MATHEMATICAL DETERMINATION OF SUBSIDENCE BREAK-POINTS IN ABANDONED OLD UNDERMINED REGIONS IN GIS SPACE FOR PURPOSES OF PEOPLE AND PROPERTIES PROTECTION

Vladimir Sedlak, PhD, Prof. Ing.
University of Security Management in Kosice, Slovakia

Abstract

For the environment protection, protection of people lives and their properties especially, it is necessary to recognize the mining subsidence, i.e. the fall depressions which are developed above mined out underground spaces in the earth surface as a natural continuation of dynamic tectonic processes. It is necessary primarily in the abandoned old undermined regions, where in a case of not protected entrance of persons there is increased danger of personal accidents and human lives threat and damage or destruction of properties. The limits of undermined regions in many cases equal to isolines connected so called break-points occurred in the front of the subsidence borders. The theory for the estimation of polynomial break-points in the case of subsidence analysis is presented. The theory was developed as a part of the kinematics analysis procedures for the evaluation of the magnesite mineral deposit Košice-Bankov, in the east of Slovakia. Some numerical and graphical results from the break-points estimation in the magnesite mineral deposit Košice-Bankov are presented. The obtained results are transformed into GIS in for a purpose of the environment protection.

Keywords: Mining, break-points, subsidence, mathematical hypothesis, GIS

Introduction

On the present in accretive exigencies to people and its property protection, there is security one from priority needs and tasks of each state, states configuration or worldwide. Security at the people and property protection cannot be known only as a policy or militant tool (Kazanský 2011, Kříha 2011, Pána 2012, Svatoš, 2012). In the environment protection, which an unspoiled ecosystem is a condition of human living, it is needed to protect people and its property against the negative industrial influences. The mining activity influence on the environment belongs to these negative industrial influences.

The gradual subsidence development at the Košice-Bankov mine region in Slovakia was monitored by geodetic measurements from the beginning of mine underground activities in the magnesite mineral deposit. The analysis of time factor of the gradual subsidence development continuing with underground exploitation allows production of more exact model situations in each separate subsidence processes and especially, it provides an upper degree in a prevention of deformations in the earth surface (Sedlák 1997, Staňková and Černota 2010a,b). Possibility in improving polynomial modelling subsidence is conditioned by the knowledge to detect position of so-called *break-points*, i.e. the points in the earth surface in which the subsidence border with a zone of breaches and bursts start to develop over the mineral deposit exploitation. It means that the break-points determine a place of the subsidence where it occurs to the expressive fracture of the earth continuous surface consistence.

Research overview

Problems of mine damages in the earth surface, dependent on the underground mine activities at the magnesite mineral deposit, did not receive a systematic research attention in Slovakia till 1976. After there, the requirements for a scientific motivation in the subsidence development following out from rising exploitations and from introducing progressive mine technologies were taken in consideration.

The monitoring station Košice-Bankov is one part from the large monitoring station for deformation measurement in Košice Depression which is aimed for the monitoring recent geotectonic movements (Košice-city) and landslides (Košice-Košická Nová Ves) and mine subsidence (Košice-Bankov) (*Fig.1 and 2*). 3D data were firstly observed by terrestrial geodetic technology (since 1976) and later by GPS (since 1997).

Fig.1: *The monitoring station: Košice Depression.*

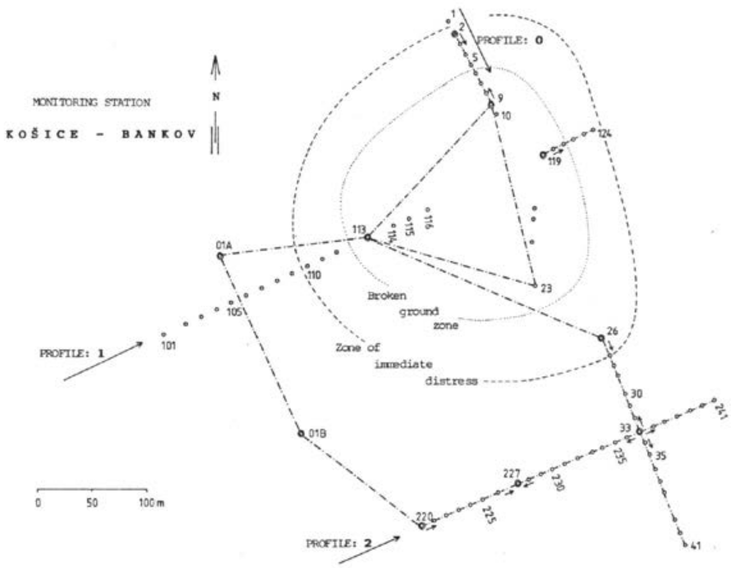

Fig.2: *The monitoring station: Košice-Bankov.*

1D Deformation analysis from levelling networks

In accordance with the general phases of the geodetic deformation analysis the project at hand was defined to contain the following phases:

1. Single epoch evaluation of the levelling data available.
2. Stability evaluation of reference benchmarks.
3. Estimation of the most likely deformation model.

The single epoch evaluation concentrates on the evaluation of the functional model, the observational data and the stochastic model. By means of the integration of hypothesis testing, including outliner detection and variance component estimation a consistent mathematical model is obtained. In the second phase of the project the assumption in the functional model of stable reference benchmarks is tested. Unstable benchmarks are removed from the set and will further be treated as objective-points. After establishing the correct functional model, the stochastic model may be improved as well. Again, we obtain a consistent mathematical model results.

To arrive at the most likely mathematical model describing the deformation pattern underlying the data is the aim of the third phase. The functional model part is restricted to 1D and 2D- and 4D-polynomials. The mathematical model is again balanced by modifications of the stochastic model.

Polynomial break-points

In the project described the third step consists again of three different steps, i.e.:

1. Estimation of 1D-polynomial model per benchmark.
2. Estimation of 3D-polynomial model per selection benchmarks.
3. Evaluation of possible external height-information available.

When evaluating the estimated time-dependent polynomials per benchmark it become more and more apparent, that such a polynomial could not accurately describe the behaviour of these benchmarks which came under the influence of the mineral deposit extraction some time after the start of the exploration. Such behaviour was described by higher order polynomials, whereas it was actually due to a break in the trend of the subsidence. Allowing the polynomial function to have a so-called *break-point*, which is defined as, may solve this problem, which is defined: *A point in time at which a benchmark, due to the mineral deposit extraction, enters the subsidence area.*

The estimation of polynomial break-points is a part of the procedure developed to establish the most likely mathematical model, describing the subsidence behaviour of a specific benchmark in time. The procedure is based on the concept of least-squares estimation and multiple hypothesis testing.

Hypothesis testing

In general, the mathematical model under null-hypothesis may be modelled in terms of observation equations (Sedlák, 1997, 1998, 2007)

$$H_o : E\{\underline{y}\} = \boldsymbol{A}.\boldsymbol{x}; \qquad D\{\underline{y}\} = \boldsymbol{Q}_y \,, \tag{1}$$

where $E\{.\}$ is mathematical expectation; $\boldsymbol{y}$ is m-by-1 vector of observations; $\boldsymbol{A}$ is m-by-n design matrix; $\boldsymbol{x}$ is n-by 1 vector of unknowns; $D\{.\}$ is mathematical dispersion; $\boldsymbol{Q}_y$ is m-by-m variance covariance matrix of the observations; and underlined stands for stochastic. Moreover, m equals the number of observations and n the number of unknowns.

The validity of the null-hypothesis may be tested against the widest possible alternative hypothesis, by means of the test-statistic

$$\boldsymbol{T} = \hat{\boldsymbol{e}}^T.\boldsymbol{O}_y^{\ 1}.\hat{\boldsymbol{e}} \,, \tag{2}$$

where $\hat{\boldsymbol{e}}$ is m-by-1 vector of least-squares corrections of the observations.

In case of a rejection of the null-hypothesis, one will try to detect the cause of rejection by formulating a (number of) possible alternative hypothesis. In general, the model under the alternative hypothesis may be written as a linear extension of the model under the null-hypothesis

$$H_o : E\{\underline{y}\} = \boldsymbol{A}.\boldsymbol{x} + \boldsymbol{C}.\boldsymbol{L}; \qquad D\{\underline{y}\} = \boldsymbol{Q}_y \,, \tag{3}$$

where $\boldsymbol{C}$ is m-by-q matrix; $\boldsymbol{L}$ is q-by-1 vector; and $\boldsymbol{CL}$ describes the assumed model error. The dimension of the linear extension of the functional model q may vary from $g = 1$ to $q = m - n$.

The validity of the alternative hypothesis may be tested by the test-statistic

$$\boldsymbol{T}_q = \hat{\boldsymbol{e}}^T.\boldsymbol{Q}_y^{\ 1}.\boldsymbol{C}.\left[\boldsymbol{C}^T.\boldsymbol{Q}_y^{\ 1}.\boldsymbol{Q}_{\hat{e}}.\boldsymbol{Q}_y^{\ 1}.\boldsymbol{C}\right]^{\ 1}.\boldsymbol{C}^T.\boldsymbol{Q}_y^{\ 1}.\hat{\boldsymbol{e}} \,, \tag{4}$$

in which $\boldsymbol{Q}_{\hat{e}}$ is the covariance matrix of the least-squares residuals. Under the null-hypothesis the test-statistic $\boldsymbol{T}_q$ has a central distribution χ^2 with q degrees of freedom, i.e. $\chi^2\ (q,0)$.

If $q = 1$ the $\boldsymbol{C}$-matrix reduces to a m-by-1 vector $\boldsymbol{c}$, and vector $\boldsymbol{L}$ reduces to a scalar, causing Eq.(4) to reduce to

$$\boldsymbol{T}_1 = \left(\boldsymbol{c}^T.\boldsymbol{Q}_y^{\ 1}.\hat{\boldsymbol{e}}\right)^2.\left(\boldsymbol{c}^T.\boldsymbol{Q}_y^{\ 1}.\boldsymbol{Q}_{\hat{e}}.\boldsymbol{Q}_y^{\ 1}.\boldsymbol{c}\right)^{\ 1} , \tag{5}$$

which is described as $\chi^2(1,0)$ under the null-hypothesis. A well-known application of Eq.(5) is found in the method of data-snooping, where the data are checked for possible measurement errors by computing the so-called conventional alternative hypotheses. These hypotheses are of the form $\boldsymbol{c}_i^T =$ $[0...010...0]$,in which 1 is found at the position j.

In the Košice-Bankov case of estimation and testing it is custom to compute, next to the overall model test all test-statistics under indication w-

test-statistic for the conventional alternative hypotheses. In the present paper we will use all three types of tests, Eq.(2),(4) and (5).

The mathematical model under $\mathbf{H_o}$

Given benchmark, its height at the various epochs as computed after the stability analysis of the reference benchmarks from, together with their covariance matrix, the starting point for the evaluation of the benchmarks subsidence behaviour. The general form of 1D time-dependent polynomials of order n for the benchmarks heights is given as

$$H_k = a_o t_k^o + a_1 t_k^1 + a_2 t_k^2 + \ldots + a_n t_k^n, \qquad (6)$$

where H_k is height of the benchmark as determined at epoch k; a_i is unknown coefficient, $i = 0,\ldots,n$; t_k^i is measurement time of epoch k to the power i.

The mathematical model under the null-hypothesis assumes a linear subsidence, which is represented by a time dependent polynomial of order 1. The assumption is based on the fact that in the Košice-Bankov case a large number of benchmarks show a natural, linear subsidence.

Alternative hypotheses considered

The assumptions are:

1. The polynomial order before the break-point is restricted to a maximum of one ($n_1 \leq 1$), which is also the case under the null-hypothesis. This assumption is based on the fact that a possibly natural subsidence in the Košice-Bankov case shows at the most a linear behaviour.
2. The polynomial order before the break-point does not exceed the polynomial order after the break-point, i.e. $n_2 \geq n_1$.
3. The function is required to be continuous in its break-point, meaning that the function values of both polynomials before and after the break-point should be the same.

Results of testing for polynomial break-points

Identification of a polynomial break-points

The aim of the procedure is to arrive at a consistent mathematical model, i.e. both the functional and the statistic model. In short the procedure is as follows. First a least-squares adjustment of the mathematical model under the null-hypothesis is performed. The validity of this model is tested by the application of the overall model test, given in Eq.(2).

Depending on the test result, the next steps are following:

1. Accept H_o*:* The estimated slope-coefficient (a_1) is tested for its significance. If the parameter is significant, the functional model is replaced by a constant polynomial with implies stability of the benchmark considered.

2. Reject H_o*:* Test all alternative hypotheses as described above for their validity and determine the most likely alternative hypothesis.

Depending on the most likely hypothesis selected, the following actions are taken:

a) w-test: Remove the observation concerned, i.e. the benchmark height at the epoch which was identified by the largest *w-test* value.

b) 01- or 02-test: Adapt the mathematical model under the selected alternative hypothesis to be the new mathematical model under the null-hypothesis. Possibly more parameters are needed to describe the benchmarks behaviour accurately. Hence, the null-hypothesis is again tested for its validity. In case of the rejection the alternative hypotheses mentioned before are once more tested.

c) B-test: Adapt a break-point at the epoch which was identified by the largest *B-test* value. The order of the polynomial before and after the break-point is now determined for each part separately.

First consider the case where the dimensions of the hypotheses considered are equal. In our procedure this occurs when all w*-tests* or when all B*-tests* are compared. Since those test-statistics T^i are all of the form of Eq.(5) and thus all have the same central distribution with one degree of freedom, i.e.

$$T^i \approx \chi^2(1,0)\ \forall\ i \qquad (7)$$

and the largest value implies the most likely alternative hypothesis. Hence, in this case the most likely alternative hypothesis is the one for which

$$T^i > T^j\ \forall\ j \neq i\,, \qquad (8)$$

where the indices *i* and *j* refer to hypothesis i and j respectively.

However, at a certain point in the procedure the most likely alternative hypothesis should be selected from a number of hypotheses with different dimensions. This is the case when it is necessary to discriminate between, for instance, the 01- and 02*-tests*. Although the related test-statistics χ^2 are again all χ^2 distributed, the number of degrees of freedom differs, i.e. we compare test-statistics of the form of Eq.(5) with test-statistics of the form of Eq.(4). Therefore the largest value does not automatically refer to the most likely alternative hypothesis.

In order to deal with this problem in the present case, a practical solution may be found, comparing the test quotients, which are defined as $T_q^i / \chi_\alpha^2(q_i,0)$, where T_q^i is test-statistics of the form of Eq.(4), referring to the i-the alternative hypothesis; $\chi_\alpha^2(q_i,0)$ is a critical value $(l = 5\%)$ of the central χ^2 distribution with q_i degrees of freedom for a certain choice of a_i.

Here it should be noted that the test quotients might only be used if the significance levels a_i of the tests involved are matched through an equal power. Those test quotients that are less than 1 are not taken into account, since the hypothesis in question is certainly not more likely than the null-hypothesis. For the order test quotients it is assumed that the most likely alternative hypothesis is the one, which is rejected strongest, i.e. differs most from 1. Hence, the most likely hypothesis is the one for which $T_q^i / \chi_\alpha^2(q_i, 0) >$ $T_q^j / \chi_\alpha^2(q_j, 0) \forall\ j \neq i$.

Results in the Košice-Bankov case

It will be clear that both polynomials with and without a break-point may result from the procedure described in the previous paragraph. In this section examples of estimated polynomials in the Košice-Bankov case are presented and discussed. In the following the test quotient belonging to the overall model test is denoted by OM *-test* (Overall Model test).

- *Benchmark No.8*: The behaviour of this benchmark caused the original null-hypothesis to be rejected. The validation of the alternative hypotheses, as specified before identified an extra parameter for the polynomial to be the most likely alternative hypothesis *(Tab.1).* After the adaptation of this alternative hypothesis as the new null-hypothesis, the overall model test value became 0.9733, which is clearly smaller than its critical value of 1.5479 (the significance level of α=5% to derive deviation mean height values). Hence a quadratic polynomial model was accepted.
- *Benchmark No.109* (*Fig.3*): This benchmark is a typical example of the break-point estimation (see table of test-quotients) at the point in time of 1986 (autumn). After adapting the model including a polynomial break-point as the null-hypothesis, the order of the polynomial after the break-point was determined to be of the order two.
- *Benchmark No.112* (*Fig.4*): This benchmark is a clear break-point. The null-hypothesis with the polynomial determined to be of order two can be again considered of the null-hypothesis in time of 1986 $\div$ 1988. And the polynomial is determined to be of the order three after time of 1988.
- *Benchmark No.113*: For this benchmark the original null-hypothesis, assuming a linear subsidence, was accepted. The overall model test-statistics was determined to be of 0.4681 which is clearly smaller than the critical value of 0.8497. However, the first epoch (spring 1986) was considered as a break-point possibility. And the alternative hypothesis after the break-point was accepted as the polynomial of order two.

Fig.5 shows panoramic view to the subsidence with some places of the break-points in Košice-Bankov.

Tab.1: *Test-quotient overview.*

Point No.	QUOTIENTS *w -test*	*OM -test*	*B -test*	*01 -test*	*02 -test*	BREAK-POINT *[%]*
8	1.8247	0.7791	1.9952	2.1890	1.5206	**0**
109	7.6910	2.2386	7.7961	4.3813	5.1463	**100**
112	6.1754	2.0023	7.0129	4.1992	4.9026	**100**
113	6.0701	1.9077	6.5099	4.0558	4.2159	**70**

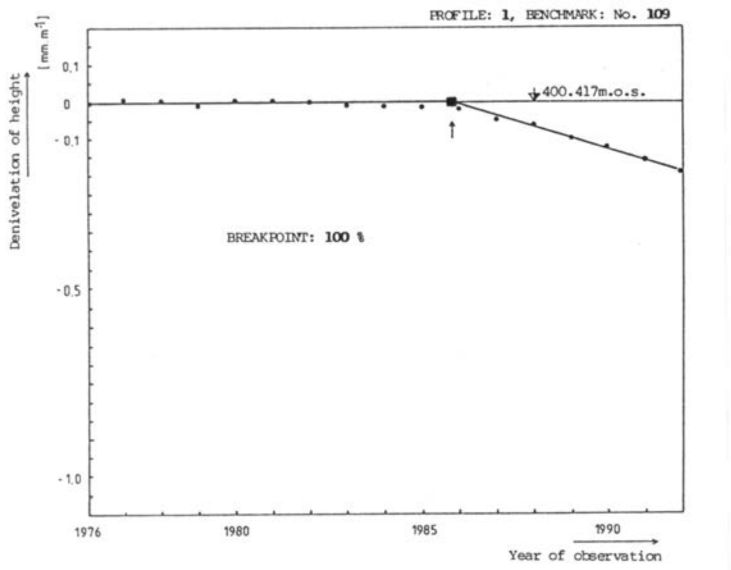

Fig.3: *Polynomial model - Benchmark No.109*

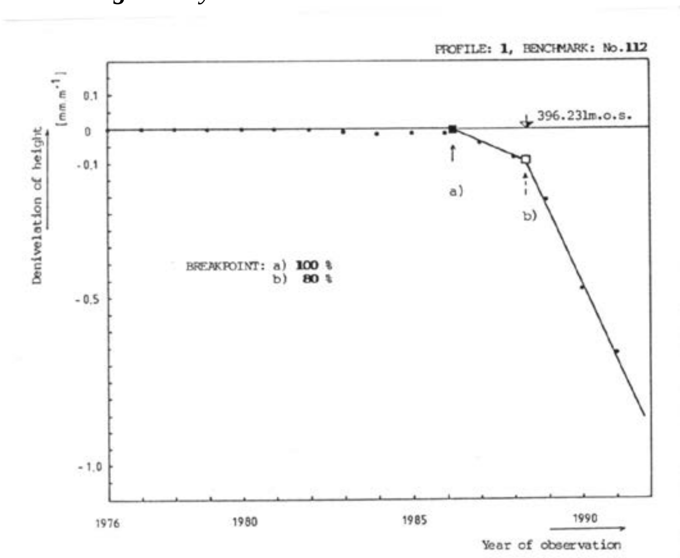

Fig.4: *Polynomial model - Benchmark No.1112.*

Fig.5: *The mine subsidence: Košice-Bankov (panoramic view).*

GIS applications

GIS of interested area is based on the next decision points:

- Basic and easy data presentation;
- Basic database administration;
- Wide information availability.

The best viable solution is to execute GIS project as the Free Open Source application available on Internet. The general facility feature is free code and data source viability through the HTTP and FTP protocol located on the project web pages. Inter among others features range simple control, data and information accessibility, centralized system configuration, modular stuff and any OS platform (depends on PHP, MySQL and ArcIMS port) (Sedlák 2000, 2005; 2007).

Network based application MySQL is in a present time the most preferred database system on Internet. It is because, that MySQL company is a member of Open Source (based on GPL license), the price of this product is less than the prices of others commercial databases (i.e. Oracle, MS SQL Server, etc.), it has high-speed responses, uses fast data storing (in a binary file up to 1 TB - in 1 single file, supports unlimited quantity of s data files) etc. This database is relational database with relational structure and supports SQL language. At the present time MySQL 4.0 is released and supports transaction data processing, full text searching and procedure executing. PHP, which stands for "PHP: Hypertext Pre-processor" is a widely used Open Source general purpose scripting language that is especially suited for Web development and can be embedded into HTML. Its syntax draws upon C, Java, and Perl, and is easy to learn. The main goal of the language is to allow web developers to write dynamically generated web pages quickly, but you can do much more with PHP.

Database part of GIS for the mine subsidence Košice-Bankov application runs on MySQL database, because it is free distributed for non-commercial projects (*Fig.6*). PHP supports native connections to many databases, for example MySQL, MSSQL, Oracle, Sybase, AdabasD, PostgreSql, mSQL, Solid, Informix. PHP supports also older database systems: DBM, dBase, FilePro ...PHP can communicate with databases with ODBC interface and this feature represents PHP to work with desktop applications supporting ODBC interface. PHP cans attend to another Internet services, because includes dynamics libraries of some Internet protocols (i.e. HTTP, FTP, POP3, SMTP, LDAP, SNMP, NNTP, etc.).

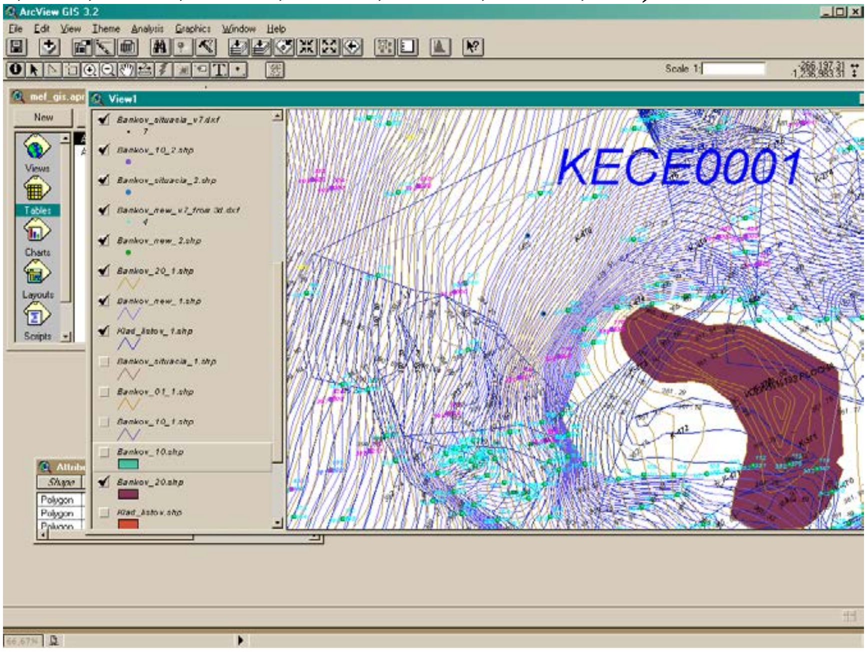

Fig.6: *The mine subsidence: Košice-Bankov (GIS-MySQL).*

Conclusions

The examples of chosen benchmarks taken from the monitoring Košice-Bankov station can give an overview of some resulting polynomial models, representing trends in the deformation developments over an extracted mine space. The presented theory of the estimating the subsidence polynomial break-points follows out from a consideration of 1D deformation model of monitoring points. Similar 3D deformation model analysis at the polynomial break-points can be taken into consideration. It will be the subject of a future research of the estimated differential polynomial points in the subsidence.

The presupposed possible subsidence development in the Košice-Bankov magnesite mine is not confirmed. The Košice-Bankov magnesite mine is abandoned since 1990. 3D terrain model of the subsidence in this mine region is expressed in MySQL database. MySQL is very convenient graphical software for many applications into GIS where land surveying, mine surveying and other geodetic data are occurred.

References:

KAZANSKÝ,R., 2011: *Bezpečnostná politika - teória konfliktov*. (in Slovak). [Security politics – theory of conflicts]. 1st ed.. Banská Bystrica: Matej Bell University in Banská Bystrica, 2011, ISBN 978-80-557-0250-6.

Kriha,J., 2011: *Ochrana místních záležitostí veřejného pořádku ve světle teritoriálních souvislostí – aktuální edukační přiležitosti a výzvy II – monitoring stavu a dynamiky dílčí součinnostní veřejnopořadkové praxe*. (In Czech). [Protection of local public order in light of the territorial context - the current educational opportunities and challenges II - monitoring the state and dynamics of parts synergy of public order practice]. In: Auspicia, No.2/2011, pp.230-232, 2011, ISSN 1214-4967.

PÁNA,L.,2012: *Various different looks at global problems*. In: Kremenchuk Mykhailo Ostrohradskij National University, pp.151-153, 2012, ISSN: 2227-3549.

SEDLÁK,V., 1997: *Modelling subsidence deformations at the Slovak coalfields*. In: Kuwait Journal of Science & Engineering, ISSN 1024-8684, 1997, Vol.24, No.2, 339-349.

SEDLÁK,V., 1998: *Measurement and Prediction of Land Subsidence Above Long-wall Coal Mines, Slovakia.* Belmont: James W.Borchers, U.S. Geological Survey, (ed.): Land Subsidence/Case Studies and Current Research, 1998, 576pp. ISBN 0-89863-197-1.

SEDLÁK,V., 2000: *GPS measurement of geo-tectonic recent movements in East Slovakia*. In: Ravenna, C.N.R. (ed.): Proceedings: 6th Int. Symp. On Land Subsidence "SISOLS 2000 – Land Subsidence", Vol.II., 24-29. Sep. 2000, Ravena, p.139-150. ISBN 88-8722-06-1.

SEDLÁK,V.: 2005: *Possibilities at modelling surface movements in GIS in the Košice Depression, Slovakia*. In: RMZ, Materials and Geoenvironment. ISSN 1408-7073, 2005, Vol.51, No.4. p.2127-2133.

SEDLÁK,V., 2007: *Determining mine subsidence break-points for GIS in undermined regions*. In: Budapest, ISM (ed.): Proceedings of the XIII. International Congress of International Society for Mine Surveying, 24-28 September, 2007, Budapest, p.001/6. ISBN 978-963-9038-18-9.

STAŇKOVÁ,H. & ČERNOTA,P., 2010(a): *A principle of forming and developing geodetic bases in the Czech Republic*.. In: Technica, Geodesy and Cartography 2010, ISSN 1392-1541, Vol.36, No.3 pp.103-112, print / ISSN 1648-3502 online.

STAŇKOVÁ,H. & ČERNOTA,P., 2010(b): *Spatial transformation between S-JTSK and ETRS89 systems*. In: Ostrava, VŠB-TU Ostrava (ed.): VŠB – Technical University of Ostrava Annual Proceedings of Science and Technology at VŠB-TUO, IV(2010), pp.174-179, Ostrava 2010, ISBN: 978-80-248-2367-6.

SVATOŠ,R., 2012: *Kriminologie*. (in Czech). [Criminology]. 1st ed., Plzeň: Aleš Čeněk, s.r.o., 2012, ISBN 978-80-7380-389-6.