DEFAULT AND FRAGILITY IN THE PAYMENT SYSTEM

Paula Lourdes Hernandez Verme
Departamento de Economía y Finanzas, Universidad de Guanajuato, Mexico

Abstract:

I present a model of the payment system where agents may choose whether or not to default on debt. Our model also features debt-financed purchases of goods, unsecured debt cleared through third parties, and debt settlement requiring final payment using fiat money. I compare the merits of three alternative settlement rules: a strict net settlement rule, a net settlement rule with debt forgiveness and a gross settlement rule. I find that net settlement is superior to gross settlement and that only a net settlement rule with debt forgiveness gives the correct incentives for a unique, stable and optimal stationary equilibrium.

Key Words: Default, payments system

Resumen:

Presentamos un modelo de sistema de pagos donde los agentes pueden elegir si desean o no dejar de pagar la deuda. Nuestro modelo también cuenta con compras de bienes financiadas por deuda, la deuda no se garantiza a través de terceros, y el pago de la deuda requiere un pago final usando dinero fiduciario. Compararemos los méritos de tres reglas de solución alternativas: una regla de liquidación neta estricta, una regla de liquidación neta con perdón de deuda y una regla de liquidación bruta. Encontraremos que la liquidación neta es superior a la de liquidación bruta y que sólo una regla de liquidación neta con perdón de deuda da los incentivos correctos para un equilibrio único, estable y óptimo.

Introduction

What are the optimal rules for the settlement of debt when debtors may choose to default? Gross settlement? Net settlement? Or net settlement with debt forgiveness in times of crisis? Which of these settlement rules generates stationary equilibria that are unique, stable, and optimal?

This is not an obscure operational question. In modern economies, most personal and business purchases and all financial transactions are conducted with debt, from credit cards and personal checks to large institutional wire transfers, not with fiat money. In general, these transactions are cleared and settled in central clearinghouses and all go through the Central Bank system. In the U.S., CHIPS and Fedwire are the largest of these central clearinghouses, and all their transactions go through the Federal Reserve System in one way or another. In particular, every day, Fedwire and CHIPS together settle close to $4 trillion in debt (if not more,) with only $47.6 billion in pre-funding (transferable deposits.) The latter implies a velocity of 228.57, even though CHIPS argues that its own velocity is over 500[39]. The absolute and relative scale of these transactions, together with the high

[39] Another helpful point of comparison is against the value of economic activity: the annual GDP in the U.S is close to $13 trillion, and thus, the value of the GDP would be exhausted in only 3.25 days by transactions in the payments system.

level of coordination observed among banks[40], mean that a problem that starts and/or affects a member of the Payment System could be eventually transmitted through associated banks, generating a liquidity crisis in the system that could lead to systemic failure. Under such circumstances, the rest of the economy would be affected significantly as well, producing chaos as a byproduct that could only worsen the problem.

The importance of a healthy Payment System, particularly in the U.S., can be seen in the essential role of the elastic provision of intraday liquidity by the Federal Reserve System in times of crisis, as for example in the Crash of October 1987 and September 11th of 2001. Clearly, without a well defined payments system through which implement the so much needed monetary policy, the effects of such crises would have been even more devastating.

Previous theoretic analysis of settlement issues often presumed perfect enforcement [e.g., Freeman (1996), Lacker (1997)] or exogenous default probabilities[41] [Freeman (1999), Fujiki, Green and Yamazaki (1999), Freixas, Parigi and Rochet (2000)]. Rochet and Tirole (1996), Angelini (1998), Kahn and Roberds (2001) take an additional step and study the moral hazard introduced by the provision of credit during the settlement process –agents whose assets are accepted as payment will acquire riskier assets than is optimal. In this important line of research agents do not choose whether to default; default occurs only when forced by low asset returns.

As Zhou (2000) has argued persuasively, these are good but not yet sufficient guides for settlement policy. Without modeling the decision of whether to default or not, we cannot discuss the effect of settlement rules on incentives to default and how default decisions may be strategically interdependent and may thus threaten the stability of the payments system. What induces the repayment of debt, and how does the repayment decision depend on the actions of others? How are these decisions affected by the rules of settlement?

We take up Zhou's challenge and study the settlement risk of non secure debt contracts where delivery versus payment (DVP) is not possible, and agents may choose nontrivially not to deliver promised payments, especially if they believe that others will do likewise. In this form of settlement risk, agents who suspect that they will not be able to collect on IOUs receivable may have less reason to repay IOUs payable, a strategic complementarity that may lead to the instability and multiplicity of equilibria and even the collapse of the payments system. Most of the previous work[42] has emphasized the disadvantages of Net Settlement, and has implicitly assumed that all transactions in Gross Settlement were DVP. It is our intention to demonstrate the clear advantages of Net Settlement in the absence of DVP.

To address these questions, we build on a basic payments model [Freeman (1996)], which features as implications debt-financed purchases of goods, debt cleared through third parties (central clearinghouses,) and debt settlement requiring final payment using fiat money. In this setup, there is incomplete contract enforcement: there is no collateral and DVP is not possible, which leads to increased counterparty risk. To this we add a nontrivial default option: after issuing and receiving debt, agents may choose whether to relocate inside or outside of the reach of the collection efforts, an action that relieves them of the need to repay debts, but which limits their own collection of debts receivable and may entail other costs. The inherently intertwined nature of the repayment and collection of debt introduces potential strategic complementarities. In particular, default is an equilibrium outcome and it may be optimal or not, depending on the settlement rule in place.

[40] According to McAndrews and Potter (2002), 63% to 76% of banks' payments depend on receipts from other banks.
[41] It can be argued also that in this collection of papers, there is a temporary failure to repay that is fixed later in the period, and not default in the strictest of terms.
[42] See Kahn, McAndrews and Roberds (2003.)

Using this framework we evaluate the welfare and stability properties of three settlement rules: gross settlement, strict net settlement (with unwinding), and net settlement with debt forgiveness in times of crisis. Major implications of the model include the superiority of net settlement to gross settlement, the instability of both gross and strict net settlement, and the optimality and stability of net settlement with debt forgiveness. It is shown that debt forgiveness -reducing the obligations of those who repay debt when others are defaulting- is a way to eliminate the strategic complementarity underlying the potential instability of the payments system.

Kiyotaki and Moore (1997) studied strategic complementarity in interrelated local credit markets with no central clearing market. Camera and Li (2008) studies similar strategic complementarity and multiplicity of equilibria in intermediated credit markets, but again without our emphasis on a central payments system[43]. Even closer to our payments aims is Kahn, McAndrews, and Roberds (2003). Our work differs in its modeling of fiat money as the payments instrument[44], its proposal of net settlement with debt forgiveness, which works without collateral and is optimal when defaults occur, and its study of local stability of equilibria.

The Payment System in Practice

Figure 1 describes how a payment order is processed, and then cleared and settled in the absence of DVP. The issuer holds a monetary obligation (IOU payable) to be paid. She then sends her order for this obligation to be paid, which is processed by her own bank and, thus, through the central clearinghouse. The clearing process starts after the payment order is sent to the clearing house, and most generally follows three steps: transmission, reconciliations of the records with the payment order, and confirmation. Immediately after, the settlement process starts, involving generally transfers of funds or transfers of financial instruments. Finally, after both clearing and settlement, the obligation is discharged.

The alternative rules for the settlement of debt together with its finality play a key role in the determination of the overall functioning of the different Payment Systems. With respect to the rules of settlement, the three that are most commonly used today are Gross Settlement (GS), Net Settlement (NS) and Hybrid rules that combine both Gross and Net settlement rules. Gross rules settle obligations individually, while Net rules settle obligations on a net basis (it could be bilateral netting, but the most commonly used is multilateral netting.) With respect to Settlement Finality, Figure 2 shows the most common schemes.

In practice, there are many combinations of settlement and finality. In Table 1 you can appreciate the different systems, both privately-owned and Central-Bank-owned in all the countries that are members of the Committee on Payment and Settlement Systems (CPSS) at the Bank for International Settlements (BIS). The most common combination of rules and finality in the CPSS countries seems to be Real-Time Gross Settlement (RTGS), with 17 systems in the sample (20 for all GS.) All net settlement systems together add up to 13, while there are 4 systems with hybrid rules, including CHIPS among them. Thus, the predominance of Gross Settlement systems in practice is evident. However, it is worth pointing out that, as for the daily distribution of payments of most RTGS systems, and Fedwire in particular, most transactions take place in what is called the peak hour (about 4:30 pm)[45]. So, even though Fedwire's and other GS systems' finality is supposed to be, technically, in real time and with DVP, in practice most of their transactions seem to be very close to the end of the day, both in volume and in value, making it somehow more comparable with most Net

[43] In their search model, fiat money and credit are alternative means of purchasing goods and debt is not settled using fiat money. Despite the models' differences in form, the models offer similar implications about the multiplicity of credit market equilibria.
[44] The modeling of fiat money as the payments instrument will be needed for any extensions to international payments systems, as in Hernandez-Verme (2004) and Fujiki (2003).
[45] See McAndrews and Rajan (2000) for more details.

Settlement and Hybrid systems. Under this consideration, the potential advantages of Net Settlement over Gross Settlement seem to play a more important role than one would have otherwise expected.

The Environment

Consider a closed, endowment economy. In each period a constant number of two-period-lived households are born. There is a generation of initial old endowed with a constant stock of M units of fiat money.

There are I outer islands. A continuum of households with unit mass populates each outer island. In addition, there are I different endowment-goods, each good being island-specific. Goods are not storable neither can they be produced. There exist no means of enforcing contracts in the outer islands.

There is a Central Island where the civil and monetary authority is located together with a central clearinghouse. Although contracts may be written anywhere, they can only be enforced only within the Central Island and through the clearinghouse.

Endowments and Preferences

Travel and endowments are built on Freeman and Tabellini (1998). Each young household born in island i, for i=1, 2, 3,…, I, is endowed with w units of the island-i-specific good. Old households have no endowment of goods.

A household j born in island i derives utility from consuming goods both when young and when old. Households are ex-ante identical within an island and across islands, in the sense that they have the same utility function: $u\left(c_1^j\right)+v\left(c_2^j\right)+\Gamma^j$, with $u''<0<u'$ and $v''<0<v'$, where both $u(\cdot)$ and $v(\cdot)$ satisfy the Inada conditions. The variable c_1^j denotes the consumption of household j when young, and c_2^j denotes its consumption when old. All the households in this economy face a symmetric problem. The good with which each household is endowed is not the good the household wants to consume: young households born in island i travel to and wish to consume only the good specific to island i+1; old households born in island i travel to and wish to consume only the good specific to island $i+I/2$ (modulo I).

Each period has two parts. During the first part of each period, intra-generational travel takes place: young households meet with neighbouring young households in the outer islands, while old households meet with other old households in the Central Island. During the second part of each period, the old travel to a distant outer island. Table 2 describes the structure of the trade patterns.

The variable Γ^j represents a nonpecuniary utility derived by old households, the value of which depends on whether the household visits the Central Island. By nonpecuniary we mean a utility not derived from the return from the assets held by a household. Without loss of generality, we set $\Gamma^j=\gamma^j$ if household j travels to the Central Island and 0 if it does not. By setting the utility of not going to the Central Island to 0, γ^j represents net locational utility, the net nonpecuniary benefit of living where contracts can be enforced ("the Pale")[46]. We think of γ^j as including punishments (if $\gamma^j<0$) or rewards (if $\gamma^j>0$) of being under the law, both incidental (e.g. the ability to visit nearby family) and man-made (e.g. criminal prosecution).

[46] We could alternatively measure γ in terms of goods. In this case the utility cost of a locational decision would vary with equilibrium prices affecting the marginal utility of consumption, making the model less tractable without altering the main implications. A locational goods cost may nevertheless be useful in the analysis of collateral requirements for clearinghouse payment system participants.

All households face the same distribution of locational utility, the realization of which is household-specific, and of any sign: $\gamma^j \in \left[\underline{\gamma}, \overline{\gamma}\right]$ is a stationary random variable, i.i.d. across both households and islands, with a p.d.f equal to $f\left(\gamma^j\right)$, where $\overline{\gamma} \geq 0$ always. The household learns γ^j only at the beginning of its last period of life. Notice that by allowing the net benefit to be household-specific, we allow households to be different ex-post, but we do not require it.

Those for any reason wary of the assumption of locational utility should note that absence of locational utility, $\gamma^{i,j} = 0$ always, is just one particular case where households are not only ex ante identical (they face a symmetric problem) but also ex post identical (they obtain the same amount of utility, but from different goods). It will become clear, once we discuss equilibria, that this case shares most of the equilibrium implications of the general model.

The Planner's Problem

Let $\tilde{c}_2$ denote the consumption of a household who does not to travel to the Central Island, and $\hat{c}_2$ be the consumption of an old household who travels to the Central Island. A benevolent planner will choose c_1, $\tilde{c}_2$, $\hat{c}_2$ and γ^* in order to maximize the expected utility of a typical household:

$$u\left(c_1\right) + \int_{\underline{\gamma}}^{\gamma^*} v\left(\tilde{c}_2\right) \cdot f\left(\gamma^j\right) \cdot d\gamma^j + \int_{\gamma^*}^{\overline{\gamma}} v\left(\hat{c}_2\right) \cdot f\left(\gamma^j\right) \cdot d\gamma^j + \int_{\gamma^*}^{\overline{\gamma}} \gamma^j \cdot f\left(\gamma^j\right) \cdot d\gamma^j , \tag{1}$$

restricted by the availability of resources in the economy:

$$w \geq c_1 + \int_{\underline{\gamma}}^{\gamma^*} \tilde{c}_2 \cdot f\left(\gamma^j\right) \cdot d\gamma^j + \int_{\gamma^*}^{\overline{\gamma}} \hat{c}_2 \cdot f\left(\gamma^j\right) \cdot d\gamma^j , \tag{2}$$

and by

$$\underline{\gamma} \leq \gamma^* \leq \overline{\gamma} . \tag{3}$$

Accordingly, three conditions must hold in a Social Optimum[47]. The first condition, given by

$$\tilde{c}_2 = \hat{c}_2 \equiv c_2 , \tag{4}$$

states that old households consume equal amounts whether they travel to the Central Island or not. The second condition, given by

$$\frac{u'\left(c_1\right)}{v'\left(c_2\right)} = 1 \tag{5}$$

says that this allocation is not only optimal but it satisfies the Golden Rule (it maximizes utility over stationary allocations). Finally, the third condition for a social optimum is $\gamma^* \geq 0$, where

$$\gamma^* = 0, \quad \text{if } \underline{\gamma} < 0 , \tag{6a}$$

$$\gamma^* = \underline{\gamma} > 0, \quad \text{if } \underline{\gamma} \geq 0 . \tag{6b}$$

[47] Notice that, given the general form of preferences that we use in our model, this need not be the only Pareto Optimal allocation. However, this allocation constitutes a useful departing point for comparing equilibria under alternative rules for the settlement of debt.

Condition (6a) describes the unique socially optimal cut-off value when travel to the Central Island is costly for some $\left(\underline{\gamma} < 0\right)$ and (6b) when travel to the Central Island is costly for none $\left(\underline{\gamma} \geq 0\right)$. Let $\pi \equiv \int_{\gamma^*}^{\overline{\gamma}} f\left(\gamma^j\right) \cdot d\gamma^j$ denote the fraction of old households who travel to the Central Island. Interestingly, when some households are hurt (in non pecuniary ways) by travel to the Central Island $\left(\underline{\gamma} < 0\right)$ it is best not only for the household but also for the economy as a whole if this household does not travel to the Central Island and, thus $\pi < 1$. When instead all enjoy travel to the Central Island $\left(\underline{\gamma} \geq 0\right)$, then optimality involves no avoidance of the Central Island because only "good things" can happen to a household there and $\pi = 1$. We formalize these results in the following propositions.

<u>Proposition 1</u>: When travel to the Central Island is costly for some $\left(\underline{\gamma} < 0\right)$, the social optimum allocation requires that these old households not travel to the Central Island $\left(\gamma^* = 0\right)$.

<u>Proof:</u> When $\left(\underline{\gamma} < 0\right)$, $\gamma^* = 0 < \overline{\gamma}$ is an interior solution. Thus, $\pi \equiv \int_{\gamma^*}^{\overline{\gamma}} f\left(\gamma^j\right) \cdot d\gamma^j < 1$ and some households do not travel to the Central Island. *Q.E.D.*

<u>Proposition 2</u>: When travel to the Central Island is costly for none $\left(\underline{\gamma} \geq 0\right)$, the social optimum allocation requires all old households to travel to the Central Island $\gamma^* = \underline{\gamma}$.

<u>Proof:</u> When $\left(\underline{\gamma} \geq 0\right)$, $\gamma^* = \underline{\gamma}$ is a corner solution. Thus, $\pi \equiv \int_{\gamma^*}^{\overline{\gamma}} f\left(\gamma^j\right) d\gamma^j = 1$ and all old households travel to the Central Island. *Q.E.D.*

The social optimal allocation described by (4), (5), and (6a, b) is unique.

Trade and Travel Patterns

a) Young households, first part of the period

During the first part of the period, each household splits into two parts: a buyer and a seller. The young buyer born in island *i* travels to island $i+1$, to purchase good *i*+1. The young seller born in island *i* remains in island *i*, waiting for buyers from island *i*-1, in order to sell part of the endowment good to them. At this point, the young buyers from island *i* have nothing of value to offer to sellers born in island *i*+1 in exchange for their good. Thus, the buyer from island *i* may issue debt (an IOU) to the seller in island *i+1* in exchange for good *i*. We assume that the total debt of each household can be observed in the outer islands, so that in equilibrium young households cannot borrow infinite, unrepayable amounts[48] and cannot exhaust their endowments by issuing IOUs. This promise, the IOU (i.e. the IOU payable,) must be repaid next period on the Central Island though the central clearinghouse, since this is the only time when people will get together. As we will see, only fiat money is useful to old agents in making purchases in the outer islands. Therefore, old agents will require that debt be repaid using fiat money. Let *r* denote the gross real interest rate promised on the debt issued. Then sellers accept the IOUs (IOUs receivable.)

Once these transactions take place, the young buyers go back home with the good they purchased. At this stage, the buyer and seller of each household are reunited and they consume this good.

[48] We abstract from a moral hazard problem in borrowing, so that that we can focus on the incentives to repay debt.

b) Young and old households, second part of the period

During the second part of the period, the young households from island *i* sell the remainder of their endowment good to old households from island (*i*-*I*/2) in exchange for fiat money. At this point, the young households accept only fiat money because next period they will need it either to settle debt or to purchase goods. The old households consume the good they purchased.

c) Old households, first part of the period

At the beginning of the period, the old household *j* observes the realization of the random variable γ^j, which is private information. Notice that ex-ante preferences are identical, but they are different ex-post. We now define γ^* as the cut-off value for a typical old household, such that $\gamma^* \in \left[\underline{\gamma}, \overline{\gamma}\right]$ holds. When $\gamma^j < \gamma^*$, then the old household *j* chooses not to travel to the Central Island, and when $\gamma^j \geq \gamma^*$ the old household *j* chooses to travel to the Central Island.

If an old household chooses to go to the Central Island during the first part of the period, it carries fiat money from the previous period. In addition, this household must repay its debt, even though only other households who owe him and travel to the Central Island pay him back. Debt is settled by third parties in the Central Island. Finally, this old household gets the utility γ^j and consumes $\hat{c}_2$ goods during the second part of the period.

If an old household chooses not to go to the Central Island during the first part of the period, it still carries fiat money from the previous period. However, this household does not repay its debt nor does get paid for the debt it accepted the previous period. Notice that the identity of a household is unknown in the outer islands, and also that contracts are not enforceable there either. This household consumes $\tilde{c}_2$ goods during the second part of the period by exchanging it for fiat money.

As before, we define π to be the fraction of old households traveling to the Central Island.

Throughout the paper, we focus on stationary allocations.

Alternative Rules of Settlement and Equilibria

A Net Settlement Rule with Debt Forgiveness

Under a net settlement rule with debt forgiveness, any loss from a default is shared by all participants in the payments system. This means that if $\pi \in [0,1]$ is the fraction of old households traveling to the Central Island, an old household traveling to Central Island would have to pay only a fraction π of its gross debt. In addition, IOUs receivable can also be used as means for repaying IOUs payable. As a result, the old household's holdings of fiat money would need to be enough to cover only its net debt. A real life example of such a loss-sharing arrangement is the Lamfalussy rule[49].

1.1.1.1.a) The Household's problem

Let b_t denote the nominal value of debt issued by the household (IOUs payable,) s_t the nominal value of debt accepted by the household (IOUs receivable,) m_t the household's nominal holdings of fiat money, and p_t the price level at time *t*. Let also *R* be the effective (net of defaults) gross real interest rate paid on the household's loan. Then, if the subscript *F* refers to net settlement

[49] Notice, however, that the Lamfalussy rule has an additional requirement to post collateral.

with debt forgiveness, a typical household will choose $\left(\frac{b_t}{p_t}\right)_F$, $\left(\frac{s_t}{p_t}\right)_F$, $\left(\frac{m_t}{p_t}\right)_F$ and γ_F^* in order to maximize its lifetime utility (1), subject to the following budget constraints:

$$c_{1,F} = \left(\frac{b_t}{p_t}\right)_F, \tag{7}$$

$$\left(\frac{s_t}{p_t}\right)_F + \left(\frac{m_t}{p_t}\right)_F = w, \tag{8}$$

$$\tilde{c}_{2,F} = \left(\frac{m_t}{p_t}\right)_F, \tag{9}$$

$$\hat{c}_{2,F} = R_F \cdot \left(\frac{s_t}{p_t}\right)_F - \pi_F \cdot r_F \cdot \left(\frac{b_t}{p_t}\right)_F + \left(\frac{m_t}{p_t}\right)_F, \tag{10}$$

and

$$\left(\frac{b_t}{p_t}\right)_F \geq 0, \quad \left(\frac{s_t}{p_t}\right)_F \geq 0, \quad \left(\frac{m_t}{p_t}\right)_F \geq 0, \quad \underline{\gamma} \leq \gamma_F^* \leq \overline{\gamma}. \tag{11}$$

Equation (7) states that consumption when young is purchased using debt. Equation (8) states that the young household's endowment good can either be sold to other young households in exchange for debt or sold to old households in exchange for fiat money. Equation (9) states that when an old household chooses not travel to the Central Island, its consumption is purchased only with fiat money balances. Notice that the gross promised return from the household's debt is given by $r_F \cdot \left(\frac{b_t}{p_t}\right)_F$ and π_F is the fraction of old households who are on the Central Island. Under this rule of settlement, the household must pay only the fraction π_F of its IOUs payable, and the household's net debt is equal to $R_F \cdot \left(\frac{s_t}{p_t}\right)_F - \pi_F \cdot r_F \cdot \left(\frac{b_t}{p_t}\right)_F$. Thus, equation (10) indicates that when an old household chooses to travel to the Central Island, its consumption is paid with net debt and fiat money. Equation (11) describes the range of possible values for each one of the choice variables.

The first order conditions determining the household's choice of debt, money and default are given by

$$u'(c_{1,F}) = \pi_F{}^2 \cdot r_F \cdot v'(\hat{c}_{2,F}) \tag{12}$$

$$(R_F - 1) \cdot \pi_F \cdot v'(\hat{c}_{2,F}) = (1 - \pi_F) \cdot v'(\tilde{c}_{2,F}) \tag{13}$$

$$\gamma_F^* \geq v(\tilde{c}_{2,F}) - v(\hat{c}_{2,F}) \tag{14}$$

Equation (12) describes the household's choice between consumption purchased with debt $\left(c_1{}^F\right)$ and consumption purchased with fiat money and net debt $(\hat{c}_{2,F})$. Next, equation (13) describes the intra-temporal choice of the household when old: the trade-off between consuming when traveling to the Central Island and consuming when not traveling to the Central Island. Finally, equation (14) describes the household's choice of when to default.

b) Market Clearing

The market for loans clears when total borrowing equals total lending:

$$\left(\frac{b_t}{p_t}\right)_F = \left(\frac{s_t}{p_t}\right)_F . \qquad (15)$$

Also, from feasibility of the settlement of debt in the Central Island we obtain

$$R_F = \pi_F \cdot r_F \qquad (16)$$

That is, the effective interest rate on loans equals the promised interest rate on loans adjusted by the fraction of old households who travel to the Central Island.

c) Equilibrium and Optimality under a Net Settlement Rule with Debt Forgiveness

Under a net settlement rule with debt forgiveness the equilibrium conditions satisfy the optimality conditions (4), (5) and (6a) or (6b) hold. This can be shown very easily, and we proceed in two steps. First, by using the aggregate consistency condition (16) in the budget constraints (9) and (10), we obtain the following equilibrium condition

$$\tilde{c}_{2,F} = \hat{c}_{2,F} = \left(\frac{m_t}{p_t}\right)_F . \qquad (17)$$

Combining the equilibrium condition (17) with the first order condition (14) we obtain

$$\gamma_F^* \geq 0 , \qquad (18)$$

implying the following equilibrium condition

$$\gamma_F^* = 0, \ \text{ if } \underline{\gamma} < 0 \qquad (18a)$$

$$\gamma_F^* = \underline{\gamma} > 0, \ \text{ if } \underline{\gamma} \geq 0 \qquad (18b)$$

Thus, from the equilibrium conditions (17) and (18a) or (18b) it follows directly that the optimality conditions (4) and (6a) or (6b) hold in equilibrium.

As a second step, we proceed to show that the optimality condition (5) holds in equilibrium. By combining the equilibrium condition (17) with the intra-temporal first order condition (13), the following condition can be easily derived

$$(\pi_F \cdot r_F - 1) = \frac{(1-\pi_F)}{\pi_F} . \qquad (19)$$

Notice that two cases may arise, depending on whether or not travel to the Central Island is costly for some. We describe both cases below.

c.1) Case 1: travel to the Central Island is costly for some, $\underline{\gamma} < 0$. The equilibrium condition (18a) implies not only that π_F is unique but also that $\pi_F < 1$, i.e.: a positive fraction of the population defaults. In particular, it follows directly from (19) that $\pi_F \cdot r_F = \frac{1}{\pi_F} > 1$, implying that the equilibrium $r_F = \frac{1}{\pi_F^2} > 1$ is unique (given that the equilibrium π_F is unique). By combining the inter-temporal first order condition (12) with the equilibrium conditions (17), (18a) and (19), it is fairly easy to show that the following condition holds in equilibrium

$$\frac{u'(c_{1,F})}{v'(\hat{c}_{2,F})} = \pi_F^2 \cdot r_F = 1 . \qquad (20)$$

The inter-temporal equilibrium condition (20) clearly implies that the inter-temporal optimality condition (5) is satisfied. Thus, (4), (5) and (6a) are satisfied in equilibrium when traveling to the Central Island is Costly for some, and this equilibrium is unique, stable and optimal.

c.2) Case 2: travel to the Central Island is costly for none, $\underline{\gamma} \geq 0$. The equilibrium condition (18b) implies that $\pi_F = 1$. Thus, this is a unique equilibrium that displays universal repayment. In addition, it follows from (19) that $r_F = 1$ in equilibrium. In addition, the combination of the inter-temporal first order condition (12) and the equilibrium conditions (17), (18b) and (19), yields the equilibrium condition (20), which holds as well in this case. The latter implies that the inter-temporal optimality condition (5) is satisfied. Therefore, (4), (5) and (6b) are satisfied in equilibrium when traveling to the Central Island is costly for none, and this equilibrium is unique, stable and optimal.

Summarizing, the equilibrium resulting from a rule of net settlement with debt forgiveness is always unique, stable and optimal.

Strict Net Settlement

Under this system, the loss-sharing rule in place is settlement unwinding: if a bank defaults, its transactions are removed from that day's transactions, net positions are recalculated for the remaining banks, and the settlement of recalculated positions proceeds as in normal settlement[50]. Under a strict settlement rule, an old household traveling to the Central Island would have to pay all of its gross debt. But the IOUs receivable are accepted as payment of IOUs payable. Thus, the old households need to bring enough real money balances to pay for their net debt. In this case, the subscript *N* refers to the Strict Net Settlement rule, but we maintain the general notation used before.

1.1.1.2.a) The household's problem

Under this rule of settlement, a typical household chooses $\left(\frac{b_t}{p_t}\right)_N$, $\left(\frac{s_t}{p_t}\right)_N$, $\left(\frac{m_t}{p_t}\right)_N$ and γ_N^* in order to maximize its expected lifetime utility (1), subject to the budget constraints (7), (8), (9), (11) and:

$$\hat{c}_{2,N} = R_N \cdot \left(\frac{s_t}{p_t}\right)_N - r_N \cdot \left(\frac{b_t}{p_t}\right)_N + \left(\frac{m_t}{p_t}\right)_N . \tag{21}$$

Notice that a household's net debt is given by $R_N \cdot \left(\frac{s_t}{p_t}\right)_N - r_N \cdot \left(\frac{b_t}{p_t}\right)_N$ under this rule of settlement. The first order conditions for this problem are given by

$$u'\left[\left(\frac{b_t}{p_t}\right)_N\right] = \pi_N \cdot r_N \cdot v'\left[R_N \cdot \left(\frac{s_t}{p_t}\right)_N - r_N \cdot \left(\frac{b_t}{p_t}\right)_N + \left(\frac{m_t}{p_t}\right)_N\right], \tag{22}$$

$$(R_N - 1) \cdot \pi_N \cdot v'\left[R_N \cdot \left(\frac{s_t}{p_t}\right)_N - r_N \cdot \left(\frac{b_t}{p_t}\right)_N + \left(\frac{m_t}{p_t}\right)_N\right] = (1 - \pi_N) \cdot v'\left[\left(\frac{m_t}{p_t}\right)_N\right], \tag{23}$$

$$\gamma_N^* \geq v\left[\left(\frac{m_t}{p_t}\right)_N\right] - v\left[R_N \cdot \left(\frac{s_t}{p_t}\right)_N - r_N \cdot \left(\frac{b_t}{p_t}\right)_N + \left(\frac{m_t}{p_t}\right)_N\right]. \tag{24}$$

Equation (22) describes the household's inter-temporal choice of consumption, which is also its choice between goods purchased with debt $(c_{1,N})$ and goods purchased with fiat money if it travels

[50] See BIS (2003) and Kahn, McAndrews and Roberds (2003.)

to the Central Island $(\hat{c}_{2,N})$. Next, equation (23) describes the household's intra-temporal choice: how much would the household consume if it chose to travel to the Central Island $(\hat{c}_{2,N})$ versus how much it would consume if it chose not to travel to the Central Island $(\tilde{c}_{2,N})$. Finally, (24) describes the household's choice of its cut-off value, γ_N^*.

Notice that the market clearing conditions (15) and (16) also hold under this rule.

b) Equilibria and Optimality under a Strict Net Settlement Rule

We combine the aggregate consistency conditions (15) and (16) with the first order conditions (22), (23) and (24). We can then show that equilibria under a strict net settlement rule are described by a system of three nonlinear equations in $\left(\frac{s_t}{p_t}\right)_N$, r_N, and γ_N^*, as follows:

$$u'\left[\left(\frac{s_t}{p_t}\right)_N\right] = \pi_N \cdot r_N \cdot v'\left[w + (\pi_N \cdot r_N - r_N - 1)\cdot\left(\frac{s_t}{p_t}\right)_N\right], \tag{25}$$

$$(\pi_N \cdot r_N - 1)\cdot \pi_N \cdot v'\left[w + (\pi_N \cdot r_N - r_N - 1)\cdot\left(\frac{s_t}{p_t}\right)_N\right] = (1-\pi_N)\cdot v'\left[w - \left(\frac{s_t}{p_t}\right)_N\right], \tag{26}$$

$$\gamma_N^* \geq v\left[w - \left(\frac{s_t}{p_t}\right)_N\right] - v\left[w + (\pi_N \cdot r_N - r_N - 1)\cdot\left(\frac{s_t}{p_t}\right)_N\right]. \tag{27}$$

It is obvious that equilibria under a Strict Net Settlement rule without universal repayment must have a net positive nominal interest rate.

<u>Lemma 1</u>: the expected return to debt, $\pi_N \cdot r_N$, exceeds the return to fiat money, 1, in all equilibria with a strict net settlement rule displaying default $(\pi_N < 1)$.

<u>Proof:</u> Any equilibrium with $\pi_N < 1$ and $\pi_N \cdot r_N < 1$ violates the equilibrium condition (26). Thus, it must be the case that $\pi_N \cdot r_N > 1$ in any equilibrium that features $\pi_N < 1$. *Q.E.D.*

b.1) Multiplicity of Equilibria. Under a strict net settlement rule, strategic complementarities are present in the following sense: if a household believes that few other households will show up at the Central Island, this household may not want to show up either. The more other old households default, the more a given household would want to default. The reason is that under a strict net settlement rule, the default of other households does reduce what a given household receives in the Central Island (IOUs receivable) but it does not reduce what the household owes (IOUs payable.) Thus, not only does a strict net settlement rule readily allow multiple, Pareto ranked steady state equilibria, but also, some of these equilibria may be locally unstable.

b.2) Equilibria in the absence of locational utility. The inherent instability of equilibria under strict net settlement may be illuminated with a look at the special case of the absence of any locational utility, $\underline{\gamma} = \overline{\gamma} = 0$. A household's choice of a cut-off locational decision γ_N^* implies a specific probability π_N that it will go to the Central Island. In Figure 3, $\pi_N(j)$ denotes this probability as a result of household *j*'s strategy, while $\pi_N(-j)$ denotes this probability as a result of the strategies of the other households. In the absence of any locational utility there are two equilibria: one in which all debt is repaid $(\pi_N = 1)$ and another with universal default $(\pi_N = 0)$, as illustrated in Figure 3.

However, the equilibrium in which all debt is repaid is unstable. If any single household will fail to pay its debt, the value of IOUs receivable falls below the value of IOUs payable for the other households. In the absence of any nonpecuniary returns to travel to the Central Island, all other households will now also choose to default.

b.3) Equilibrium when travel to the Central Island is always beneficial. Consider next equilibria when travel to the Central Island is always beneficial: $\underline{\gamma} > 0$. The universal desirability of travel to the Central Island now makes the equilibrium with a high π_N stable. Figure 4a illustrates this possibility. There is a stationary equilibrium in a situation where $\pi_N(j) = \pi_N(-j)$. For illustration purposes, Figure 4a depicts an example where households have a log linear utility function, $\underline{\gamma} \geq 0$ and $\bar{\gamma} > 0$. Notice that there are three steady state equilibria. In the first equilibrium, $\pi_N = 0$ and all the households default; this equilibrium is locally stable. In the second equilibrium (an interior equilibrium), $0 < \pi_N < 1$ and there is some default; this equilibrium is locally unstable. Finally, in the third equilibrium, $\pi_N = 1$ and there is no default. This equilibrium is locally stable: it can tolerate small departures from the strategy of never defaulting because travel to the Central Island has nonpecuniary benefits.

<u>Proposition 3</u>: When $\underline{\gamma} > 0$, one strict net settlement equilibrium features universal repayment $(\pi_N = 1)$, and this equilibrium is optimal.

<u>Proof:</u> If $\underline{\gamma} > 0$, optimality involves $\gamma_N^* = \underline{\gamma}$ and $\pi_N = 1$. When $\pi_N = 1$, using the budget constraints (9) and (18), we obtain that the optimality condition (4) holds in equilibrium. In addition, given that $\hat{c}_{2,N} = \tilde{c}_{2,N}$ and that $\pi_N = 1$, the equilibrium condition (26) yields $r_N = 1$. As a consequence, the optimality condition (5) also holds in equilibrium. Finally, (6b) can be easily obtained as one possible outcome from the equilibrium condition (27). Thus, one strict net settlement equilibrium features $\gamma_N^* = \underline{\gamma}$, and this equilibrium is optimal and locally stable. *Q.E.D.*

However, equilibria under a strict net settlement rule are typically not unique. Thus, Proposition 3 only states that the “best” equilibrium under a strict net settlement rule, the one with no default, is optimal. The remainder of the equilibria under a strict net settlement rule display $\gamma_N^* > \underline{\gamma}$ and a strictly positive rate of default which is obviously larger than the social optimum. Therefore, the remaining equilibria are Pareto dominated by the best equilibrium. Following this principle, and due to the presence of strategic complementarities under a strict net settlement rule, it is possible to establish a Pareto ranking of these equilibria according to their rate of default: equilibria with a higher rate of default are Pareto dominated by equilibria with a lower rate of default. Therefore, when $\underline{\gamma} > 0$ so that default is always undesirable, there is an equilibrium from the strict net settlement rule that is as optimal as net settlement with debt forgiveness. Net settlement with debt forgiveness, however, avoids a multiplicity of equilibria, while strict net settlement does not.

b.4) Equilibrium when travel to the Central Island is costly to some. Consider next equilibria when travel to the Central Island is costly for some $(\underline{\gamma} < 0)$, but beneficial for a positive fraction of households.

<u>Proposition 4:</u> No strict net settlement equilibria are optimal when $\underline{\gamma} < 0$.

<u>Proof:</u> Suppose $\underline{\gamma} < 0$, so that optimality requires $\gamma_N^* = 0$. Notice that $\pi_N < 1$ for any γ_N^* such that $\underline{\gamma} < \gamma_N^* \leq 0$. Obviously, optimality under these circumstances involves some default. Using the budget constraints (9) and (18), we obtain

$$\tilde{c}_{2,N} = \left(\frac{m_t}{p_t}\right)_N > \hat{c}_{2,N} = (\pi_N - 1) \cdot r_N \cdot \left(\frac{s_t}{p_t}\right)_N + \left(\frac{m_t}{p_t}\right)_N . \tag{28}$$

Evaluating the equilibrium condition (27) using (28), we obtain

$$\gamma_N^* = v(\tilde{c}_{2,N}) - v(\hat{c}_{2,N}) > 0 . \tag{29}$$

Thus, $\gamma_N^* = 0$, the optimal solution when $\underline{\gamma} < 0$, is not an equilibrium. Also, any allocation with $\underline{\gamma} < \gamma_N^* < 0$ is not an equilibrium under a strict net settlement rule. Instead, all equilibria under a strict net settlement rule display $\gamma_N^* > 0$ and the rate of default observed in equilibrium is larger than at the social optimum. *Q.E.D.*

Figure 4b illustrates the case of a log-linear utility where $\underline{\gamma} < 0$.

Figure 5a illustrates the nonoptimality of even the best of multiple equilibria when households have a Constant Relative Risk Aversion (CRRA) utility function and traveling to the Central Island is costly for some, but beneficial for a positive fraction of households.

It may also be the case that the costs of travel to the Central Island are sufficiently high that no one wishes to go there –even those with nonpecuniary benefits find these outweighed by the pecuniary losses caused by the default of others. Figure 5b illustrates this possibility also when households have a Constant Relative Risk Aversion utility function, so that we can compare with Figure 5a. In this case there is a unique equilibrium. In this equilibrium, which is stable, all households default. This equilibrium with $\pi_N = 0$ is not optimal, optimality requiring that $\gamma_N^* = 0$ and thus $0 < \pi_N < 1$.

Strict Gross Settlement

Under a (strict) gross settlement rule, IOUs receivable cannot be used to pay IOUs payable, and there is unwinding. As a result, old households need to carry enough fiat money to pay for their gross debt. As before, the household's gross debt is given by $r_G \cdot \left(\frac{b_t}{p_t}\right)_G$, where the subscript G indicates a gross settlement rule.

a) The household's problem

Under a gross settlement rule, a typical household chooses $\left(\frac{b_t}{p_t}\right)_G$, $\left(\frac{s_t}{p_t}\right)_G$, $\left(\frac{m_t}{p_t}\right)_G$ and γ_G^* in order to maximize its lifetime utility (1), subject to the budget constraints (7), (8), (9), (11), (18) and the following additional constraint:

$$\left(\frac{m_t}{p_t}\right)_G \geq r_G \cdot \left(\frac{b_t}{p_t}\right)_G . \tag{30}$$

The inequality (30) reflects that, under a gross settlement rule, each household must carry enough real money balances to pay its gross debt.

Let μ be the Lagrange multiplier associated with constraint (30). Then, the first order conditions for this problem are given by

$$u'\left[\left(\frac{b_t}{p_t}\right)_G\right]-\pi_G\cdot r_G v'\left[R_G\cdot\left(\frac{s_t}{p_t}\right)_G-r_G\cdot\left(\frac{b_t}{p_t}\right)_G+\left(\frac{m_t}{p_t}\right)_G\right]=\mu\cdot r_G\,, \tag{31}$$

$$(1-\pi_G)\cdot v'\left[\left(\frac{m_t}{p_t}\right)_G\right]+\mu=\pi_G\cdot(R_G-1)\cdot v'\left[R_G\cdot\left(\frac{s_t}{p_t}\right)_G-r_G\cdot\left(\frac{b_t}{p_t}\right)_G+\left(\frac{m_t}{p_t}\right)_G\right], \tag{32}$$

$$\gamma_G^*\geq v\left[\left(\frac{m_t}{p_t}\right)_G\right]-v\left[R_G\cdot\left(\frac{s_t}{p_t}\right)_G-r_G\cdot\left(\frac{b_t}{p_t}\right)_G+\left(\frac{m_t}{p_t}\right)_G\right], \tag{33}$$

and (30). The market clearing conditions (15) and (16) also hold.

b) Equilibria and Optimality under Strict Gross Settlement

After combining the aggregate consistency conditions (15) and (16) with the first order conditions (31), (32) and (33), we can show that equilibria are described by the following system of three nonlinear equations in $\left(\frac{s_t}{p_t}\right)_G$, r_G, γ_G^* and μ:

$$u'\left[\left(\frac{s_t}{p_t}\right)_G\right]-\pi_G\cdot r_G\cdot v'\left[w+(\pi_G\cdot r_G-r_G-1)\cdot\left(\frac{s_t}{p_t}\right)_G\right]=\mu\cdot r_G\,, \tag{34}$$

$$(1-\pi_G)\cdot v'\left[w-\left(\frac{s_t}{p_t}\right)_G\right]+\mu=\pi_G\cdot(\pi_G\cdot r_G-1)\cdot v'\left[w+(\pi_G\cdot r_G-r_G-1)\cdot\left(\frac{s_t}{p_t}\right)_G\right], \tag{35}$$

$$\gamma_G^*\geq v\left[w-\left(\frac{s_t}{p_t}\right)_G\right]-v\left[w+(\pi_G\cdot r_G-r_G-1)\cdot\left(\frac{s_t}{p_t}\right)_G\right], \tag{36}$$

$$\text{If } w>(r_G+1)\cdot\left(\frac{s_t}{p_t}\right)_G\text{, then }\mu=0\,, \tag{37a}$$

$$\text{If } w=(r_G+1)\cdot\left(\frac{s_t}{p_t}\right)_G\text{, then }\mu>0\,. \tag{37b}$$

Notice that equilibria under a gross settlement rule constitute (weakly) constrained strict net settlement equilibria. We then give the following two definitions.

<u>Definition 1:</u> An **<u>Unconstrained Gross Settlement Equilibrium</u>** is defined as an equilibrium resulting from a gross settlement rule where (37a) holds, and the additional constraint on real money balances does not bind.

<u>Definition 2:</u> A **<u>Constrained Gross Settlement Equilibrium</u>** is defined as an equilibrium resulting from a gross settlement rule where (37b) holds, and the additional constraint on real money balances does bind.

When (37a) holds, the gross settlement equilibrium conditions (34) – (36) are identical to those of strict net settlement, (25) – (27), leading to Proposition 5.

Proposition 5: Equilibrium allocations resulting from an unconstrained gross settlement rule are identical to those resulting from a strict net settlement rule.

When (37b) holds, gross settlement equilibria are identical to strict net settlement equilibria but with one additional constraint to be satisfied. Thus constrained gross settlement equilibria offer less utility than its strict net settlement counterpart, leading to the following proposition[51].

Proposition 6: For each constrained gross settlement equilibrium there is a Pareto superior equilibrium resulting from a strict net settlement rule.

Figure 6 illustrates one example this proposition. The proof can be found in the Appendix.

Also, the scope for multiple equilibria is preserved in the system both under unconstrained and constrained gross settlement. Gross settlement does not reduce the strategic complementarities that may lead to the breakdown of the payments system. Multiple equilibria occur under unconstrained gross settlement whenever they occur under strict net settlement, all being defined by the same conditions. By the continuity of equilibrium conditions, where multiple equilibria occur in strict net settlement, they occur in constrained gross settlement, at a minimum where the constraint does not strongly bind (values of μ close to 0).

Welfare Analysis

We now turn to compare the welfare of equilibria under the alternative settlement rules that we have discussed so far. As we see below, the welfare properties differ according to whether $\underline{\gamma} < 0$ or $\underline{\gamma} \geq 0$. The summary of our results is presented in Table 3.

It is obvious that the Social Planner's solution –and the equilibrium under net settlement with debt forgiveness, is Pareto optimal, and therefore it occupies the first place in our ranking. Additionally, when $\underline{\gamma} \geq 0$, the best equilibrium under strict net settlement, which displays universal repayment, is Pareto optimal as well.

Equilibria resulting from a strict net settlement rule –other than the best equilibrium when $\underline{\gamma} \geq 0$, are Pareto dominated by the equilibria under net settlement with debt forgiveness, thus occupying the second place in our ranking. We have also shown that equilibrium allocations that result from unconstrained gross settlement equilibria are identical to their counterpart under a strict net settlement rule and, thus, they yield the same welfare.

Finally, constrained gross settlement equilibria are Pareto dominated by unconstrained gross settlement equilibria, yielding lower welfare and therefore occupying the last place in our ranking.

A Collateral Requirement

We now introduce collateral into our model and examine the properties of equilibria under the three alternative settlement rules examined in this paper. Young households who borrow $\frac{b_t}{p_t}$ goods at *t* must put aside k_t units of fiat money. k_t is a nominal variable that is exogenously determined by the laws and rules governing the financial system. The collateral is set apart before the end-of period *t*, immediately after the old households exchange fiat money for part of the young households' endowment good. At this point, the collateral is shipped to the clearinghouse on the Central Island, where it is kept until *t+1*. Later, during the first part of the period *t+1*, the collateral will be divided equally among the old households who travel to the Central Island. We will now describe how the inclusion of collateral affects the results obtained under each of the three rules of settlement discussed in the previous section.

[51] This utility loss when households are forced by gross settlement is fairly general and noted widely in the literature, including Angelini (1998) and Kahn and Roberds (2001).

A Collateral Requirement in a Net Settlement System with Debt Forgiveness: the Lamfalussy Rule.

The budget constraints (8) and (10) faced by households under a net settlement rule with debt forgiveness are replaced by the following equations, where the subscript *FC* indicates a net settlement rule with debt forgiveness and collateral:

$$w = \left(\frac{s_t}{p_t}\right)_{FC} + \left(\frac{m_t}{p_t}\right)_{FC} + \left(\frac{k_t}{p_t}\right), \tag{38}$$

$$\hat{c}_{2,FC} = R_{FC} \cdot \left(\frac{s_t}{p_t}\right)_{FC} - \pi_{FC} \cdot r_{FC} \cdot \left(\frac{b_t}{p_t}\right)_{FC} + \left(\frac{m_t}{p_t}\right)_{FC} + \left(\frac{1}{\pi_{FC}}\right) \cdot \frac{k_t}{p_t}. \tag{39}$$

Notice that (39) and feasibility rule out universal default $(\pi_{FC} = 0)$ as a possible equilibrium. Moreover, feasibility will also rule out equilibria with values of π_{FC} that are close enough to zero. The uniqueness of the equilibrium under this settlement rule is preserved after the collateral requirement is introduced. However, the equilibrium is not Pareto optimal because collateral is costly. The equilibrium probability of default might be reduced or not with respect to the no collateral case depending on whether traveling to the Central Island is costly or not.

a) Equilibrium and Optimality when Travel to the Central Island is always beneficial $(\underline{\gamma} \geq 0)$.

In this case, $\gamma^*_{FC} = \underline{\gamma}$, $\pi_{FC} = 1$, and there is no default in equilibrium. Thus, condition (6b) for optimality is satisfied. However, $\hat{c}_{2,FC} > \tilde{c}_{2,FC}$ and $\frac{u'(c_{1,FC})}{v'(\hat{c}_{2,FC})} > 1$ in equilibrium, and conditions (4) and (5) for optimality are not satisfied. Thus the equilibrium displays the optimal probability of default –with universal repayment, but it is not Pareto optimal due to the costly collateral.

b) Equilibrium and Optimality when Travel to the Central Island is costly for some $(\underline{\gamma} < 0)$.

In this case, $\underline{\gamma} < \gamma^*_{FC} < 0$, implying a lower fraction of the population defaulting in equilibrium compared to the Golden Rule allocation. However, in this case also $\hat{c}_{2,FC} > \tilde{c}_{2,FC}$ and $\frac{u'(c_{1,FC})}{v'(\hat{c}_{2,FC})} > 1$ in equilibrium, and conditions (4) and (5) for optimality are not satisfied as well. Thus, this equilibrium is suboptimal: introducing collateral reduces default beyond the optimal level (the Golden Rule), but a collateral requirement is costly.

A Collateral Requirement in a Strict Net Settlement Rule

In the case of a strict net settlement rule, equations (8) and (21) are replaced by the following equations, where the subscript *NC* indicates a strict net settlement rule with collateral requirements

$$w = \left(\frac{s_t}{p_t}\right)_{NC} + \left(\frac{m_t}{p_t}\right)_{NC} + \left(\frac{k_t}{p_t}\right), \tag{40}$$

$$\hat{c}_{2,NC} = R_{NC} \cdot \left(\frac{s_t}{p_t}\right)_{NC} - r_{NC} \cdot \left(\frac{b_t}{p_t}\right)_{NC} + \left(\frac{m_t}{p_t}\right)_{NC} + \left(\frac{1}{\pi_{NC}}\right) \cdot \frac{k_t}{p_t}. \tag{41}$$

Notice that equation (41) and feasibility rule out universal default $(\pi_{NC} = 0)$ as a possible equilibrium here as well, together with equilibria displaying values of π_{NC} that are close enough to zero.

The introduction of a collateral requirement in a strict net settlement rule reduces the scope for multiple equilibria generated by strategic complementarities. Moreover, the equilibria with higher probability of default disappear.

a) Equilibrium and Optimality when Travel to the Central Island is always beneficial $(\underline{\gamma} \geq 0)$.

In Figure 7 we present the results from a numerical example using a log-linear utility function. This figure is comparable with Figure 4, where collateral was not required. We do observe that once the collateral requirement is introduced, the two equilibria with higher probability of default disappear. Moreover, the equilibrium is unique and globally stable, displaying $\gamma^*_{NC} = \underline{\gamma}$ and $\pi_{NC} = 1$, as in the universal repayment equilibrium with no collateral. However, this equilibrium is suboptimal, given that collateral is costly.

b) Equilibrium and Optimality when Travel to the Central Island is costly for some $(\underline{\gamma} \geq 0)$

With log-linear preferences in the absence of a collateral requirement there were typically two equilibria: one with $\pi = 0$ and $\pi = 1$; however, when the costs of travel to the Central Island were sufficiently high, the equilibrium with $\pi = 1$ disappeared, and there was only one equilibrium with universal default.

The introduction of a collateral requirement in a strict net settlement rule when travel to the Central Island is costly reduces the number of equilibria. In Figure 8a we present the results of numerical examples for the case when travel to the Central Island is costly for some but the cost is not sufficiently high. In this case, introducing collateral eliminates the equilibrium with universal default $(\pi = 0)$, and the equilibrium with $\pi = 1$ remains the unique equilibrium. However, this equilibrium is suboptimal because of mainly two reasons: (i) zero default is not optimal in this when traveling to the Central Island is costly for some; and (ii) collateral is costly.

In Figure 8b we present the results of numerical examples for the case when the cost of travel to the Central Island is sufficiently high. In this case, after introducing collateral the number of equilibria is preserved: there is still a unique equilibrium. However, there is no universal default anymore: the fraction of the population who default in equilibrium is significantly lower, lower even than the optimal one. Thus, this equilibrium is suboptimal for the same reasons that we stated before: (i) zero default is not optimal when traveling to the Central Island is costly for some; and (ii) collateral is costly.

A Collateral Requirement with a Gross Settlement Rule

We have showed in Section 3.3 that Unconstrained Gross Settlement Equilibria are identical to their counterpart under a Strict Net Settlement rule. We can expect then that these unconstrained gross settlement equilibria will be affected in a similar way as strict net settlement equilibria when a collateral requirement is introduced, and the results from Section 4.2 still hold.

Constrained Gross Settlement Equilibria are Pareto inferior to Unconstrained Gross Settlement, and we can expect that the introduction of a collateral requirement in this case will have more extreme consequences, but they move in the same direction.

The Crash of 1987

The Stock Market Crash of 1987 can be listed among the worst crises that have been recorded in U.S. history: approximately $1 trillion of financial wealth was wiped-out, with the near-failure of the clearing and settlement system that underpins the stock and commodities markets and integrates with the overall payment system. Moreover, the fall in the Dow Jones index between October 16 and 19 of 1987 was significantly higher than the decline observed between October 28 and 29 of 1929. For this section, we rely on the accounts by Brimmer (1989) and Wigmore (1998). Both authors agree on the list of factors that initiated and contributed to the crisis, as we describe below.

The years 1985, 1986 and January-October of 1987 had shown significant increases in the S&P500, in the order of 26%, 15% and 39%, respectively. These increases translated into an initial overvaluation of the stocks in the S&P500 of the order of 27% by the end of September of 1987. Until September of 1987, general expectations and confidence about the future were very high and had translated into significant purchases of the stocks of the S&P500.

However, a set of circumstances were put together mostly during October 11 and 19 of 1987. Among the most important, we can list the following: growing expectations of renewed inflation, a depreciation of the US dollar, and increase in the interest rate that made bond yields more competitive with common stocks, a fading of foreigner investors' confidence, an increased pessimism about the near-term profitability for U.S. corporations, the "unwillingness" of Japan and Germany to stimulate their economies, and a growing concern over federal budget deficits.

The immediate result was a break in expectations that lead to a massive avalanche of sales of the stocks in the S&P500, motivated by a desire to "sell ASAP and take profits while you can." Therefore, there was an enormous volume of sell orders accompanied by a shrinking demand, accelerating the fall in their prices and motivating enormous amounts of margin calls. Under normal circumstances, these margin calls would have been covered with marginal payments at the next day by the latest. However, the extreme reduction in the value of brokers and customers portfolios prevented the receipt of balancing payments through the Clearing and Settlement Services (CSS.)

a) A Bank's Reaction Function

As pointed by McAndrews and Potter (2002), there is usually a very high level of payment coordination among the interbank participants of the Payment System. Let P_t^i denote the payments to be made by Bank *i* at time *t*, while R_t^i represents the receipts by Bank *i* from other banks at time *t*. We now follow McAndrews and Potter (2003) analysis of Bank *i*'s reaction function, *i*=1,2,…*I*, as to how much P_t^i relies on R_t^i. Bank *i*'s reaction function is given by

$$P_t^i = a + b \cdot R_t^i . \tag{42}$$

From (42), the coefficient *b* would indicate the strength of the dependence of payments on receipts. Using different panel equations based on information for the year 2001, McAndrews and Potter obtain estimated values of *b* such that $b \in [0.632, 0.765]$, illustrating the high degree of coordination in the functioning of the Payment System[52].

b) A Bank's Beliefs

In simple terms: if Bank *i* fails to make its payments, the rest of banks in the system have not enough receipts to make their own payments, and there is typically a domino effect that reduces the liquidity and payments in the system. The latter could be interpreted as a significant negative shock that reduces the incentives of banks to interact with a central clearing house, in that banks believe that most other banks will not repay their debts. Under these extremes circumstances, a bank may believe that a renewed interaction with a clearinghouse would cause them to repay their own debt at a loss

[52] See also Bech and Garrat (2006.)

because they will not be paid back by other banks. Given these beliefs, banks will find that it to their benefit to cut their link from central clearinghouses.

c) An Interpretation of the Crash of 1987 using our Framework.

The negative shock mentioned in the previous section can be interpreted as bad realization of the shock γ^j, such that $\underline{\gamma} \le \gamma^j < 0$. The optimal rate of default associated with this realization of γ^j is positive given that $\gamma^* = 0$. In this case, strict net settlement and gross settlement yield a set of multiple equilibria displaying strategic complementarities and suboptimal rates of default.

The most important actions and policies implemented by the Federal Reserve System to try to contain the crisis of 1987, according to Wigmore (1998) were: flooding the system with liquidity, bending the legal lending-limits to banks, and persuading the major participants in the Payment System as to get them to provide loans needed by other bank that had been more affected by the crisis. We argue that these actions and policies took the form of debt forgiveness in that they attempted to give all the banks the right incentives to avoid the intrinsic sub optimality of equilibria with rates of default higher than optimal –obtained usually from strict net settlement and gross settlement under these circumstances.)

Conclusions

We have addressed in this model the question of settlement failure, the danger that agents may choose not to repay debts, an option introducing potential strategic complementarities, the existence of which depends on the rules of settling debt. Net settlement with debt forgiveness is shown to free equilibrium behavior from strategic complementarities and lead to a unique, stable and optimal equilibrium. In contrast, strict net settlement rules may result in a multiplicity of equilibria, the best of which may not be optimal and the worst of which features the collapse of the settlement process. Gross settlement, if lacking debt forgiveness, is shown to be no better than strict net settlement, with the same multiplicity of equilibria. When gross settlement forces agents to hold excessive money balances, it is strictly worse than net settlement.

Our model features a deliberately general nonpecuniary (opportunity) cost to default. We find that if this cost is set to zero, the complete collapse of debt markets is the only stable equilibrium under strict net settlement. Nonpecuniary costs to default make possible stable equilibria with less default. Our results with this very general utility cost of default suggest that debt forgiveness will remain playing a central role in preventing excessive default.

A natural extension is to introduce a collateral requirement. We show in Section 4 that introducing collateral reduces the scope for multiplicity of equilibria, as equilibria with universal default disappear. However, it is possible for collateral to generate rates of default that are lower than optimal. The latter, together with the fact that collateral is costly to maintain, lead to the sub optimality of the resulting allocations.

Interesting topics to continue in this line of research are: the determination optimal collateral requirements and the analysis of endogenous default in open economies under alternative exchange regimes.

References:

1. Angelini, Paolo (1998): “An Analysis of Competitive Externalities in Gross Settlement Systems.” Journal of Banking and Finance, vol. 22, no. 1, pp. 1-18
2. Bank for International Settlements (2003): “A Glossary of Terms used in Payments and Settlement System.” Prepared by the Committee on Payment and Settlement Systems.
3. Bank for International Settlements (2007): “Statistics on Payment and Settlement Systems in Selected Countries.” Prepared by the Committee on Payment and Settlement Systems.
4. Bech, Morten L. and Garrat, Rod (2006): “Illiquidity in the Interbank Payment System following Wide-Scale Disruptions.” FRBNY Staff Report 239.

5. Brimmer, Andrew F. (1989): “Distinguished Lecture on Economics in Government: Central Banking and Systemic Risks in Capital Markets.” Journal of Economic Perspectives, vol. 3, no. 2, Spring, pp.3-16.
6. Camera, Gabriele and Li, Yiting (2008): “Another Example of a Credit System that Coexists with Money.” Journal of Money Credit and Banking, forthcoming.
7. Freeman, Scott (1996): “The Payments System, Liquidity and Rediscounting,” American Economic Review, vol. 86, no. 5, pp 1126-1138.
8. Freeman, Scott (1999): “Rediscounting under aggregate risk,” Journal of Monetary Economics 43, pp 197-216.
9. Freeman, Scott and Tabellini, Guido (1998): “The optimality of nominal contracts,” Economic Theory 11, pp 545-562.
10. Freixas, Xavier, and Parigi, Bruno (1998): “Contagion and Efficiency in Gross and Net Interbank Payments System.“ Journal of Financial Intermediation, no. 7, pp.3-31.
11. Freixas, Xavier, Parigi, Bruno and Rochet, Jean-Charles (2000): “Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank.” Journal of Money, Credit and Banking, vol. 32, no. 3, Part 2, pp.611-638.
12. Fujiki, Hiroshi (2003): “A Model of the Federal Reserve Act under the International Gold Standard System.” Journal of Monetary Economics, vol. 50, no. 6, pp. 1333-50.
13. Fujiki, Hiroshi, Green, Edward J. and Yamazaki, Akira (1999): “Sharing the Risk of Settlement Failure.” The Second Joint Central Bank Research Conference on Risk Measurement and Systemic Risk: Toward a Better Understanding of Market Dynamics during Periods of Stress.
14. Hernandez-Verme, Paula (2004): “International Reserves Crises, Monetary Integration and the Payments System during the International Gold Standard." Forthcoming in Macroeconomic Dynamics. Working paper version available at: http://econweb.tamu.edu/phernandez/research.htm.
15. Kahn, Charles M., McAndrews, James and Roberds, William (2003): “Settlement Risk under Gross and Net Settlement.” Journal of Money, Credit and Banking, vol. 4, pp.591-608.
16. Kahn, Charles M. and Roberds, William (2001): “Real-Time Gross Settlement and the Costs of Immediacy.” Journal of Monetary Economics, vol. 47, no. 2, pp. 299-319.
17. Kiyotaki, Nobuhiro, and Moore, John (1997): “Credit Cycles.” Journal of Political Economy, vol. 105, no. 2, pp. 211-248.
18. Lacker, Jeffrey M. (1997): “Clearing, Settlement and Monetary Policy.” Journal of Monetary Economics, vol. 40, no. 2, pp.347-381.
19. McAndrews, James and Potter, Simon (2002): “Liquidity Effects of the Events of September 11, 2001.” FRBNY Economic Policy Review, November, pp.59-79.
20. McAndrews, James and Rajan, Samira (2000): “The Timing and Funding of Fedwire’s Funds Transfers.” FRBNY Economic Policy Review, July, 99.17-32.
21. Rochet, Jean-Charles and Tirole, Jean (1996): “Controlling Risk in Payments Systems.” Journal of Money, Credit, and Banking, vol. 28, no. 4, Part 2, pp.832-862.
22. Wigmore, Barrie A. (1998): “Revisiting the October 1987 Crash.” Financial Analysts Journal, January/February, pp. 36-48. Association for Investment Management and Research.
23. Zhou, Ruilin (2000): “Understanding intraday credit in large-value payment systems,” Economic Perspectives, Federal Reserve Bank of Chicago, 3rd quarter, pp 29-44.a