alpha}{2\xi\omega_n}$	$\frac{2\xi}{\varepsilon\omega_n}$	

Proposed PID design methodology for first order systems

First order systems and systems that can be approximated as first order systems, are characterized, *mainly*, by time constant *T*. Time constant is a characteristic time that is used as a measure of speed of response to a step input and governs the approach to a steady-state value after a long time. The general form of first order system's transfer function in terms of time constant T, is given by Eq.(18).

Testing PID gains design based on plants time constant for different first order systems, show that, if the three PID gains (K_P, K_I, K_D) are set equal to plant's time constant, a smooth response curve without or with minimum overshoot is resulted, but for some systems, will result in response with ,relatively, big settling time, to speed up response and reduce (*remove*) overshoot only both derivative gain K_D ,and integral gain K_I are *softly* tuned, by multiplying K_D by factor α, where α =[0.1:3], and by multiplying K_I by factor ε, where ε =[0.1:2].

Based on plant's time constant the expressions listed in Table 2, are proposed to calculate PID gains in terms of time constant for first order system and tuning limits for K_D and K_I.

Based on Eqs.(7)(8), the derived formulae for calculating PID controller gains in terms of derivate time T_D and integral time T_I , to be as given by Eq.(19) equal to unity and tuned values as given in Table 2 the divisor *N* is chosen in the range 2 to 20.

$$G(s) = \frac{1}{Ts+1} \tag{18}$$

$$T_I = \frac{K_P}{K_I}, \quad T_I = \frac{T}{T} = 1 \quad , \qquad T_D = \frac{K_D}{K_P}, \quad T_I = \frac{T}{T} = 1 \tag{19}$$

Table 2 Proposed formulae for PID gains calculation for first order system, and softening ranges

Plant	**PID parameters**					
	$\mathbf{K_P}$	$\mathbf{K_I}$	$\mathbf{K_D}$	$\mathbf{T_D}$	$\mathbf{T_I}$	**N**
T	T	$\alpha * T$	$\varepsilon * T$	1	1	$2 \div 20$
Tuning limits	T	$\alpha * T$, $\alpha = 0.1 \div 3$	$\varepsilon * T$, $\varepsilon = 0.1 \div 2$	ε, $\varepsilon = 0.1 \div 2$	$1/\alpha$ $\alpha = 0.1 \div 3$	

Proposed PID design method for first order plus delay time (FOPDT) process

A large number of industrial plants can approximately be modeled by a first order plus time delay (FOPTD) (Katsuhiko Ogata, 2010)(Saeed Tavakoli et al, 2003) FOPDT models are the combination of a first-order process model with dead-time ,it transfer function is given by Eq.(20) and it response curve is shown in Figure 1, this s-shape curve with no overshoot *is called reaction curve, it* is characterized by two constants ; the delay time L, and time constant T, these two constants can be determined by drawing a tangent line at the inflection point of the s-shaped curve, and finding the intersection of the tangent line with time axis and steady state level *K*, (see Figure 1), then the transfer function of these-shaped curve can be approximated by first order system with transport lag and given by Eq.(20):

$$\frac{C(s)}{R(s)} = \frac{Ke^{-Ls}}{Ts+1} \tag{20}$$

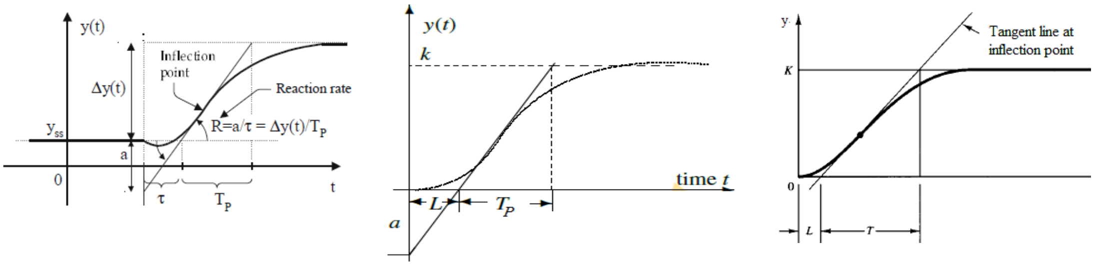


Figure 1 s-shaped curve with terminology (Farhan A. Salem, 2013)

To create a first order plus dead time model and plot corresponding step response, the InputDelay or OutputDelay properties of MATLAB built-in function tf ,can be used as follows; for system transfer function give by Eq.(21), the MATLAB Function and result is written below, result step response is shown in figure 2:

$$G(s) = \frac{Ke^{-Ls}}{Ts+1} = \frac{2e^{-0.3s}}{s+1} = e^{-0.3s}\frac{2}{s+1} \quad \Rightarrow \quad L = 0.3, \quad T = 1 \tag{21}$$

>> G = tf(2,[1 1],'InputDelay',0.3), step(G)

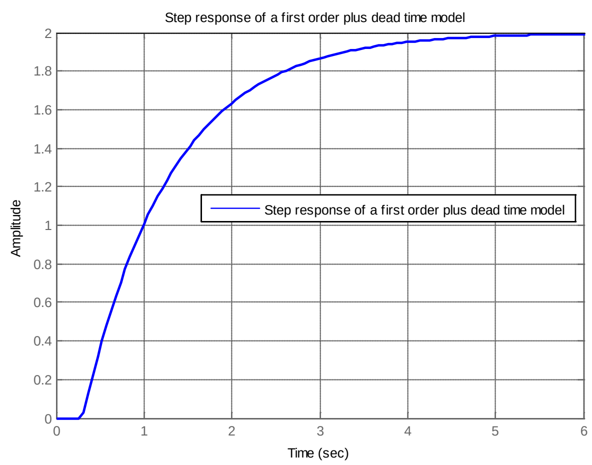


Figure 2 Step response of first order plus delay time (FOPDT) process

Based on plant's delay time L, time constant T, and steady state level K, (see Eq. (1)). The formulae listed in Table 3, are proposed to calculate PID gain in terms L, T, K for (FOPDT) process and tuning limits for K_D and K_I. Based on Eqs.(7)(8), the derived formulae for calculating PID controller gains in terms of derivate time T_D and integral time T_I , to be as given by in Table 3, the divisor N is chosen in the range 2 to 20.

Table 3 Proposed formulae for PID gains calculation for first (FOPDT) system, and softening ranges

Plant	**PID parameters**					
	K_P	**K_I**	**K_D**	**T_I**	**T_D**	**N**
T,K,L	T	$L*T$	$\frac{L}{T}$	$1/L$	$\frac{L}{T^2}$	$2 \div 20$
Tuning limits	T	$\varepsilon * L * T$, $\varepsilon = 0.1 \div 3$	$\alpha \frac{L}{T}$ $\alpha = 0.1 \div 3$	$\frac{1}{\varepsilon * \mathrm{L}}$	$\alpha \frac{L}{T^2}$	

Testing proposed PID design method

The proposed PID design method is to be tested and verified for different systems including first, second, third and fourth order systems and first-order process with dead-time, the numerical results and response curves are plotted and some are compared with other PID design methods including Ziegler-Nicols and CHR. Testing results show, for all systems, applying proposed PID design method, is resulted in smooth without overshoot response, but for some systems with, relatively, big settling time constant, to speed up the response and reduce (remove) the overshoot tuning factor for both K_D and K_I are introduced.

Testing proposed PID design method for both first order systems and first-order process with dead-time. To verify proposed PID design method, simulink model shown in Figure 3, with four different systems is built, three systems are of first order system and the fourth system is first-order process with dead-time, the calculated PID gains values applying proposed formulae are listed in Table 4, and the tuning values of derivative gain K_D, $K_{I,}$ to speed up response and reduce overshoot**,** as well as, both resulted responses for each system (calculated PID gains and after tuning K_D or K_I or both) are be plotted, and shown in Figure 4, Figure 5, and Figure 6

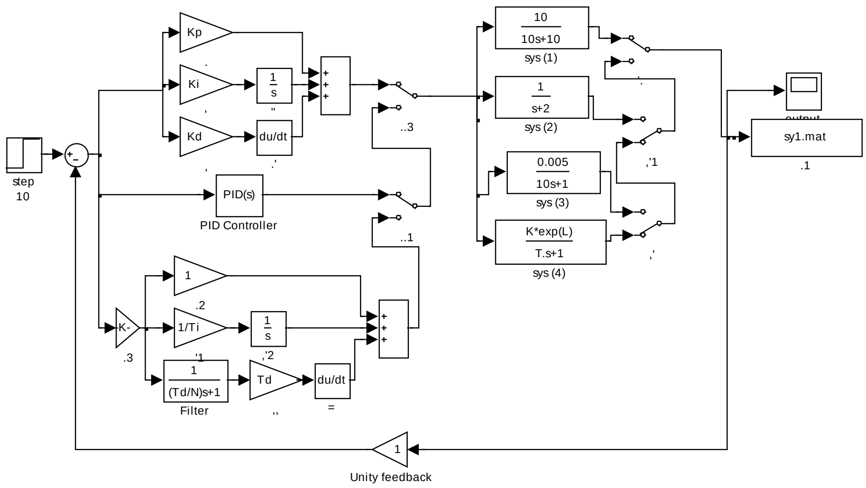
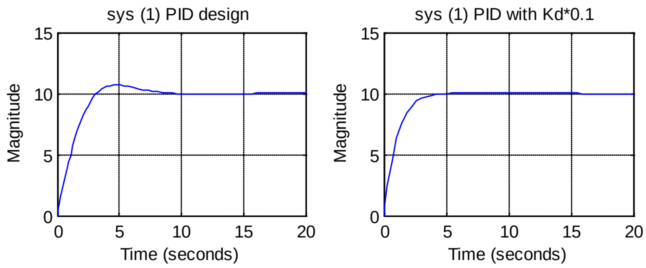


Figure 3 Simulink model of first order systems for verifying proposed PID design

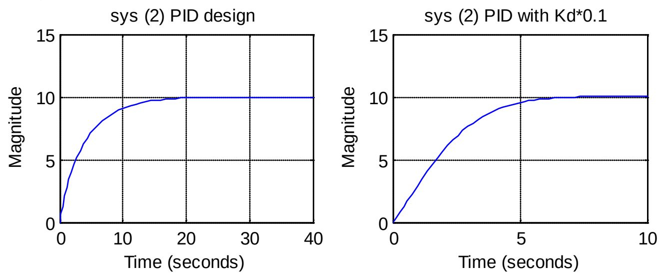
Table 4

System	**Plant's parameters**	**PID parameters**					
	Time constant T	**K_P**	**K_I**	**K_D**	**T_D**	**T_I**	**N**
Sys(1)	1	1	1	1	1	1	2
Sys(1), tuned *only* K_D value		1	0.5	1	2	1	
Sys(2)	0.5	0.5	0.5	0.5	0.5	0.5	2
Sys(2), tuned K_D and K_D value		0.5	1	1	2	0.5	2
Sys(4) FOPDT	**Parameters L, T, K**	1	0.3	0.3	3.3333	0.3000	
	L=0.3,T=1,K=1	1	0.60	0.6	6.6666	0.6	2



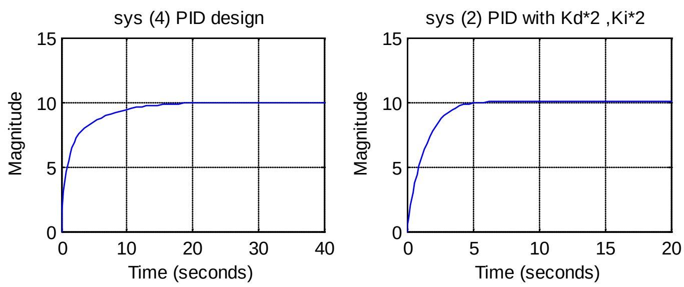
(a) Applying calculated PID gains (b) after tuning *only* Kd

Figure 4 System (1) response applying proposed PID design



(a) Applying calculated PID gains (a) after tuning K_D and K_I

Figure 5 System (2) response applying proposed PID design



(a) Applying calculated PID gains (a) after tuning K_D and K_I

Figure 6 System (4) FOPDT response applying proposed PID design

Since large number of industrial plants can approximately be modeled by a first order plus time delay (FOPTD), the proposed PID method is tested to control other three processes given by Eq.(22), the PID gins calculated are listed in Table 5, the resulted responses are shown in Figure 7(a)(b)(c) , these response curves show , a smooth responses without or with minimum overshoots are obtained.

$$G_1(s) = \frac{2e^{-0.3s}}{s+1}, \qquad G_2(s) = \frac{5e^{-s}}{1.5s+1}, \qquad G_3(s) = \frac{0.4e^{-1.8s}}{0.9s+1} \tag{22}$$

Table 5

FOPDT System	**Plant's parameters**	**PID gins**		
	T, K, L	**K_P**	**K_D**	**K_I**
G_1(s)	T=1, K=2, L=0.3	1	0.3	0.3
Tuned K_D and K_D value		1	0.9	0.45
G_2(s)	T=5, K=5, L=1.5	1.5	0.6667	1.5
Sys(2), tuned K_D and K_D value		1.5	0.3333	0.75
G_3(s)	T=0.9. K=0.4, L=1.8	0.9	2	1.62
Tuned K_D and K_D value		0.9	6	4.86

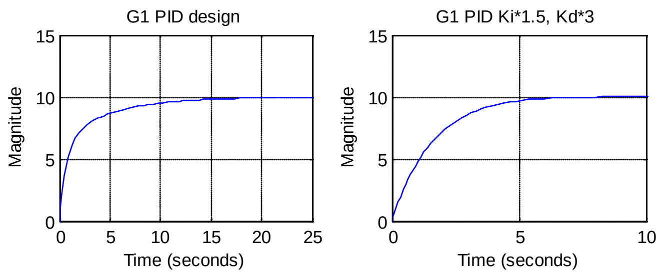


Figure 7 (a) Closed loop step response resulted from applying proposed PID parameters to G_1 (s), original calculated PID gains (left) and softly tuned (right)

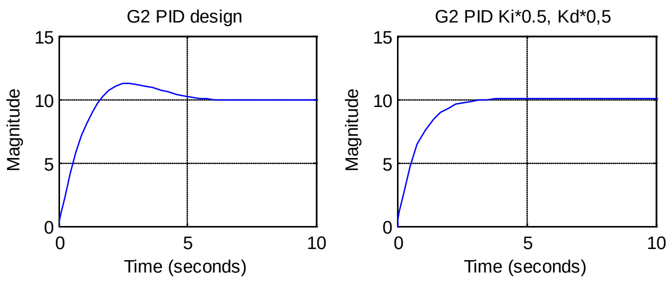


Figure 7 (b) Closed loop step response resulted from applying proposed PID parameters to G_2 (s), original calculated PID gains (left) and softly tuned (right)

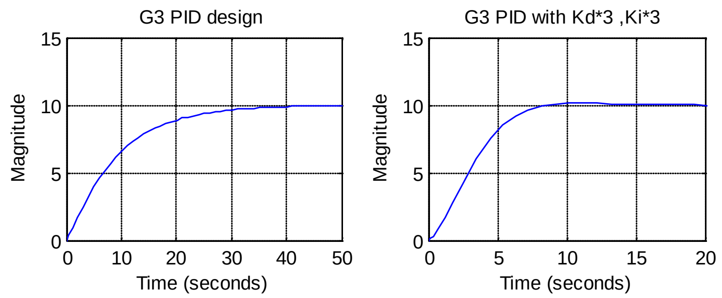


Figure 7 (c) Closed loop step response resulted from applying proposed PID parameters to G_3 (s), original calculated PID gains (left) and softly tuned (right)
Figure 7(a)(b)(b).

Testing proposed PID design method for second order systems, to test the proposed methodology, simulink model shown in Figure 8, with three different second order systems with no zeros is built**,** the calculated PID gains values applying proposed formulae are listed in table 6, as well as resulted response are be plotted, and shown in Figure 9, Figure 10, Figure 11, also the soft tuning factor to speed up response and reduce overshoot

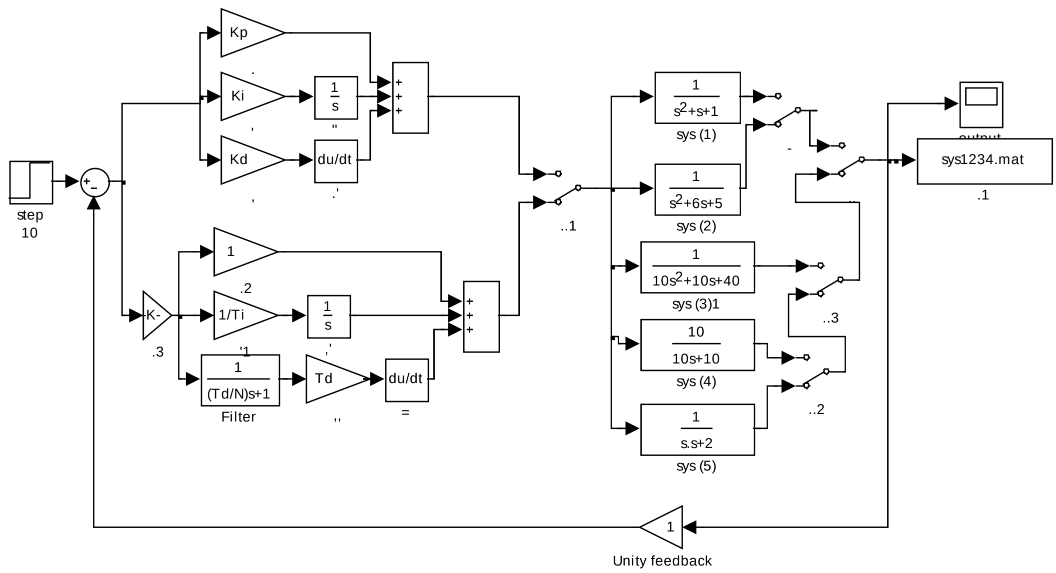
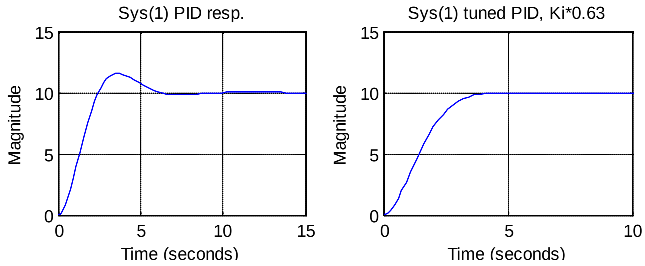


Figure 8 (b) Simulink model of second order systems for verifying proposed PID design

Table 6 testing results of second order systems

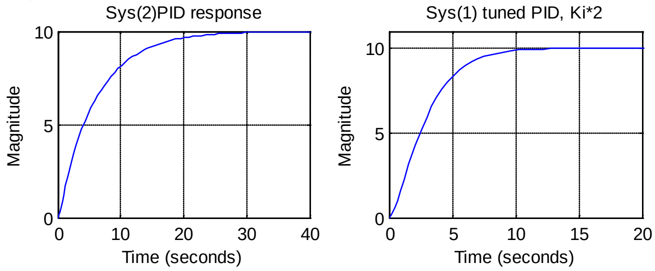
System	Plant parameter		PID parameters		
parameter	**ζ**	**ω_n**	**K_P**	**K_I**	**K_D**
Sys(1)	0.5	1	1	1	1
Sys(1), tuned *only* K_I value			1	0.6300	1
Sys(2)	1.3416	2.2361	1	0.8333	0.1667
Sys(2), tuned *only* K_I value			1	1.6667	0.1667
Sys(3)	0.2500	2	1	4	1

Sys(2), tuned *only* K_I value	1	8	1



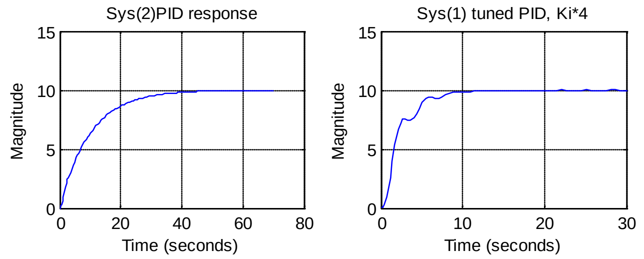
(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 9 System (1) responses applying proposed PID design



(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 10 System (2) response applying proposed PID design.



(a) Applying calculated PID gains (a) after tuning *only* KI

Figure 11 System (3) response applying proposed PID design

Testing proposed PID design method for fourth order plant given by Eq.(23), this system can be approximated as second order system with two dominant poles given by P1,2= -1,-2, and given by Eq.(24), the step response of open loop original fourth order and approximated second order systems are shown in figure 12

$$G(s) = \frac{10}{(s+1)(s+2)(s+3)(s+4)} \quad (23)$$

$$G(s) = \frac{0.8333}{(s+1)(s+2)} \quad (24)$$

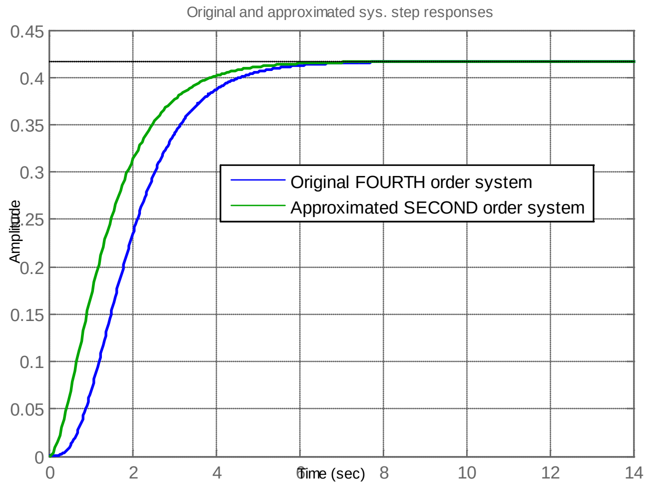


Figure 12 Dominant poles approximation

Calculating damping ratio and undamped natural frequency of approximated second order system, (ζ=1.0607, ω_n=1.4142), and applying both proposed and Ziegler-Nicols method for PID design, results in PID gains shown in Table 7, response curves of both methods when subjected to step input of 10, are shown in figure 13, the curves show, applying proposed PID design resulted in smooth response without overshoot.

Table 7

Design Method	K_P	K_I	K_D
Ziegler-Nichols	7.4274	4.8864	2.8224
Proposed method	1	0.6667	0.3333

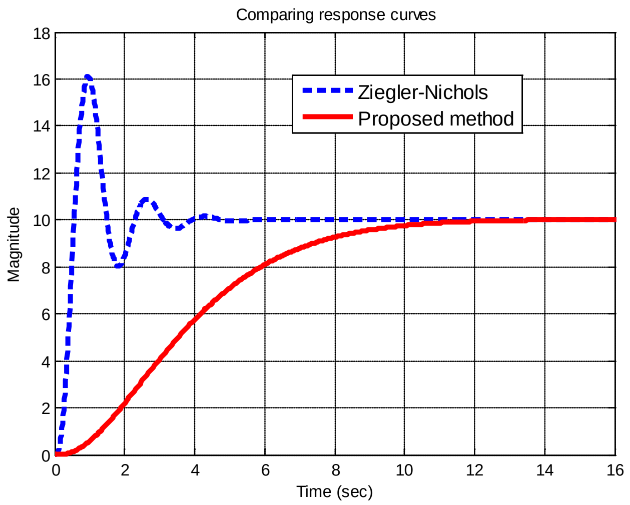


Figure 13

Testing proposed PID design method for fourth order plant transfer function given by Eq.(25), applying three different PID controller design methods, particularly, Ziegler Nichols frequency response, Ziegler-Nichols step response, and Chein-Hrones-Reswick design methods, will result in PID gains shown in Table 8 (Robert A. Paz, 2001), , as shown in this table different values of PID gain are obtained and correspondingly different system's responses (see figure 14(a)), when subjected to step input of 10. Comparing shown response curves, show that the Chein-Hrones-Reswick design is, with less overshoot and oscillation (than Ziegler-Nicols), all three method allmostly, result in the same settling time, Applying the proposed method, based on plant's dominant poles approximation, result in smooth response curve without overshoot, and zero steady state error, shown in figure 14.

$$G(s) = \frac{10000}{s^4 + 126s^3 + 2725s^2 + 12600^s + 10000} \qquad (25)$$

Table 8

Design Method	K_P	K_I	K_D
Ziegler Nichols Frequency Response	14.496	45.300	1.1597
Ziegler-Nichols Step Response	11.1524	34.3786	0.9045
Chein-Hrones-Reswick	5.5762	5.0794	0.4522
	1	0.8632	0.1231
Proposed method	ζ=3.0677	ω_n= 2.6481	

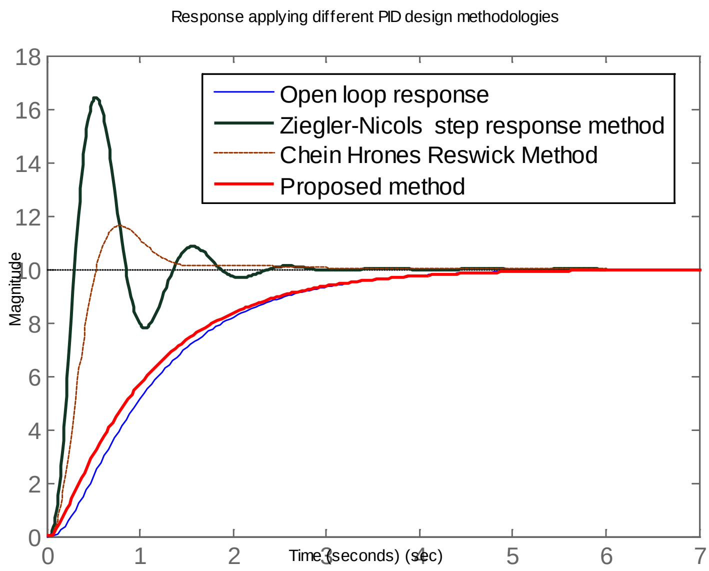


Figure 14 System step responses obtained applying different design methodologies

Testing proposed PID design method for third order plant given by Eq.(26). Approximating system to second order system with two dominant poles, calculating damping ratio and undamped natural frequency ,and applying both proposed and Ziegler-Nicols method for PID design, results PID gains shown in Table 9, response curves of both methods when subjected to step input of 10, are shown in figure 15, the curves show, applying proposed PID design resulted in smooth response without overshoot

$$G(s) = \frac{1}{(s+1)(s+3)(s+5)} \qquad (26)$$

Table 9

Design Method	K_P	K_I	K_D
Ziegler-Nichols	115.2	177.2	18.3
Proposed method	1	3.75	0.125

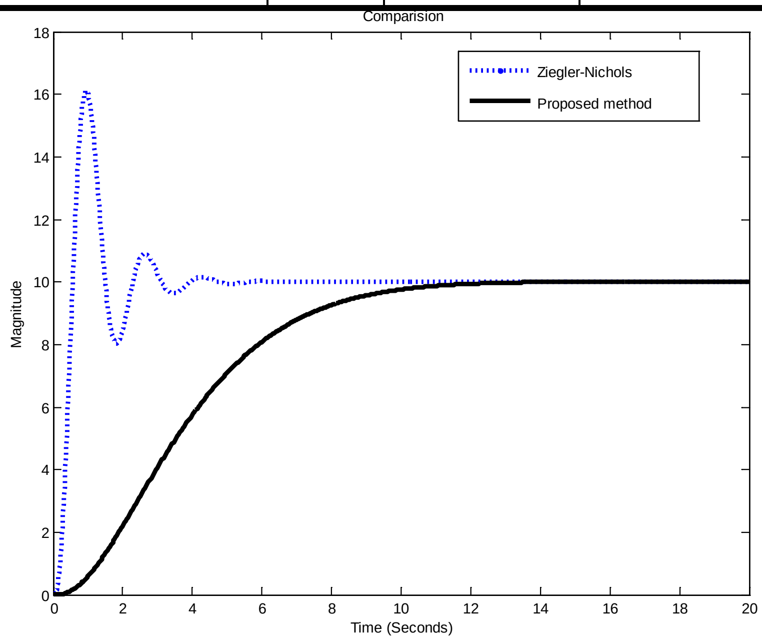


Figure 15 PID design for third order system , applying Ziegler-Nicols method and proposed method

Conclusion

A new simple and efficient model-based PID design method, based on Plant's parameters, is proposed, the proposed method was test for different systems including first, second, third and fourth order systems and first-order process with dead-time, the numerical results and response curves are plotted and some are compared with other PID design methods.

Analysis of testing and simulation results show that an important design compromise in the form of acceptable stability and medium fastness smooth and without overshoot response, is achieved, to speed up the response and reduce (remove) the overshoot, a gains tuning factor is introduced.

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