ESTIMATION OF POPULATION DENSITY BASED ON LINE TRANSECT DATA WITH AND WITHOUT THE SHOULDER CONDITION

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Abstract:

In this paper, new nonparametric estimators for the population density D using line transect sampling are proposed and studied. One estimator is developed when the shoulder condition is assumed to be true and another one when this assumption is violated. The mathematical and numerical properties of these estimators are investigated and compared -via simulation technique- with other existing estimators. A new technique is suggested to combine the proposed estimators. This technique relies on testing the shoulder condition, which in turn produces two new estimators. The simulation results show the good potential performances of the different proposed estimators.

Key Words: Line transect method, Shoulder condition, Estimation of abundance, Kernel method, Smoothing parameter, Boundary effect

Introduction

Line transect method is a popular and convenient technique used to estimate the density (abundance) of a biological population D, since it is direct, cost efficient and can be carried out on foot, or from a variety of land, air, or watercraft. Assume that the population size is N and the sampled area is A then the population density is $D = N / A$. In line transect method, an area of known boundaries and size is divided into non-overlapping strips, each with known length. Then an observer moves on the middle of line of the strip and records the perpendicular distance x from the centerline to a detected object within the strip. The total length of lines $l_1, l_2, \ldots, l_k$ is denoted by L.

The detection function $g(x)$ represents the probability of detecting an object given that its perpendicular distance is x. The logical assumptions on $g(x)$ are (Burnham et al., 1980) $g(x)$ must be monotonically decreasing and objects directly on the transect line will never be missed (i.e., $g(0)$ = 1).

Suppose that the observer detected n objects with perpendicular distances $X_1, X_2, \ldots, X_n$. These perpendicular distances form a random sample of size n that follows a specific pdf $f(x)$. /Burnham and Anderson (1976) introduced the basic relationship between $g(x)$ and $f(x)$, which is given by

$$f(x) = \frac{g(x)}{\int_0^w g(t)dt}, \qquad 0 \le x \le w \tag{1.1}$$

where w is a truncated distance. They gave the fundamental relationship between $f(0) = \left[\int_0^w g(x)dx\right]^{-1}$ and the population abundance, D, which can be expressed as

$$D = \frac{E(n)\,f(0)}{2L} \quad , \tag{1.2}$$

where n is the number of detected objects, $E(n)$ is the expected value of n, and L is the total length of the transect lines.
The estimation of D can be accomplished via the estimation of $f(0)$ by (Burnham et al., 1980)

$$\hat{D} = \frac{n\hat{f}(0)}{2L} \quad , \tag{1.3}$$

where $\hat{f}(0)$ is an estimator of $f(x)$ evaluated on the transect line (i.e., at $x = 0$). As Equation (1.3) demonstrates, the crucial problem in line transect sampling is to estimate $f(0)$ by $\hat{f}(0)$. This leads us to obtain the estimation of density D by $\hat{D}$. Moreover, the estimation of D is equivalent to estimate the number of objects N in a specific known area A. Therefore, the estimation of N can be accomplished by $\hat{N} = A\,\hat{D}$.

The estimator $\hat{f}(0)$ can be obtained by using a parametric approach or a nonparametric approach. The first one assumed that the form of the probability density function $f(x;\theta)$ is known with unknown parameter θ (θ may be a vector). A good statistical method – such as the maximum likelihood method - can be used now to estimate θ and then $f(0;\theta)$. While the parametric method performs well when the form of $f(x;\theta)$ is chosen correctly, its performance is not satisfactory otherwise (Buckland et. al., 2001). As an alternative method to the parametric approach, recent works has focused on employing the nonparametric approach to estimate the parameter $f(0)$ and consequantely the parameter D or N. A popular method is the kernel method which becomes an important tool in wildlife sampling (See for example, Chen, 1996, Mack and Quang, 1998 and Eidous, 2005).

Some Estimators of $f(0)$

The condition $f'(0) = 0$ is known in line transect literature as the shoulder condition assumption, which means that the probability of detecting an object in a narrow area around the centerline remains certain. In this section, we presented some existing estimators for $f(0)$.
Chen (1996) suggested the classical kernel estimator for $f(0)$, which is given by

$$\hat{f}_k(0) = \frac{2}{nh}\sum_{i=1}^{n} K\left(\frac{X_i}{h}\right), \tag{2.1}$$

where h is called the smoothing (or bandwidth) parameter, which controls the smoothness of the fitted density curve and K is the kernel function assumed to be symmetric and satisfies,

$$\int_{-\infty}^{\infty} K(t)dt = 1 \ , \quad \int_{-\infty}^{\infty} tK(t)dt = 0, \quad \int_{-\infty}^{\infty} t^2 K(t)dt = C \neq 0 < \infty.$$

The optimal formula of h can be obtained by minimizing the asymptotic mean square error (AMSE) of $\hat{f}_k(0)$ (see Chen, 1996). Barabesi (2001) proposed a new estimator for $f(0)$ based on local parametric estimation technique. His estimator is given by

$$\text{(2.2)} \quad \hat{f}_B(0) = \hat{f}_k(0)\left[\frac{h^2}{\hat{\gamma}^2} + 1\right]^{\frac{1}{2}}$$

where $\hat{f}_k(0)$ is the classical kernel estimator and $\hat{\gamma}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$. The two estimators $\hat{f}_k(0)$ and $\hat{f}_B(0)$ are developed under the assumption that $f'(0) = 0$. Barabesi's estimator (2.1) can be consider as a special case of Eidous and Alshakhatreh (2011)'s estimator.

Mack et al. (1999) introduced the boundary kernel estimator for $f(0)$ under the assumption that the shoulder condition is not satisfied (i.e. $f'(0) \neq 0$). Their estimator is given by

$$\hat{f}_{ME}(0) = \frac{1}{nh}\sum_{i=1}^{n} K^*\left(\frac{X_i}{h}\right), \tag{2.3}$$

where K^* is a kernel function satisfying

$$\int_0^\infty K^*(u)du = 1\ ,\ \int_0^\infty uK^*(u)du = 0 \text{ and } \int_0^\infty u^2 K^*(u)du = d \neq 0\,.$$

Note that the assumptions about K^* are little different from those about K. Here all integrals are defined on $(0,\infty)$. According to Mack et al. (1999), the boundary kernel function that minimize the AMSE of $\hat{f}_{ME}(0)$ is

$$K^*(u) = 6\left(1 - 3u + 2u^2\right)I_{(0,1)}(u)\,, \tag{2.4}$$

where $I_B(u)$ is an indicator function of a real set B.

Eidous (2011) proposed a new estimator for $f(0)$ without requiring the assumption $f'(0) = 0$. He named his estimator "additive histogram estimator". The additive histogram estimator is given by

$$\hat{f}_{EE}(0) = \frac{1}{nh}\sum_{j=1}^{4}\sum^{n} k_j I_j(X_i)\ . \tag{2.5}$$

where the constant k_i's are $k_1 = \frac{107}{60}$, $k_2 = 0.4$, $k_3 = \frac{-59}{60}$, $k_4 = \frac{41}{120}$, h is the bin width, and $I_j(x)$ is the indicator function defined by

$$I_j(x) = \begin{cases} 1, & 0 < x < jh \\ 0, & o.w \end{cases}.$$

The Proposed Estimators

Let $X_1, X_2, \ldots, X_n$ be a random sample of perpendicular distances of size n. Under the assumption that $f'(0) = 0$, we propose the following estimator for $f(0)$,

$$\hat{f}_{P1}(0) = \frac{2}{nh}\sum_{j=1}^{4}\sum_{i=1}^{n} r_j K\left(\frac{X_i}{jh}\right), \tag{3.1}$$

where $r_1 = 47/50$, $r_2 = 127/200$, $r_3 = -91/150$ and $r_4 = 61/400$. Let $D_v = \sum_{j=1}^{4} j^v r_j$, then $D_1 = 1$, $D_2 = 0.46$, $D_3 = -0.6$. Also, let $T(r_1,\ldots,r_4) = \int_0^\infty K^2(u)du \sum_{j=1}^{4} jr_j^2 + 2\sum_{j=1}^{3}\sum_{l=j+1}^{4} r_j r_l \int_0^\infty K(u/j)K(u/l)du$ and the kernel function

$K(t)$ to be Gaussian function, i.e. $K(t) = N(0,1)$, then $T(r_1,\ldots,r_4) = 0.1847$. The optimal value of h can be obtained by minimizing the $AMSE$ of $\hat{f}_{P1}(0)$, which gives $h = 1.206\,\hat{\sigma}\, n^{-\frac{1}{5}}$ when $f(x)$ is assumed to be half normal with scale parameter σ^2. The illustrations for the use of the above notations are given below. The asymptotic properties of Estimator (3.1) are stated in the following lemma.

Lemma (3.1). Suppose that $f(x)$ is defined on $[0,\infty)$ and has a continuous second derivative at $x = 0$. Under the assumption that $h \to 0$ and $nh \to \infty$ as $n \to \infty$, the expected value and the variance of $\hat{f}_{P1}(0)$ are,

$$E\left(\hat{f}_{P1}(0)\right) = f(0)D_1 + 2hf'(0)D_2\int_0^\infty uK(u)du + h^2 f''(0)D_3\int_0^\infty u^2K(u)du + o(h^2)$$

$$\cong f(0) - 0.6h^2 f''(0)\int_0^\infty u^2K(u)du, \tag{3.2}$$

and

$$\mathrm{var}\left(\hat{f}_{P1}(0)\right) = \frac{4f(0)}{nh}T(r_1,\ldots,r_4) + o(n^{-1}h^{-1}).$$

$$\cong \frac{0.7388\,f(0)}{nh}\ . \tag{3.3}$$

The proof of Lemma (3.1) is given below together with the proof of Lemma (3.2). Note that, because $D_1 = 1$ then $\hat{f}_{P1}(0)$ is asymptotically ($h \to 0$ as $n \to \infty$) unbiased estimator for $f(0)$ and since $f'(0) = 0$ then the convergence rate for bias of $\hat{f}_{P1}(0)$ is $O(h^2)$. Also note that the variance of $\hat{f}_{P1}(0)$ converges to zero as $nh \to \infty$ when $n \to \infty$.

On the other hand if the shoulder condition is not true (i.e. $f'(0) \neq 0$), then the bias of $\hat{f}_{P1}(0)$ is of order h, which is significantely larger than the order h^2 as $h \to 0$. If $f'(0) \neq 0$, we propose the following estimator for $f(0)$,

$$\hat{f}_{P2}(0) = \frac{2}{nh}\sum_{j=1}^{4}\sum_{i=1}^{n} r_j K\left(\frac{X_i}{jh}\right), \tag{3.4}$$

where $r_1 = 43/30$, $r_2 = 7/10$, $r_3 = -31/30$ and $r_4 = 19/60$. Now if $D_v = \sum_{j=1}^{4} j^v r_j$, then $D_1 = 1$, $D_2 = 0$ and $D_3 = -0.6$. Also $T(r_1,\ldots,r_4) = 0.2742$ when the kernel function K is chosen to be Gaussian function (i.e. $K(t) = N(0,1)$). Assume that $f(x) = \frac{1}{\theta}e^{-x/\theta}$, $x \geq 0$ and $K(t)\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ then the optimal formula for estimating the smoothing parameter h is $h = 1.248\,\hat{\theta}\, n^{-\frac{1}{5}}$, where $\hat{\theta} = \overline{X}$.

Lemma (3.2). Suppose that $f(x)$ is defined on $[0,\infty)$ and has a continuous second positive derivative at $x = 0$. Under the assumption that $h \to 0$ and $nh \to \infty$ as $n \to \infty$, the expected value and variance of $\hat{f}_{P2}(0)$ are,

$$E\left(\hat{f}_{P2}(0)\right) = f(0) - 0.6\,h^2 f''(0)\int_0^\infty u^2K(u)du + o(h^2), \tag{3.5}$$

and

$$\mathrm{var}\left(\hat{f}_{P2}(0)\right)=\frac{1.0896\, f(0)}{nh}+o\left(n^{-1}h^{-1}\right). \tag{3.6}$$

Proof of Lemma (3.1) and Lemma (3.2):

Let $\hat{f}_{Pi}(0)$, $i=1,2$ be the proposed estimators, where $\hat{f}_{P1}(0)$ is the Estimator (3.1) and $\hat{f}_{P2}(0)$ is the Estimator (3.4). The expected value of $K(X/jh)$ is

$$EK(X/jh)=\int_0^\infty K(x/jh)\,f(x)dx$$

$$= jh\int_0^\infty K(u)\left(f(0)+jhuf'(0)+(jhu)^2 f''(0)/2+(jhu)^3 f'''(0)/6+\cdots\right)du$$

$$= jhf(0)/2+(jh)^2 f'(0)R_1+(jh)^3 f''(0)R_2/2+(jh)^4 f'''(0)R_3/6+\cdots$$

where $R_z=\int_0^\infty u^z K(u)du$. Therefore, the expected value of $\hat{f}_{Pi}(0)$, $i=1,2$ is

$$E(\hat{f}_{Pi}(0))=\frac{2}{h}\sum_{j=1}^{4} r_j E\left[K(X/jh)\right]$$

$$=\frac{2}{h}\sum_{j=1}^{4} r_j\left[jhf(0)/2+(jh)^2 f'(0)R_1+(jh)^3 f''(0)R_2/2+(jh)^4 f'''(0)R_2/6+\cdots\right]$$

$$= f(0)D_1+2hf'(0)R_1D_2+h^2 f''(0)R_2D_3+o(h^2).$$

Now, for estimator $\hat{f}_{P1}(0)$ we indicated that $D_1=1$, $D_2=0.46$, $D_3=-0.6$ and since $f'(0)=0$, we obtain,

$$E\left(\hat{f}_{P1}(0)\right)= f(0)-0.6\,h^2 f''(0)R_2+o(h^2).$$

Also, for estimator $\hat{f}_{P2}(0)$ we obtained $D_1=1$, $D_2=0$, $D_3=-0.6$, which gives (without assuming that $f'(0)=0$),

$$E\left(\hat{f}_{P2}(0)\right)= f(0)-0.6\,h^2 f''(0)R_2+o(h^2).$$

Note that the bias of the two estimators is of order h^2.

We turn to the variance of $\hat{f}_{Pi}(0)$, $i=1,2$. Suppose that $h\to 0$ and $nh\to\infty$ as $n\to\infty$ then the variance of $\hat{f}_{Pi}(0)$ is

$$Var(\hat{f}_{Pi}(0))=\frac{4}{nh^2}\mathrm{var}\left[\sum_{j=1}^{4} r_j K(X/jh)\right]$$

$$=\frac{4}{nh^2}\left(E\left[\sum_{j=1}^{4} r_j K(X/jh)\right]^2-\left[\sum_{j=1}^{4} r_j EK(X/jh)\right]^2\right). \tag{3.7}$$

By substituting the expression of $EK(X/jh)$ in the second term of (3.7), then we obtain

$$Var(\hat{f}_{Pi}(0))=\frac{4}{nh^2}E\left[\sum_{j=1}^{4} r_j K(X/jh)\right]^2+o(n^{-1}h^{-1})$$

$$=\frac{4}{nh^2}E\left[\sum_{j=1}^{4}\sum_{l=1}^{4} r_j r_l K(X/jh)K(X/lh)\right]+o(n^{-1}h^{-1})$$

$$= \frac{4}{nh^2} E\left[\sum_{j=1}^{4} r_j^2 K^2(X/jh) + 2\sum_{j<l}^{4} r_j r_l K(X/jh)K(X/lh)\right] + o(n^{-1}h^{-1})$$

$$= \frac{4}{nh^2}\left[\sum_{j=1}^{4} r_j^2 E\left(K^2(X/jh)\right) + 2\sum_{j=1}^{3}\sum_{l=j+1}^{4} r_j r_l E\left(K(X/jh)K(X/lh)\right)\right] + o(n^{-1}h^{-1})$$

(3.8)

Now,

$$E\left(K^2(X/jh)\right) = \int_0^\infty K^2(x/jh)f(x)dx$$

$$= jh\int_0^\infty K^2(u)\left(f(0) + jhuf'(0) + (jhu)^2 f''(0)/2 + \ldots\right)du$$

$$= jhf(0)\int_0^\infty K^2(u)du + o(h). \qquad \textbf{(3.9)}$$

Also,

$$E\left(K(X/jh)K(X/lh)\right) = \int_0^\infty K(x/jh)K(x/lh)f(x)dx$$

$$= h\int_0^\infty K(u/j)K(u/l)\left(f(0) + huf'(0) + (hu)^2 f''(0)/2 + \ldots\right)du$$

$$= hf(0)\int_0^\infty K(u/j)K(u/l)du + o(h). \qquad \textbf{(3.10)}$$

By substituting (3.9) and (3.10) back into (3.8), we obtain

$$Var\left(\hat{f}_{Pi}(0)\right) = \frac{4hf(0)}{nh^2}\left[\int_0^\infty K^2(u)du\sum_{j=1}^{4} jr_j^2 + 2\sum_{j=1}^{3}\sum_{l=j+1}^{4} r_j r_l \int_0^\infty K\left(\frac{u}{j}\right)K\left(\frac{u}{l}\right)du\right] + o(n^{-1}h^{-1})$$

$$= \frac{4f(0)}{nh}T(r_1,\ldots,r_4) + o\left(n^{-1}h^{-1}\right).$$

Now, for estimator $\hat{f}_{P1}(0)$**,** $T(r_1,\ldots,r_4) = 0.1847$**. This gives,**

$$Var\left(\hat{f}_{P1}(0)\right) = \frac{0.7388 f(0)}{nh} + o(n^{-1}h^{-1}).$$

Also, for estimator $\hat{f}_{P2}(0)$**,** $T(r_1,\ldots,r_4) = 0.2742$**. Therefore,**

$$Var\left(\hat{f}_{P2}(0)\right) = \frac{1.0896 f(0)}{nh} + o(n^{-1}h^{-1}).$$

This completes the proof. Note that the variance of $\hat{f}_{P1}(0)$ and $\hat{f}_{P2}(0)$ is of order $n^{-1}h^{-1}$ (or the convergence rate for the variance of $\hat{f}_{P1}(0)$ and $\hat{f}_{P2}(0)$ is $O(n^{-1}h^{-1})$).

Combining the Estimators

In this section, we propose another two estimators for $f(0)$ that combining the two estimators $\hat{f}_{P1}(0)$ and $\hat{f}_{P2}(0)$. Let F_0 = the class of all *pdf*s that satisfy $f'(0) = 0$ and F = the class of all *pdf*s that are differentiable at 0, and consider the following test,

$$H_0 : f \in F_0 \quad \text{vs.} \quad H_1 : f \in F \setminus F_0. \qquad (4.1)$$

According to Mack (1998), H_0 is reject (i.e., the shoulder condition is not true) if | T | > -Z$_{\alpha/2}$, where Z$_{\alpha/2}$ represents the α/2th quantile of the standard normal distribution. The test statistics T is defined by

$$T = S'(0)\sqrt{\frac{nb^3}{2\hat{f}_k(0)}} \quad ,$$

where $S'(0)$ is estimated by $\hat{S}'(0) = \frac{[F_n(2b) - 2F_n(b)]}{b^2}$, $b = \hat{\sigma}\ n^{-\frac{1}{4}}$, $\hat{\sigma} = \sqrt{\sum_{i=1}^{n} x_i^2 / n}$ and $F_n(u) = \frac{\# x_i \in [0,u]}{n}$ is the empirical cumulative distribution function. Based on testing (4.1), we propose the following two estimators for $f(0)$,

$$\hat{f}_{P3}(0) = \begin{cases} \hat{f}_{P1}(0) & if \quad f \in F_0 \\ \hat{f}_{P2}(0) & if \quad f \notin F_0 \end{cases}, \tag{4.2}$$

and

$$\hat{f}_{P4}(0) = \alpha\ \hat{f}_{P1}(0) + (1-\alpha)\ \hat{f}_{P2}(0), \tag{4.3}$$

where the estimators $\hat{f}_{P1}(0)$ and $\hat{f}_{P2}(0)$ are given by (3.1) and (3.4) respectively. The parameter $\alpha \in [0,1]$ represents the weight of $\hat{f}_{P1}(0)$ in the final estimator $\hat{f}_{P4}(0)$. In this study, we suggest to choose α to be the $p-value$ of the Test (4.1). A large $p-value$ supports the hypothesis $H_0 : f \in F_0$ and then $\hat{f}_{P1}(0)$ is more appropriate (has larger weight) than $\hat{f}_{P2}(0)$ to estimate $f(0)$. The $p-value$ of Test (4.1) is

$$p - value = 2\,pr(Z < -|T|)$$

$$= 2\Phi(-|T|)\ ,$$

where Φ is the standard normal distribution function. The p-value indicates how strong H_0 is supported by the data. The properties of these proposed estimators are studied via simulation in the next section.

Simulation Study

To compare among the performances of the different estimators, a simulation study was performed. The data are simulated from densities that satisfy $f'(0) = 0$ (e.g. half normal) and from densities that do not satisfy $f'(0) = 0$ (e.g. negative exponential).

The smoothing parameter h for the different estimators is computed by using the formula $h = A\hat{\sigma}\ n^{-\frac{1}{5}}$, where $A = 0.933$ for $\hat{f}_k(0)$ and $\hat{f}_B(0)$; and $A = 1.206$ for the proposed estimator $\hat{f}_{P1}(0)$. Also, $h = B\hat{\theta}\,n^{-\frac{1}{5}}$, where $B = 3.438$ for estimator $\hat{f}_{ME}(0)$; $B = 3.122$ for estimator $\hat{f}_{EE}(0)$; and $B = 1.248$ for the proposed estimator $\hat{f}_{P2}(0)$.

All results in tables (1) and (2) depend on simulated 1000 samples of sizes $n = 50,\ 100,\ 200$. The data generated from three different families of models which are commonly used in line transect studies (see Barabesi, 2001 and Eidous, 2009). The first model is the exponential power (EP) family (Pollock,1978)

$$f(x) = \frac{1}{\Gamma\left(1+\frac{1}{\beta}\right)} \exp\left(-x^{\beta}\right), \quad x \geq 0,\ \beta \geq 1,$$

with detection function $g(x) = \exp\left(-x^{\beta}\right)$. The hazard rate (HR) family (Hayes and Buckland,1983)

$$f(x) = \frac{1}{\Gamma\left(1-\frac{1}{\beta}\right)} \left(1-\exp\left(-x^{-\beta}\right)\right), \quad x \geq 0,\ \beta > 1,$$

with detection function $g(x) = \left(1-\exp(-x^{-\beta})\right)$, and the beta (BE) model (Eberhardt,1968)

$$f(x) = (1+\beta)(1-x)^{\beta},\ 0 \leq x < 1\ \beta \geq 0,$$

with detection function $g(x) = (1-x)^{\beta}$. In our simulation design, these three families were truncated at some distance w. Four models were selected from EP family with parameter values $\beta = 1.0,\ 1.5,\ 2.0,\ 2.5$ and corresponding truncation points given by $w = 5.0,\ 3.0,\ 2.5,\ 2.0$. Four models were selected from HR family with parameter values $\beta = 1.5,\ 2.0,\ 2.5,\ 3.0$ and corresponding truncation points given by $w = 20,\ 12,\ 8,\ 6$. Moreover, four models were selected from BE model with parameter values $\beta = 1.5,\ 2.0,\ 2.5,\ 3.0$ and $w = 1$ for all cases. The considered models cover a wide range of perpendicular distance probability density functions which vary near zero from spike to flat. The shoulder condition do not satisfy for BE model with different values of β and for EP model with $\beta = 1$. Also, despite the shoulder condition is satisfied for HR model, this model decreases sharply away from the original point (i.e. $x = 0$) when $\beta = 1.5$, 2.0. This case may be occur in practice when the visibility away from the transect line is not distinct due to - may be – fog and tall grass.

For each considered estimator and for each sample size, the relative bias

$$RB = \frac{E\left(\hat{f}(0)\right) - f(0)}{f(0)},$$

and the relative mean error

$$RME = \frac{\sqrt{MSE\left(\hat{f}(0)\right)}}{f(0)},$$

are computed. The relative bias of the different estimators are presented in Table (1), while the relative mean errors are given in Table (2).

Results and Conclusions

(1) The proposed estimator $\hat{f}_{P1}(0)$ performs well as a general estimator. Despite that this estimator is developed under the constraints $f'(0) = 0$, it performs well even for models with $f'(0) \neq 0$. Also, the results of Barabesi's estimator $\hat{f}_B(0)$ are acceptable in general.

(2) If the set of data seem to be spike at the origin, the proposed estimator $\hat{f}_{P2}(0)$ is a very competitor for the other existing estimators and can be recommended in this case.
(3) The idea of combining between the proposed estimators based on testing the shoulder condition assumption seems to be success in some cases. Among the two combining estimators, the estimators $\hat{f}_{P3}(0)$ performs better than $\hat{f}_{P4}(0)$ in general.
(4) It is not easy job to recommend a specific estimator –from those considered in this thesis- as a best estimator for all cases. However, we can close our comments and conclusions by saying that:

- The classical kernel estimator $\hat{f}_k(0)$ is recommended when the model of data has a large shoulder at the origin provided that it does not decrease sharply away the origin (e.g. EP model with $\beta = 2.5$ and HR model with $\beta = 3.0$).
- The Barabesi's estimator $\hat{f}_B(0)$ and the proposed estimator $\hat{f}_{P1}(0)$ are recommended for data models with moderate shoulder condition at the origin (e.g. EP model with $\beta = 1.5$ and HR model with $\beta = 2.0$).
- Eidous's estimator $\hat{f}_{EE}(0)$ and the proposed estimator $\hat{f}_{P2}(0)$ are recommended for data models that do not have a shoulder condition at the origin (e.g. EP model with $\beta = 1.0$ and BE model with different values of β), or even for data models that have a shoulder but decreases markedly away the origin (e.g. HR model with $\beta = 1.5$). In these two cases, the proposed estimators $\hat{f}_{P3}(0)$ and $\hat{f}_{P4}(0)$ are also perform well.

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Table 1. The Relative Bias (RB) for $\hat{f}_k(0)$, $\hat{f}_B(0)$, $\hat{f}_{P1}(0)$, $\hat{f}_{ME}(0)$, $\hat{f}_{EE}(0)$, $\hat{f}_{P2}(0)$, $\hat{f}_{P3}(0)$ and $\hat{f}_{P4}(0)$.

β	w	n	$\hat{f}_k(0)$	$\hat{f}_B(0)$	$\hat{f}_{P1}(0)$	$\hat{f}_{ME}(0)$	$\hat{f}_{EE}(0)$	$\hat{f}_{P2}(0)$	$\hat{f}_{P3}(0)$	$\hat{f}_{P4}(0)$
Exponential power (EP) model										
		50	-0.323	-0.264	-0.309	-0.070	-0.204	-0.116	-0.247	-0.168
1.0	5.0	100	-0.290	-0.242	-0.270	-0.045	-0.163	-0.100	-0.193	-0.129
		200	-0.262	-0.225	-0.234	-0.040	-0.130	-0.085	-0.171	-0.105
		50	-0.147	-0.072	-0.099	0.046	0.062	0.057	-0.089	-0.013
1.5	3.0	100	-0.130	-0.072	-0.072	0.027	0.084	0.074	-0.060	0.011
		200	-0.109	-0.063	-0.048	0.027	0.100	0.057	-0.035	0.021
		50	-0.076	0.005	-0.004	0.064	0.200	0.124	-0.001	0.065
2.0	2.5	100	-0.060	0.003	0.013	0.049	0.207	0.117	-0.005	0.035
		200	-0.049	0.001	0.036	0.029	0.204	0.099	0.028	0.065
		50	-0.036	0.049	0.037	0.063	0.298	0.150	0.039	0.094
2.5	2.0	100	-0.032	0.033	0.055	0.038	0.286	0.123	0.067	0.106
		200	-0.025	0.024	0.074	0.024	0.253	0.098	0.049	0.070
Hazard rate (HR) model										
		50	-0.363	-0.308	-0.406	0.157	-0.275	-0.053	-0.095	-0.072
1.5	20.0	100	-0.329	-0.284	-0.355	0.183	-0.228	-0.012	-0.008	-0.004
		200	-0.275	-0.238	-0.296	0.223	-0.153	0.061	0.065	0.065
		50	-0.225	-0.157	-0.228	0.166	-0.066	0.090	-0.008	0.075
2.0	12.0	100	-0.179	-0.124	-0.181	0.180	0.009	0.135	0.030	0.104
		200	-0.135	-0.091	-0.130	0.176	0.087	0.162	0.089	0.153
		50	-0.102	-0.023	-0.074	0.138	0.142	0.176	-0.023	0.103
2.5	8.0	100	-0.067	-0.005	-0.031	0.127	0.211	0.207	0.012	0.124
		200	-0.040	0.009	0.016	0.101	0.272	0.208	0.042	0.128

		50	-0.046	0.037	0.012	0.108	0.261	0.216	0.037	0.131
3.0	6.0	100	-0.023	0.042	0.044	0.065	0.330	0.215	0.069	0.141
		200	-0.007	0.043	0.078	0.048	0.354	0.199	0.096	0.143
Beta model		(BE)								
		50	-0.166	-0.093	-0.126	0.002	0.049	0.018	-0.104	-0.041
1.5	1.0	100	-0.149	-0.092	-0.095	-0.002	0.055	0.009	-0.090	-0.045
		200	-0.127	-0.083	-0.061	0.003	0.044	0.006	-0.075	-0.033
		50	-0.186	-0.115	-0.151	0.004	0.006	-0.009	-0.126	-0.061
2.0	1.0	100	-0.172	-0.117	-0.123	-0.016	0.014	-0.010	-0.113	-0.056
		200	-0.153	-0.110	-0.095	-0.007	0.004	-0.003	-0.094	-0.046
		50	-0.213	-0.144	-0.174	-0.025	-0.030	-0.028	-0.157	-0.088
2.5	1.0	100	-0.190	-0.135	-0.145	-0.018	-0.015	-0.033	-0.131	-0.070
		200	-0.176	-0.133	-0.112	-0.026	-0.016	-0.011	-0.105	-0.054
		50	-0.223	-0.155	-0.192	-0.026	-0.047	-0.033	-0.159	-0.083
3.0	1.0	100	-0.200	-0.146	-0.162	-0.019	-0.033	-0.036	-0.138	-0.075
		200	-0.180	-0.138	-0.129	-0.017	-0.025	-0.023	-0.115	-0.062

Table 2. The Relative Mean Error (RME) for $\hat{f}_k(0)$, $\hat{f}_B(0)$, $\hat{f}_{P1}(0)$, $\hat{f}_{ME}(0)$, $\hat{f}_{EE}(0)$, $\hat{f}_{P2}(0)$, $\hat{f}_{P3}(0)$ and $\hat{f}_{P4}(0)$.

β	w	n	$\hat{f}_k(0)$	$\hat{f}_B(0)$	$\hat{f}_{P1}(0)$	$\hat{f}_{ME}(0)$	$\hat{f}_{EE}(0)$	$\hat{f}_{P2}(0)$	$\hat{f}_{P3}(0)$	$\hat{f}_{P4}(0)$
Exponential power (EP) model										
		50	0.338	0.284	0.325	0.208	0.236	0.189	0.294	0.232
1.0	5.0	100	0.299	0.255	0.280	0.160	0.188	0.150	0.245	0.186
		200	0.269	0.233	0.241	0.125	0.149	0.121	0.207	0.143
		50	0.193	0.154	0.157	0.252	0.160	0.181	0.148	0.136
1.5	3.0	100	0.160	0.123	0.113	0.187	0.137	0.155	0.132	0.142
		200	0.132	0.101	0.086	0.151	0.131	0.116	0.097	0.099
		50	0.154	0.146	0.126	0.283	0.246	0.218	0.126	0.163
2.0	2.5	100	0.119	0.109	0.097	0.208	0.233	0.182	0.097	0.120
		200	0.094	0.085	0.084	0.159	0.222	0.145	0.095	0.125
		50	0.151	0.167	0.134	0.294	0.335	0.239	0.137	0.190
2.5	2.0	100	0.117	0.125	0.116	0.224	0.309	0.187	0.129	0.165
		200	0.088	0.092	0.105	0.169	0.268	0.149	0.090	0.117
Hazard rate (HR) model										
		50	0.382	0.333	0.421	0.279	0.318	0.206	0.216	0.192
1.5	20.0	100	0.339	0.297	0.365	0.246	0.256	0.134	0.144	0.140
		200	0.283	0.249	0.303	0.258	0.180	0.120	0.120	0.120
		50	0.257	0.208	0.264	0.275	0.184	0.198	0.220	0.200
2.0	12.0	100	0.202	0.159	0.205	0.243	0.136	0.184	0.183	0.172
		200	0.150	0.114	0.147	0.216	0.131	0.180	0.193	0.192
		50	0.159	0.134	0.147	0.267	0.230	0.246	0.186	0.213
2.5	8.0	100	0.113	0.097	0.100	0.224	0.247	0.244	0.142	0.189
		200	0.077	0.072	0.067	0.171	0.289	0.230	0.108	0.168
		50	0.129	0.137	0.128	0.272	0.308	0.281	0.152	0.219
3.0	6.0	100	0.094	0.106	0.100	0.197	0.353	0.255	0.131	0.192

		200	0.076	0.091	0.106	0.167	0.365	0.222	0.122	0.170
Beta model		(BE)								
		50	0.207	0.164	0.175	0.249	0.140	0.175	0.178	0.176
1.5	1.0	100	0.178	0.138	0.135	0.191	0.115	0.133	0.138	0.126
		200	0.148	0.115	0.092	0.152	0.089	0.100	0.117	0.107
		50	0.222	0.175	0.184	0.244	0.129	0.167	0.187	0.174
2.0	1.0	100	0.196	0.154	0.150	0.186	0.100	0.123	0.151	0.130
		200	0.168	0.131	0.116	0.139	0.077	0.103	0.130	0.110
		50	0.243	0.192	0.205	0.240	0.126	0.167	0.211	0.182
2.5	1.0	100	0.209	0.165	0.167	0.179	0.097	0.128	0.173	0.144
		200	0.189	0.152	0.130	0.142	0.081	0.106	0.146	0.119
		50	0.249	0.197	0.218	0.223	0.133	0.165	0.210	0.176
3.0	1.0	100	0.219	0.175	0.182	0.178	0.105	0.137	0.179	0.142
		200	0.192	0.154	0.143	0.133	0.082	0.101	0.153	0.118