CREDIT CHAINS AND MORTGAGE CRISES

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Abstract:

I examine a production economy with a financial sector that contains multiple layers of credit. Such layers are designed to constitute credit chains which are inclusive of a simple mortgage market. The focus is on the nature and contagion properties of credit chains in an economy where the financial sector plays a real allocative role and agents have a nontrivial choice of whether to default on mortgages or not. Multiple equilibria with different rates of default are observed, due to the presence of strategic complementarities. Default can trigger a financial crisis as well as constrain the purchases of factors of production, thus leading to potentially serious effects on real activity.

Key Words: Credit chains, mortgages, default

Resumen

Examiné una economía de producción con un sector financiero que contiene múltiples capas de crédito. Tales capas están diseñadas para constituir cadenas de crédito las cuales son incluyentes de un mercado hipotecario sencillo. La atención se centra en la naturaleza y las propiedades de contagio de las cadenas de crédito en una economía donde el sector financiero juega un rol real de asignación de recursos y los agentes tienen una elección no trivial ya sea por defecto de las hipotecas o no. Equilibrios múltiples con diferentes tasas de incumplimiento son observados, debido a la presencia de estrategias complementarias. El incumplimiento puede desencadenar una crisis financiera tan buena como limitar las compras de factores de producción, entonces conduce efectos potencialmente serios en la actividad real.

Introduction

In this paper, I build a dynamic stochastic general equilibrium model of credit chains in a closed-economy payments system that shares the spirit of Freeman (1996, 2001), Hernandez-Verme (2004) and Freeman and Hernandez-Verme (2008.) I must point out outright that I do not aim at replicating the facts before and after the subprime mortgage crisis. My purpose is humbler: to illustrate the presence and functioning of credit chains in the overall structure of a financial system where spatial separation is nontrivial. The following innovations in my model are of the utmost importance: the presence of a simple mortgage market, the presence of a strategic group of banks who are local monopolies in offering deposits and mortgage loans; the double role of the lending sector, which is also the productive sector in this economy; the presence of a shock that sizes down (or up) the value of the real state at the time when the mortgages are repaid, leading to a nontrivial choice of whether to default on mortgages or not.

There are three groups of strategic agents that interact in a payments system in this model economy: lenders, borrowers and banks. Moreover, one can classify borrowers in two classes in equilibrium: borrowers who default on their debt and borrowers who repay their debt. My main underlying hypothesis is that, even in the absence of a house bubble, the structure of the financial system itself makes it vulnerable to default, systemic risk and downturns in economic activity. Moreover, the analysis of welfare in equilibrium will be shown to defy conventional wisdom in terms of who wins and who loses as a result of a crisis in this economy.

The contributions of this paper with respect to the previous literature can be listed as follows:

- The presence of a shock that alters the resources available to borrowers[66].
- Borrowers choosing endogenously whether to default or not.
- The presence of banks that are local monopolies and offer both deposit and mortgage contracts.
- The introduction of a very simple mortgage market in a model of the payment system
- The linkage between production possibilities and default on debt
- The presence of strategic complementarities that may lead to the presence of multiple equilibria

In this model economy, default reduces the resources available to purchase labor, subsequently reducing output. There is always a positive rate of default in equilibrium, whether it is unique or not. Moreover, universal default and universal repayment cannot obtain in equilibrium due to the nature of the shock that continuously hits this economy and the signaling to lenders by borrowers, respectively. The equilibrium interest rates on deposits, IOUs and loans display the potential for the existence of two equilibria: one with a low rate of default and the other with a high rate of default. However, the model so far has not produced any criteria for equilibrium selection yet[67]. I must also point out that, against standard intuition, the bank obtains positive profits only when the equilibrium rates of default are sufficiently high; my interpretation of this result is that banks are risk lovers in equilibrium.

In this model, young borrowers formulate a contingent plan in light of the realization of the shock that they will experience the following period. This works through the choice of a cut-off value from the distribution of the shock, such that realizations below the cut-off will lead to defaulting while realizations above the cut-off will lead to repayment. A crisis in this model takes the form of an innovation in such cut-off value of the shock. The main results of the analysis are as follows:

1. A crisis reduces total output in this economy by reducing the resources available to purchase labor.
2. A crisis also reduces the aggregate welfare of borrowers. However, this effect can be misleading, since the borrowers who default experience welfare gains, while the “honest” borrowers who choose to repay their debt are made worse-off. Thus, contrary to conventional wisdom, the result shows that the borrowers who default do not suffer from this shock and that the ones who are in the need of help, probably from government agencies, are the “honest” borrowers.
3. The lenders in this economy (equivalent to the general public who holds deposits on banks) experience significant welfare gains as a result of a crisis. This, then, is another sector that is not in need of government assistance.
4. A crisis increases the bank’s income but reduces its costs, thus increasing the bank’s profits in equilibrium. This result seems to go against conventional wisdom, but remember that there are no investment banks in this model nor mortgage backed securities. This is consistent with the fact that commercial banks seem to be doing reasonable well in light of the subprime mortgage crisis.

Finally, I would like to mention that my initial intentions included the examination of alternative Liquidity Saving Mechanisms (LSM) and alternative rules for the settlement of debt as well, but, due to my time constraint, this has not been incorporated into the model yet.

[66] This is an improvement on Freeman and Hernandez-Verme (2008,) that used a location-specific utility shock.
[67] This is mostly due to my time constraint, but I will be working on this subject next.

The Environment

Consider a model closed economy consists of one central settlement location and $I > 2$ triplets of outer islands, indexed by $i = 1, 2, ...I$. Time is discrete and indexed by $t=1, 2, ...$. All strategic agents are overlapping generations that live for 2-periods, and population is constant. Throughout the paper, I assume that all contracts are enforceable when on the Central Island, but not on the outer islands. There is a fiat currency that circulates in this economy. For simplicity, I will label this currency as dollars.

In this model economy, each date has two parts: morning and afternoon, and transactions/actions take place sequentially in each of them. All strategic agents must travel to the central island in the morning of their old age, where contracts can be enforced. On the central island, clearing and settlement of debt will take place through a third party, which I will interchangeably call throughout the centralized settlement institution or monetary authority.

The Main Actors

There are three groups of strategic agents in each triplet i. There is a continuum of borrowers with unit mass on the first island of the triplet, which I will call Borrowers i, and a continuum of Lenders also with a unit mass on the second island of the triplet, which I will call Lenders i. Finally, there is a monopolistic bank on the third island of the triplet, which I will call Bank i.

There is also a nonstrategic agent on the central island that I will call the monetary authority, which is also the central settlement institution. Each bank must create a reserve account with the monetary authority.

The Goods

There are five goods in this economy. First, Lenders i have an endowment good when young. The size of this endowment is $x>0, \forall i$, but this good is island-specific, with a price of p_t^i dollars per good at date t. Second, Lenders i can produce a final good when old. This good is also island-specific, and its price at date t is θ_t^i dollars per good.

In the third place, young Borrowers i have an endowment good in the amount $y \in (0,1)$. This good is also island-specific, and it is attachable as down-payment for a house in Banks i in the following way: y is the fraction of the value of a house that Borrowers i can put as a down-payment when young. This good is not otherwise traded.

Fourthly, Bank i is endowed with $z>0$ of an agent- and triplet-specific good. Bank i can only use this good to partially finance mortgage loans to young Borrowers i at the price μ_t^i. This good is not traded otherwise.

The last "good" in this economy is labor. Young borrowers in island i are endowed with one unit of non storable labor. Labor is homogeneous across agents and islands, and it is traded for the nominal wage w_t in the afternoon of date t.

The Assets

There are six different assets in this economy. In the first place, we have the debt (IOUs) issued by Borrowers i to Lenders $i+1$, so that the former can purchase the endowment good of the latter. The associated promised real gross interest rate r_{t+1}^{i+1} is set on triplet $i+1$ at date t by young lenders. This loan must be repaid next period using fiat money.

Secondly, there is a fixed stock I of identical houses, such that there is one house per each Borrower. In each triplet, there is a continuum of houses with unit mass that is located in the

Borrowers' island. The price of a house in triplet i is q_t^i dollars at date t. Houses do not depreciate, other than for a shock that we will discuss in the next section.

Third, young Lenders i can accumulate physical capital by investing when young. This investment technology will yield as much capital at $t+1$ as it was invested at t. Capital is island-specific and it depreciates completely after production, and I will denote it by K_{t+1}^i. In the fourth place, there is fiat money issued by the monetary authority, and M_t denotes the nominal money supply at date t. Fiat money is used to purchase labor, to pay mortgages in full, to settle IOUs and to purchase the Lender i's final good. The return on real money balances between dates t and $t+1$ is given by $\left(w_t / w_{t+1}\right)$.

The fifth asset consists of mortgage contracts issued by Bank i to Borrowers i. The promised real gross interest rate set at date t by the bank is ρ_{t+1}^i, and mortgages must be repaid in full with fiat money the next period. Finally, young Lenders i can also make deposits in the monopolistic Bank i in the amount d_t^i at date t. The promised real gross interest rate on deposits set by the bank at date t is R_{t+1}^i, which is paid at date $t+1$.

Uncertainty

In this economy, all Borrowers i are ex ante identical. However, they expect a shock to be realized at the beginning of their old age. This shock will affect the amount of real state held by old borrowers in that it can reduce or increase the value of individual real state, but leaves the aggregate real state unchanged. I will denote the realization of the shock to household j in triplet i at date t by $\lambda_t^{i,j}$. However, I will drop the subscript and supra-scripts in the remainder of this section in order to simplify both notation and explanation.

I let λ denote the shock, which represents the fraction of the house that is left to borrowers in the morning when old. This shock is i.i.d.-distributed over borrower households, triplets and time. I use $g(\lambda)$ to denote the stationary p.d.f. of the shock, while $G(\lambda)$ denotes the associated c.d.f. The support set of this shock is given by $\left[\lambda_L, \lambda_H\right]$, where I assume that $\lambda_L \in [0,1)$ and $\lambda_H > 1$. Henceforth, for tractability, I will assume λ has a uniform distribution. Thus, $g(\lambda) = \dfrac{1}{\lambda_H - \lambda_L}$ and $G(\lambda) = \dfrac{\lambda}{\lambda_H - \lambda_L}$. The distribution of the shock is public information, but not its individual realization, which will be known only to the individual borrower.

The variable λ is a multiplicative shock that alters the resources available to old borrowers. The old borrower must pay $\rho_{t+1}^i \cdot q_t^i \cdot (1-y)$ dollars to the bank early in the morning of date $t+1$ to repay her mortgage in full, but her resources are scaled down (or up) to $\lambda_{t+1} \cdot \rho_{t+1}^i \cdot q_t^i \cdot (1-y)$. Before she sells the house, she must repair it, incurring in a cost of $\left(1-\lambda_{t+1}\right) \cdot q_{t+1}^i$ dollars $\forall \lambda_{t+1} \le 1$, to bring the house back to its original condition[68]. Next, she sells the house for q^i_{t+1}, obtaining the net the profit $\lambda_{t+1} \cdot q^i_{t+1}$. In case the borrower defaults, the bank seizes the house and it must make the same repairs before selling it.

Young borrowers anticipate this, but not the particular realization of λ_{t+1}, and they form a contingent plan in which they choose a cut-off value $\tilde{\lambda}$ of the shock from the support of the distribution, such that $\tilde{\lambda} \in \left[\lambda_L, \lambda_H\right]$. In particular, borrower households will repay their mortgage when the realization of the shock is such that $\tilde{\lambda} \le \lambda_{t+1} \le \lambda_H$, while they will default when $\lambda_L \le \lambda_{t+1} < \tilde{\lambda}$ obtains. As we will see later, a choice of repaying or defaulting on a mortgage is, at the same time, a choice of repaying or defaulting on IOU: all these transactions must take place on the central island and if the borrower chooses to repay her IOU, it must also repay her mortgage.

Default is also costly to borrowers in utility terms. Old borrowers may consume (use) their houses when old in the afternoon: $h^i_{t+1} = \{0,1\}, \forall i, t > 0$. $h^i_{t+1} = 1$ only when the mortgage has been repaid in full, but $h^i_{t+1} = 0$ if they default on the mortgage, and they must forego the utility of the house they bought the previous date.

The stock of houses left in a particular triplet after the shocks are realized and before the houses are repaired is given by $\int_{\lambda_L}^{\lambda_H} \lambda \cdot g(\lambda) \cdot d\lambda = \left(\lambda_H + \lambda_L\right) \big/ 2 \le (\ge) 1$. After the repairs, the stock of houses per triplet goes back to $\int_0^1 (1)\, dj = 1$.

One can also calculate the probability of default in a particular island by using the Law of Large Numbers. Let π^i_{t+1} denote the probability of default on a particular triplet. Then, $\pi^i_{t+1} = \int_{\lambda_L}^{\tilde{\lambda}} g(\lambda) \cdot d\lambda = G(\tilde{\lambda}) = \tilde{\lambda} \big/ \left(\lambda_H - \lambda_L\right)$. The reader may notice that I keep the supra-script i on the rate of default, since there is a potential for multiple equilibria in this economy. The promised interest rates will be reduced by the factor $\left(1 - \pi^i_{t+1}\right)$ in equilibrium.

Endowments of the Agents in Triplet i

Lenders. Lenders are endowed with $x > 0$ units each of their endowment good when young, which is island-specific and it non storable. They are also endowed with an investment technology that allows them to accumulate physical capital when young. This technology is island-specific and not transferable, and so is the capital.

When old, lenders are endowed with a technology that allows them to produce an island-specific final good, for which they need to utilize physical capital and labor to produce the specific final good i when old, for which they need physical capital and labor. This technology is island-specific and not transferable. Lenders have no other endowment when old.

Borrowers. Young borrowers are endowed with $y \in (0,1)$, where y represents the fraction of the value of a house they can put as down payment. This endowment good is island-specific and

[68] Notice that when $\lambda_{t+1} \in \left(1, \lambda_H\right]$, the borrower is entitled to a profit in the order of $\left(\lambda_{t+1} - 1\right) \cdot q^i_{t+1}$ dollars.

only attachable as a down-payment for a house. In addition, they are endowed with a technology to issue IOUs to Lenders from island $i+1$, which is island-specific and not transferable as well. Finally, also when young, borrowers are endowed with one unit of non storable labor, which is homogeneous across agents and borrowers. Borrowers have no endowment when old.

The Bank. When young, the bank is endowed with $z>0$ units of and endowment good, which will be used to partially finance mortgages. The young bank has a monopoly on the following two activities in triplet i: first, they can offer deposit contracts from young lenders, and, second, they can package mortgage contracts to be offered to young borrowers. These last activities are island-specific and non transferable. The bank has no endowments when old.

The Initial Old. At the initial date, $t=0$, there is a generation of initial old agents in each triplet i with the same size as the regular generation born at date $t\geq 0$. They hold the initial supplies of assets in this economy. Thus, the initial old borrowers hold the endowment of houses in each triplet, given by $I/I=1$. The initial money supply (the monetary base) is distributed equally among triplets in the amount of $m_0 \equiv \left(M_0/I\right)$ dollars. The money supply in a particular triplet is distributed proportionately to initial old lenders and the initial old bank. Notice that the latter implies I endogenous initial conditions, summarized by $\left\{m_0/p_0^i\right\}_{i=1}^{I}$. Finally, the initial stock of capital in triplet i is given by K_0^i, which is distributed equally among initial old lenders. The latter implies I exogenous initial conditions, which are summarized by $\left\{K_0^i\right\}_{i=1}^{I}$.

Monetary Policy and Aggregate Money Supply

The monetary policy in this model economy is very simple. For now, I assume that supply of high power money is constant, such that $M_t=M_0, \forall t\geq 0$. In addition, the I banks in this economy must hold domestic currency reserves with the central bank, where $\phi\in[0,1]$ denotes the fraction of deposits that each bank must hold each period. Moreover, in this model economy, banks give loans to young borrowers, who use them to purchase houses. Thus, the aggregate money supply in public hands at date t is given by $M_t=M_0-\sum_{i=1}^{I}\phi\cdot p_t^i\cdot d_t^i$ dollars. Notice that there is no secondary money creation role played by banks, since those funds are already allocated into the purchase of houses and do not circulate into the economy again.

Markets for Houses and Factors of Production

The characterization of the market for houses is very simple. There are I triplet-specific markets for houses in which young borrowers buy a house each either from old borrowers or the bank. The depiction of the markets for factors of production is very simple as well. In the first place, there is an integrated market for labor: labor is homogeneous across triplets, and young workers can work on any triplet they choose. The price of labor is the nominal wage at date t, given by w_t, and the relevant measure of the real wage is given by $\left(w_t/p_t^{i+1}\right)$. On the other hand, there are I island-specific markets for capital, since only $K_t^i>0$ can be used by old lenders i to produce their island-specific final good at date t.

Preferences and Actions

In this section, I discuss the preferences and potential choices of each of the three types of strategic agents, as well as the problem they each solve. The focus is on a typical generation born at triplet i at date t. Recall that the composition of each generation is constant over time: there are lenders, borrowers and one bank which will interact not only with each other, but also with either agent from other triplets as well as with agents from other generations.

The Lender from Triplet i

All lenders from triplet i are ex-ante identical. Each lender wishes to consume the final good i that they produce when old in the amount $l^{i}_{2,t+1}$ in the afternoon. When young, she allocates her endowment good as follows. First, early in the morning, she sells part of her endowment to young borrowers from island $i-1$ in exchange for IOUs in the amount of s^{i}_{t} dollars. Second, still in the morning, she deposits d^{i}_{t} units of her endowment in Bank i. Finally, before the end of the morning, she invests the remainder of her endowment into physical capital, such that K^{i}_{t+1} goods invested at date t will yield K^{i}_{t+1} units of capital in the afternoon at date $t+1$. K^{i}_{t+1} will be used at date $t+1$, together with labor, to produce the final good. The details of the lender's time line are presented in Figure 1 below.

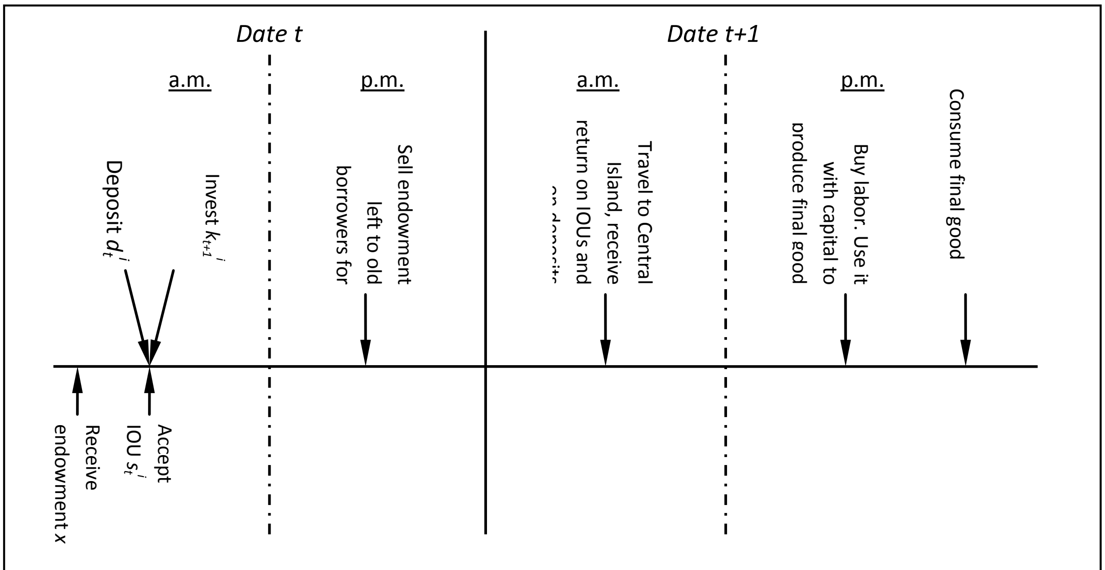

Figure 1: Time Line for Lenders in Triplet i born at date t

In the morning of the next period, all lenders must travel to the central island in order to get the repayment of their IOUs from borrowers and the return of their deposits from the bank. Next, in the afternoon, each old lender uses the return on her deposits and the payment of the IOUs she holds to purchase labor. She also sells part of the final good she will produce to old borrowers from island $i-I/2$ in exchange for $\widehat{m}^{i}_{t+1}$ dollars, and uses these proceeds to purchase labor as well. Next, before the end of the afternoon, the old lender produces her triplet-specific final good using the technology $f^{i}_{t+1} = F\left(K^{i}_{t+1}, L_{t+1}\right) = \left(K^{i}_{t+1}\right)^{\alpha} \cdot \left(L_{t+1}\right)^{1-\alpha}, \alpha \in (0,1)$, and consume part of it in the amount $l^{i}_{2,t+1}$.

The preferences of a lender from island i born at date t are given by

$$u\left(l^{i}_{2,t+1}\right)=\ln\left(l^{i}_{2,t+1}\right) \qquad (1),$$

while her budget constraints are given by

$$x=\left(\frac{s^{i}_{t}}{p^{i}_{t}}\right)+d^{i}_{t}+K^{i}_{t+1}, \quad (2a)$$

$$\left(1-\pi^{i}_{t+1}\right)\cdot p^{i}_{t+1}\cdot\left[R^{i}_{t+1}\cdot d^{i}_{t}+r^{i}_{t+1}\cdot\left(\frac{s^{i}_{t}}{p^{i}_{t}}\right)\right]+\widehat{m}^{i}_{t+1}=w_{t+1}\cdot L_{t+1}, \qquad (2b)$$

$$l^{i}_{2,t+1}+\frac{\widehat{m}^{i}_{t+1}}{\theta^{i}_{t+1}}=f^{i}_{t+1}=\left(K^{i}_{t+1}\right)^{\alpha}\cdot\left(L_{t+1}\right)^{1-\alpha}. \quad (2c)$$

Thus, the problem of a young lender from triplet i is given by

$$\underset{\frac{s^{i}_{t}}{p_{t}},d^{i}_{t},\frac{\widehat{m}^{i}_{t+1}}{\theta^{i}_{t+1}}}{Max}\ \ln\left[\left(K^{i}_{t+1}\right)^{\alpha}\cdot\left(L_{t+1}\right)^{1-\alpha}-\frac{\widehat{m}^{i}_{t+1}}{\theta^{i}_{t+1}}\right], \qquad (3a)$$

subject to:

$$K^{i}_{t+1}=x-\frac{s^{i}_{t}}{p^{i}_{t}}-d^{i}_{t} \qquad (3b)$$

$$L_{t+1}=\left(1-\pi^{i}_{t+1}\right)\cdot\left(\frac{p^{i}_{t+1}}{w_{t+1}}\right)\cdot\left[r^{i}_{t+1}\cdot\left(\frac{s^{i}_{t}}{p^{i}_{t}}\right)+R^{i}_{t+1}\cdot d^{i}_{t}\right]+\left(\frac{\widehat{m}^{i}_{t+1}}{\theta^{i}_{t+1}}\right)\cdot\left(\frac{\theta^{i}_{t+1}}{w_{t+1}}\right), \qquad (3c)$$

taking π^{i}_{t+1} and all prices as given. Notice that the proceeds from deposits and IOUs are scaled down by the rate of default π^{i}_{t+1}. As a result, the necessary first order conditions for this problem, assuming an interior solution, are summarized by

$$\left(\frac{\alpha}{1-\alpha}\right)\cdot\left(\frac{L_{t+1}}{K^{i}_{t+1}}\right)=\left(1-\pi^{i}_{t+1}\right)\cdot r^{i}_{t+1}\cdot\left(\frac{p^{i}_{t+1}}{w_{t+1}}\right), \qquad (4a)$$

$$\left(\frac{\alpha}{1-\alpha}\right)\cdot\left(\frac{L_{t+1}}{K^{i}_{t+1}}\right)=\left(1-\pi^{i}_{t+1}\right)\cdot R^{i}_{t+1}\cdot\left(\frac{p^{i}_{t+1}}{w_{t+1}}\right), \qquad (4b)$$

$$\left(1-\alpha\right)\cdot\left(\frac{K^{i}_{t+1}}{L_{t+1}}\right)^{\alpha}=\frac{w_{t+1}}{\theta^{i}_{t+1}}. \qquad (4c)$$

Equivalently, (4c) can be rewritten as $MPL^{i}_{t+1}=\left(w_{t+1}\big/\theta^{i}_{t+1}\right)$, which illustrates the fact that labor paid according to its marginal product. Notice as well that (4a) together with (4b) imply that there is no arbitrage between IOUs and deposits, and, thus, the following condition must hold for an interior solution:

$$r^{i}_{t+1}=R^{i}_{t+1}=\left(\frac{\alpha}{1-\alpha}\right)\cdot\left(\frac{L_{t+1}}{K^{i}_{t+1}}\right)\left[\frac{\left(w_{t+1}\big/p^{i}_{t+1}\right)}{\left(1-\pi^{i}_{t+1}\right)}\right] \qquad (4d)$$

The Borrower j from Triplet i

All borrowers $j \in [0,1]$ in island i are identical in every respect but for the realization of the shock that will hit them in the following date. The realizations of this shock are borrower and triplet-specific and its distribution is common knowledge. Each borrower knows that the shock will hit them next period, but they do not know what its realization will be at the moment when they are making their choices. Then, all borrowers must formulate choose a contingent plan that will be adjusted depending on the realization of the shock. In addition, I assume that borrowers do not have a bequest motive, so they must sell their houses before they die. The details of the borrower's time line are presented in the figure below.

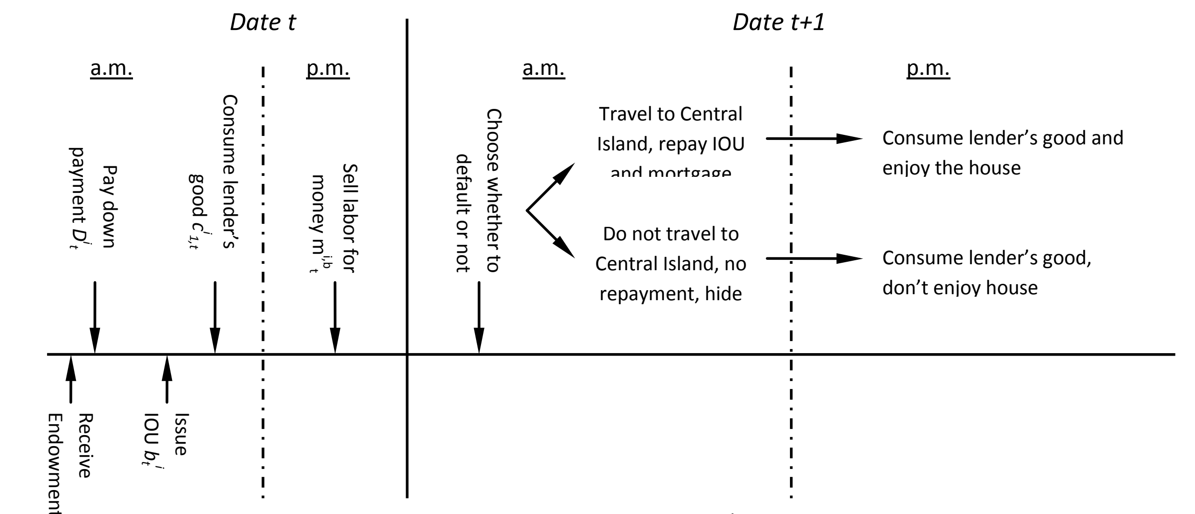

Figure 2: Time Line for Borrowers in Triplet i born at date t

Young borrowers from island i wish to consume $c_{1,t}^{i}$ units of Lenders $i+1$'s endowment good when young in the morning of date t. Since they have nothing of value to offer to lenders at this time, they issue an IOU of value $b_t^i = p_t^{i+1} \cdot c_{1,t}^i$ dollars to be re-paid using fiat money in the morning of $t+1$ on the Central Island, with the promised interest rate r_{t+1}^{i+1}. At this time, they also sign a mortgage contract with Bank i, where this Bank requires a promised real gross interest rate ρ_{t+1}^{i}. Obviously, houses are an indivisible good, and they can buy only one house. The mortgage contract requires young borrowers from island i to make the down payment D_t^i for a house when young in the morning, in the amount $D_t^i = q_t^i \cdot y$ dollars. Finally, at this point of time, they also choose the cut-off value $\tilde{\lambda}$ from the distribution of shocks, in the manner described in sub-section 2.4. In the afternoon, young borrowers sell their labor. They work for either old lenders or old banks on any island in exchange for the nominal wage w_t. By this point in time, the choices of $c_{1,t}^i$, $\left(b_t^i / p_t^{i+1}\right)$, D_t^i and the supply of labor are already inelastic.

First thing in the morning of the following period, the borrower j from triplet i is in her old age, and she learns her individual realization of the shock, $\lambda_{t+1}^{i,j}$. Regardless of the particular realization of the shock and of whether the mortgage will be repaid or not, the house must be rebuilt before it is sold again later in the morning, incurring in a cost of $\left(1-\lambda_{t+1}^{i,k}\right)\cdot q_{t+1}^{i}$ dollars. Next, based upon this realization, the borrower decides whether or not to pay the remainder of their mortgage. Recall that all repayments must be made on the Central Island, using fiat money. Thus, if a particular borrower chooses not to repay her mortgage, she cannot repay her IOUs either, since she would get caught once she is on the Central Island. Moreover, if the borrower defaults, she cannot enjoy the house $\left(h_{t+1}^{i,j}=0\right)$ or sell it later, since the house goes to the bank. In this respect, then, the choice of whether or not to default is also a choice of enjoying their house or not when old. Later, in the afternoon, the borrower would like to purchase the final good produced by old lenders in island $i+I/2$ and consume it, implying a cost of $\theta_{t+1}^{i+I/2}\cdot c_{2,t+1}^{i}$ dollars. She must pay for this good at the time of the transaction using fiat money, and she can do this regardless of her choice of whether or not to default, since nobody knows her in island $i+I/2$, nor they care about her having repaid all her debt or not.

Let me discuss briefly how borrowers implement their contingent plans. Each old borrower j learns her individual realization of the shock, $\lambda_{t+1}^{i,j}$, and she compares it against her chosen cut-off value $\tilde{\lambda}$. On the one hand, if $\lambda_{t+1}^{i,j}<\tilde{\lambda}$, the borrower decides that is not in her best interest to repay her mortgage and IOUs. Instead, she can take her money balances with her and run away to island $i+I/2$ to purchase the old lender's final good, but $h_{t+1}^{i}=0$ and her house goes back to the bank, which has to rebuild it before selling it. On the other hand, if $\lambda_{t+1}^{i,j}\geq\tilde{\lambda}$, the borrower repays her mortgage in full for $\rho_{t+1}^{i}\cdot q_{t}^{i}\cdot(1-y)$ dollars, and repays her IOU for $p_{t+1}^{i+1}\cdot r_{t+1}^{i+1}\cdot\left(b_{t}^{i}/p_{t}^{i+1}\right)$ dollars. Next, she enjoys the house $\left(h_{t+1}^{i}=1\right)$ and she rebuilds whatever was destroyed by the shock $\left(1-\lambda_{t+1}^{i,k}\right)\cdot q_{t+1}^{i}$ dollars[69], then selling it for q_{t+1}^{i}. Finally, in the afternoon, she travels to island $i+I/2$ to purchase the old lender's final good.

The lifetime utility of a borrower is given by

$$u\left(c_{1,t}^{i},c_{2,t+1}^{i},h_{t+1}^{i},\lambda_{t+1}^{i,j}\right)=\ln\left(c_{1,t}^{i}\right)+\ln\left(c_{2,t+1}^{i}\right)+h_{t+1}^{i}, \qquad (5)$$

while the budget constraints faced by a young borrower are:

$$c_{1,t}^{i}=\left(\frac{b_{t}^{i}}{p_{t}^{i+1}}\right), \qquad (6a)$$

$$D_{t}^{i}=q_{t}^{i}\cdot y \qquad (6b)$$

$$\overline{m}_{t}^{i}=w_{t}=w_{t}\cdot L_{t}+w_{t}\cdot a_{t} \qquad (6c)$$

For starters, an old borrower who chooses not to default (i.e., when $\lambda_{t+1}^{i,j}\geq\tilde{\lambda}$ obtains) will face the following constraints

[69] When $\lambda_{t+1}^{i,j}>1$, the borrower will experience an equity gain instead in the amount of $\left(\lambda_{t+1}^{i,j}-1\right)\cdot q_{t+1}^{i}$.

$$\widehat{c}^{\,i,j}_{2,t+1}=\left(\frac{w_t}{\theta^{i+I/2}_{t+1}}\right)+\lambda^{i,j}_{t+1}\cdot\left(\frac{q^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)-r^{i+1}_{t+1}\cdot\left(\frac{p^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)\cdot\left(\frac{b^i_t}{p^{i+1}_t}\right)-\rho^i_{t+1}\cdot\left(\frac{q^i_t}{\theta^{i+I/2}_{t+1}}\right)\cdot(1-y),\quad (7a)$$

$$h^{i,j}_{t+1}=1,\qquad (7b).$$

Secondly, an old borrower who chooses to default (i.e., when $\lambda^{i,j}_{t+1}<\tilde{\lambda}$ obtains) faces the following two constraints instead

$$\breve{c}^{\,i,j}_{2,t+1}=\frac{w_t}{\theta^{i+I/2}_{t+1}},\qquad (8a)$$

$$h^{i,j}_{t+1}=0\,.\qquad (8b)$$

Thus, the problem that Borrower j from triplet i must solve is

$$\underset{c^i_{1,t},\widehat{c}^i_{2,t+1},\breve{c}^i_{2,t+1},\tilde{\lambda}}{Max}\quad \ln\left(c^i_{1,t}\right)+\int_{\lambda_L}^{\tilde{\lambda}}\ln\left(\breve{c}^i_{2,t+1}\right)\cdot d\lambda+\int_{\tilde{\lambda}}^{\lambda_H}\left[\ln\left(\widehat{c}^i_{2,t+1}\right)+1\right]\cdot d\lambda\qquad (9a)$$

s.t. (6a), (7a), (7b), (8a), (8b),

$$\lambda_L\le\tilde{\lambda}\le\lambda_H,\qquad (9b)$$

and taking all prices as given. After integrating by parts and using change of variable, we can rewrite this problem as the maximization, by choosing $\left(b^i_t\big/p^{i+1}_t\right)$ and $\tilde{\lambda}$, of the following objective function[70]

$$u\left(c^i_{1,t},\breve{c}^i_{2,t+1},\widehat{c}^i_{2,t+1},\tilde{\lambda}\right)=\ln\left(\frac{b^i_t}{p^{i+1}_t}\right)+\left(\tilde{\lambda}-\lambda_L\right)\cdot\left(\frac{w_t}{\theta^{i+I/2}_{t+1}}\right)$$
$$+\ln\left[\widehat{c}^i_{2,t+1}\left(\lambda_H\right)\right]\cdot A\left(\lambda_H\right)-\ln\left[\widehat{c}^i_{2,t+1}\left(\tilde{\lambda}\right)\right]\cdot A\left(\tilde{\lambda}\right)\qquad ,\qquad (10a)$$

where:

$$\widehat{c}^i_{2,t+1}\left(\lambda_H\right)\equiv\left(\frac{w_t}{\theta^{i+I/2}_{t+1}}\right)+\lambda_H\cdot\left(\frac{q^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)-\rho^i_{t+1}\cdot(1-y)\cdot\left(\frac{q^i_t}{\theta^{i+I/2}_{t+1}}\right)-r^{i+1}_{t+1}\cdot\left(\frac{p^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)\cdot\left(\frac{b^i_t}{p^{i+1}_t}\right),\qquad (10b)$$

$$\widehat{c}^i_{2,t+1}\left(\tilde{\lambda}\right)\equiv\left(\frac{w_t}{\theta^{i+I/2}_{t+1}}\right)+\tilde{\lambda}\cdot\left(\frac{q^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)-\rho^i_{t+1}\cdot(1-y)\cdot\left(\frac{q^i_t}{\theta^{i+I/2}_{t+1}}\right)-r^{i+1}_{t+1}\cdot\left(\frac{p^i_{t+1}}{\theta^{i+I/2}_{t+1}}\right)\cdot\left(\frac{b^i_t}{p^{i+1}_t}\right),(10c)$$

$$A\left(\spadesuit\right)\equiv\left\{\spadesuit+\frac{w_t}{q^i_{t+1}}-\rho^i_{t+1}\cdot\left[\frac{q^i_t\cdot(1-y)}{q^i_{t+1}}\right]-r^{i+1}_{t+1}\cdot\left(\frac{p^i_{t+1}}{q^i_{t+1}}\right)\cdot\left(\frac{b^i_t}{p^{i+1}_t}\right)\right\}.\ (10d)$$

Of course, the condition (9b) must also hold. A couple of observations will become useful later on. First, $\widehat{c}^i_{2,t+1}$ and $A(\cdot)$ are increasing in λ. Second, $A\big/\widehat{c}_2=\theta_{t+1}\big/q_{t+1}$ is positive. Next, I proceed to discuss and evaluate the first order conditions for this problem.

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[70] For details, see the Technical Appendix.

The first order condition with respect to $\left(b_t^i \big/ p_t^{i+1}\right)$ is given by

$$\frac{1}{\left(b_t^i \big/ p_t^{i+1}\right)} - \frac{r_{t+1}^{i+1} \cdot p_{t+1}^i}{q_{t+1}^i} \cdot \left\{ \ln\left[\hat{c}_{2,t+1}^i\left(\lambda_H\right)\right] - \ln\left[\hat{c}_{2,t+1}^i\left(\tilde{\lambda}\right)\right] \right\} \geq 0 . \qquad (11a)$$

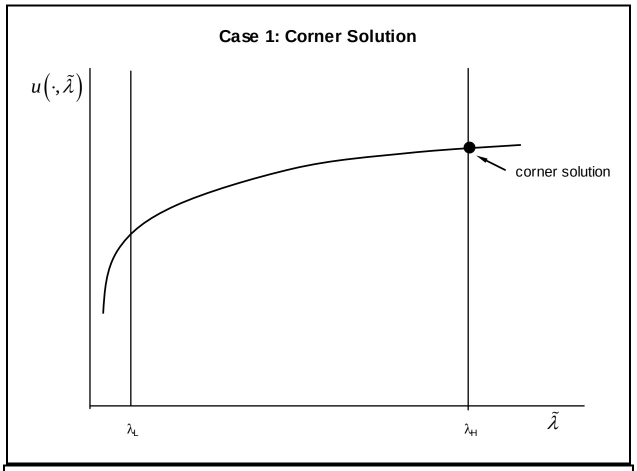
To discuss the properties of the borrower's solution for $\left(b_t^i \big/ p_t^{i+1}\right)$, we must evaluate different possibilities using (11a). I use a simple fixed point technique[71]. The results show that $\left(b/p\right)^* > 0$ does exist and it is unique, with some qualifications. First, $\left(b/p\right)^*$ does not exist for $\tilde{\lambda} = \lambda_L$. Second, $\exists \lambda_M \leq \lambda_H \left| \left(b/p\right)^* = \left(w_t \big/ \theta_{t+1}\right) \right.$.

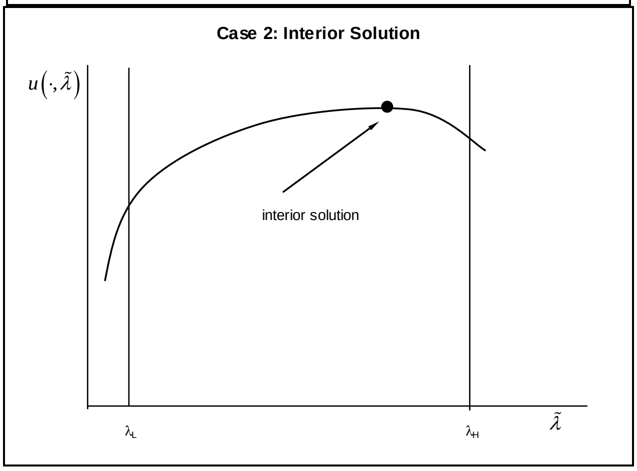
The first order condition with respect to $\tilde{\lambda}$ is given by

$$\breve{c}_{2,t+1}^i - 1 - \ln\left[\hat{c}_{2,t+1}^i\left(\tilde{\lambda}\right)\right]. \quad (11b)$$

The expression in (11b) requires additional examination. First, notice that $\breve{c}_{2,t+1}^i = \left(w_t \big/ \theta_{t+1}^{i+I/2}\right)$ is independent of $\tilde{\lambda}$ from the standpoint of the borrower and, thus, we can treat it as fixed for now. Second, the expression in (11b) is positive when $\tilde{\lambda} = \lambda_L$, implying that the latter is not a solution. Third, the expression in (11b) decreases with $\tilde{\lambda}$. Two cases may arise, based on the particular value that λ_H may take: when λ_H is sufficiently low, $\tilde{\lambda}^* = \lambda_H$ is a corner solution, while when λ_H is large enough, $\tilde{\lambda}^* < \lambda_H$ is an interior solution. The two cases are depicted in the diagrams below.

[71] For details, see the Technical Appendix.





I now proceed to evaluate the expression in (11b) for different potential values of $\tilde{\lambda} \in \left[\lambda_L, \lambda_H\right]$.

Universal Repayment. When $\tilde{\lambda} = \lambda_L$, the borrower would choose to always repay. Obviously, it must be the case that $\hat{c}^i_{2,t+1}\left(\tilde{\lambda} = \lambda_L\right) > \left(w_t \big/ \theta^{i+I/2}_{t+1}\right)$. The latter implies that the following condition must hold

$$\lambda_L > \bar{\lambda}_L \equiv \frac{\rho^i_{t+1} \cdot (1-y) \cdot q^i_t}{q^i_{t+1}} + \left(\frac{r^i_{t+1} \cdot p^i_{t+1}}{q^i_{t+1}}\right) \cdot \left[\frac{b^i_t}{p^{i+1}_t}\left(\lambda_L\right)\right]^* > 0 \quad (12a)$$

for $\tilde{\lambda} = \lambda_L$ to be a solution to this problem. Obviously, this is not the case when $\lambda_L \in \left[0, \bar{\lambda}_L\right]$, implying that $\tilde{\lambda} = \lambda_L$ cannot be a solution when the potential negative effects of the shock are significantly large. As an example, when borrowers know that their house can be totally destroyed by the shock, some positive mass of them will choose not to repay. The latter is consistent with my previous examination of (11b) that states that $\tilde{\lambda} = \lambda_L$ is not a solution.

Universal Default. Suppose now that $\tilde{\lambda} = \lambda_H$. In this case, the borrower chooses never to repay, and it must be the case that $\hat{c}^i_{2,t+1}\left(\tilde{\lambda} = \lambda_H\right) < \left(w_t \big/ \theta^{i+I/2}_{t+1}\right)$. The preceding inequality leads to the following condition

$$\lambda_H < \overline{\lambda}_H \equiv \frac{\rho^i_{t+1} \cdot q^i_t \cdot (1-y)}{q^i_{t+1}} + r^{i+1}_{t+1} \cdot \left(\frac{p^i_{t+1}}{q^i_{t+1}} \right) \cdot \left[\frac{b^i_t}{p^{i+1}_t} \left(\lambda_H \right) \right]^*, \qquad (12b)$$

which must hold for $\tilde{\lambda} = \lambda_H$ to be a solution. Condition (12b) means that, for values of λ_H that are sufficiently low, $\tilde{\lambda}^* = \lambda_H$ is a solution, which is illustrated in Case 1. However, for values of λ_H that are high enough, i.e. for values such that $\lambda_H \geq \overline{\lambda}_H$, $\tilde{\lambda} = \lambda_H$ is not the solution, but instead $\lambda_L < \tilde{\lambda} < \lambda_H$ is the interior solution for this problem. The latter can be explained by the fact when borrowers know that the equity in their houses can increase significantly, a positive mass of them will choose to repay in order to rip-off these benefits.

Some Default. Finally, consider again the case of the interior solution such that $\tilde{\lambda}^* \in \left(\lambda_L, \lambda_H \right)$. In this case, $\hat{c}^i_{2,t+1}\left(\tilde{\lambda} \right) = \breve{c}^i_{2,t+1}$ must hold, implying that $\tilde{\lambda}^*$ must satisfy the following condition

$$\tilde{\lambda}^* = \frac{\rho^i_{t+1} \cdot q^i_t \cdot (1-y)}{q^i_{t+1}} + r^i_{t+1} \cdot \left(\frac{p^i_{t+1}}{q^i_{t+1}} \right) \cdot \left[\frac{b^i_t}{p^{i+1}_t} \left(\tilde{\lambda}^* \right) \right]. \qquad (12c)$$

The Bank from Triplet i

The Bank in triplet i is a monopoly and she starts with zero reserves when young. She packages and offers deposit contracts to young lenders and mortgage contracts to young borrowers. In addition, the bank must hold currency reserves from t to $t+1$: a fraction $\phi \in (0,1)$ of the dollar-value of the young lenders' deposits. Let χ^i_t denote the amount of dollar-reserves held in the reserves account in the monetary authority. Figure 3 below illustrates the details of the bank's time line.

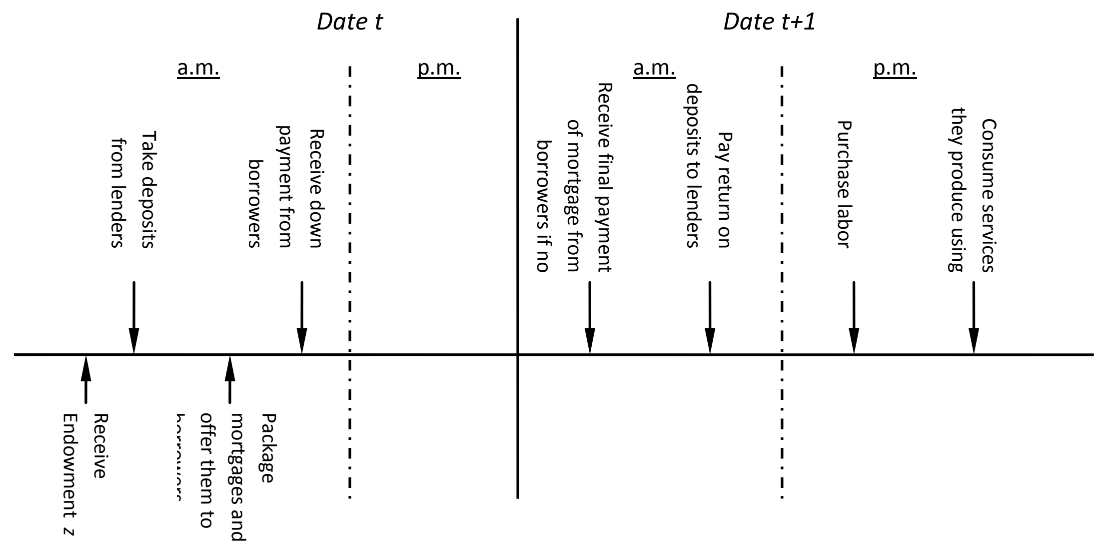


Figure 3: Time Line for Bank in Triplet i born at date t

Deposit contracts. The Bank offers deposits contracts to young lenders born on triplet i. Each young lender must deposit $p^i_t \cdot d^i_t$ dollars early in the morning of date t, and keep them in the bank until the afternoon of date $t+1$. In turn, the Bank promises the real gross interest rate R^i_{t+1} per unit of deposits, to be paid in the afternoon of date $t+1$. The Bank has the monopoly in triplet i in offering these deposit contracts. The return on deposits will be paid to the lenders once they are on the central island, at the same time the mortgages are repaid.

Mortgage contracts. Young lenders would like buy a house worth q^i_t in the morning of date t. They aim at borrowing from the Bank for this purpose. The mortgage contract specifies the following: 1) the young lender must put a down payment of $D^i_t = y \cdot q^i_t$ dollars. The bank lends the remainder $\Lambda_t = (1-y) \cdot q^i_t$ dollars and it purchases the house. The lenders must repay $\rho^i_{t+1} \cdot \Lambda_t = \rho^i_{t+1} \cdot (1-y) \cdot q^i_t$ dollars in the morning of date $t+1$, after which they own the house and they can sell it after repairing it at the price q^i_{t+1}. In the case of default, the Bank i appropriates the house instead, and she can sell it after making the repairs also at the price q^i_{t+1}. Of course, ρ^i_{t+1} denotes the real gross interest rate on loans set by the Bank. It is worth pointing out that the Bank has the monopoly in pooling the resources available, given by her endowment good and the fraction $(1-\phi)$ of the deposits in her vaults, and this must also be feasible. While the mortgage contract is signed at the Bank, all repayments must take place on the Central Island in the morning of date $t+1$.

With respect to her preferences, the Bank wishes to consume a^i_{t+1} units of services when old in the afternoon. Such services are produced only by the Bank, one-to-one, from labor purchased. Obviously, $\left(w_{t+1} \big/ p^i_{t+1}\right) \cdot a^i_{t+1}$ denotes Bank i's real cost of purchasing $a^i_{t+1} > 0$ units of services. Her utility function takes the following form:

$$u\left(a^i_{t+1}\right) = \ln\left(a^i_{t+1}\right), \qquad (13)$$

while the Bank's budget constraints are given by

$$\mu^i_t \cdot z + p^i_t \cdot d^i_t = q^i_t \cdot (1-y) + \chi^i_t \qquad (14a)$$

$$\chi^i_t = \phi \cdot p^i_t \cdot d^i_t . \qquad (14b)$$

Combining (14a) and (14b), I obtain

$$a^i_{t+1} = \frac{\left(\tilde{\lambda}^2 - \lambda_L^{\ 2}\right) \cdot q^i_{t+1}}{2 \cdot w_{t+1}} + \left(\lambda_H - \tilde{\lambda}\right) \cdot \left[\frac{\rho^i_{t+1} \cdot q^i_t \cdot (1-y)}{w_{t+1}}\right] - R^i_{t+1} \cdot \left[\frac{p^i_{t+1} \cdot d^i_t}{w_{t+1}}\right] + \left(\frac{\chi^i_t}{w_{t+1}}\right). \qquad (14c)$$

I now can express the Bank's problem as a constrained profit-maximization

$$\underset{R^i_{t+1}, \rho^i_{t+1}}{Max} \ \ln\left(a^i_{t+1}\right)$$

s.t. (14c,)

taking $\tilde{\lambda}, \lambda_L, \lambda_H$ and all other prices as given. The first order conditions of this problem with respect to R^i_{t+1} and ρ^i_{t+1} are, respectively,

$$-\frac{p_t^i \cdot d_t^i}{a_{t+1}^i \cdot w_{t+1}} < 0 \text{ (15a)}$$

$$\left(\lambda_H - \tilde{\lambda}\right) \cdot \left[\frac{q_t^i \cdot (1-y)}{a_{t+1}^i \cdot w_{t+1}}\right] > 0 . \quad \text{(15b)}$$

On the one hand, the condition in (15a) implies a left-corner solution for R_{t+1}^i, i.e. the Bank will pay the lowest interest rate on deposits that lenders would be willing to accept, which I will denote by $\hat{R}_{t+1}^i$. In this case, we can obtain $\hat{R}_{t+1}^i$ from equation (4d), such that

$$\hat{R}_{t+1}^i\left(\tilde{\lambda}\right) = \hat{r}_{t+1}^i\left(\tilde{\lambda}\right) = \frac{\alpha \cdot L_{t+1} \cdot w_{t+1}}{(1-\alpha) \cdot K_{t+1}^i \cdot p_{t+1}^i} \cdot \left(\frac{\lambda_H - \lambda_L}{\lambda_H - \tilde{\lambda}}\right), \quad \text{(16a)}$$

where it is apparent that $\hat{R}_{t+1}^i\left(\tilde{\lambda}\right)$ is increasing in $\tilde{\lambda}$, from the Bank's standpoint.

On the other hand, the condition in (15b) indicates that the Bank will charge the maximum possible interest rate on loans that would be acceptable by borrowers who will not default. This interest rate $\hat{\rho}_{t+1}^i$ will be such that $\hat{c}_{2,t+1}^i\left(\tilde{\lambda}\right) = \breve{c}_{2,t+1}^i = \left(w_{t+1} \big/ \theta_{t+1}^{i+I/2}\right)$ obtains. Using equation (10c), one can solve for $\hat{\rho}_{t+1}^i$ and obtain the following

$$\hat{\rho}_{t+1}^i\left(\tilde{\lambda}\right) = \frac{\tilde{\lambda} \cdot q_{t+1}^i}{q_t^i \cdot (1-y)} - \frac{\hat{R}_{t+1}^i\left(\tilde{\lambda}\right) \cdot p_{t+1}^i}{q_t^i \cdot (1-y)} \cdot \frac{b_t^i}{p_t^{i+1}} . \quad \text{(16b)}$$

However, it is unclear whether the equilibrium interest rate on loans increases with $\tilde{\lambda}$ or not. To summarize, equations (16a) and (16b) represent the Bank's profit-maximizing choices.

General Equilibrium

In this section, I summarize the conditions that must hold in a general equilibrium. I proceed by blocks. First, I present the four conditions related to the financial and asset markets. The market for IOUs must clear at date t, which happens when

$$\frac{s_t^i}{p_t^i} = \frac{b_t^{i-1}}{p_t^i}\left(\tilde{\lambda}\right) \quad \text{(17a)}$$

for all t and i. Next, the market for mortgages clears at t when

$$\mu_t^i \cdot z + (1-\phi) \cdot p_t^i \cdot d_t^i = q_t^i \cdot (1-y), \quad \text{(17b)}$$

while the markets for money and for reserves clear, respectively, when

$$M_t = M_0 - \sum_{i=1}^{I} p_t^i \cdot \phi \cdot d_t^i = I \cdot w_t \quad \text{(17c)}$$

$$\phi \cdot p_t^i \cdot d_t^i = \chi_t . \quad \text{(17d)}$$

Secondly, I present the markets for factors of production. The market for labor at date t clears when

$$I \cdot L_t + \sum_{i=1}^{I} a_t^i = I \quad \text{(18)}$$

holds, and, due to the particular structural characteristics of capital creation, the market for physical capital always clears.

In the third place, I present the conditions for the different markets for goods. The market for the lender's endowment good clears when

$$x = K^i_{t+1} + \frac{b^{i-1}_t}{p^i_t}\left(\tilde{\lambda}\right) + d^i_t \quad (19a)$$

is satisfied, while the market for the lender's final good clears when

$$\begin{aligned} \left(K^i_{t+1}\right)^{\alpha}\left(L_{t+1}\right)^{1-\alpha} &= l^i_{2,t+1} + \left(\frac{w_{t+1}}{\theta^{i+I/2}_{t+1}}\right) \\ &+\left(\theta^{i+I/2}_{t+1}\right)^{-1}\int_{\tilde{\lambda}}^{\lambda_H}\left[\lambda^i_{t+1}\cdot q^i_{t+1} - \hat{\rho}^i_{t+1}\left(\tilde{\lambda}\right)\cdot q^i_t\cdot(1-y) - \hat{R}^i_{t+1}(\lambda)\cdot\frac{b^{i-1}_t}{p^{i+1}_t}\left(\tilde{\lambda}\right)\right]\cdot d\lambda \end{aligned} \quad (19b)$$

Now, I turn to discuss the properties of some of the key variables in the model in a symmetric equilibrium.

A Symmetric General Equilibrium

In this model economy, the structure at heart has been designed so that the agents that belong to a particular type but were born in different triplets face the same problem and the same conditions but for the individual realizations of the shock that affects old borrowers. In comes naturally, then, to pay attention to a symmetric equilibrium.

A symmetric equilibrium is defined as one in which all triplet-specific variables x^i are equal to their counterparts from all other triplets $x^j, j \neq i$. For example, $\tilde{\lambda}^i_{t+1} = \tilde{\lambda}_{t+1}, \forall i$, and so on, with the exception of the realizations of individual-specific shocks. This exercise will allow me to point out the potential for strategic complementarities that underlies this economy.

The reader will notice that the main equilibrium quantities and interest rates are a function of $\tilde{\lambda}$ in equilibrium. In what follows, I describe the properties of the gains/losses of the different agents in equilibrium when there is an innovation in $\tilde{\lambda}$.

Labor Purchases by Old Lenders and Total Output

In equilibrium, I obtain that there is no closed-form solution for the labor purchases by old lenders $L_{t+1}\left(\tilde{\lambda}\right)$. Moreover, $L_{t+1}\left(\tilde{\lambda}\right)$ is properly defined iff $K_{t+1} < \alpha \cdot x$ [72]. The following equation defines implicitly the labor purchases by old lenders

$$-\left[\frac{\alpha\cdot x - K_{t+1}}{(1-\alpha)\cdot K_{t+1}}\right] = \frac{1}{L_{t+1}\left(\tilde{\lambda}\right)} + \left[\frac{q_{t+1}}{2\theta\cdot\left(\lambda_H - \lambda_L\right)}\right]\cdot\frac{\left(\lambda_H - \tilde{\lambda}\right)^2}{(1-\alpha)\cdot K_{t+1}}\cdot\left[L_{t+1}\left(\tilde{\lambda}\right)\right]^{\alpha}. \quad (20a)$$

Equation (21a) illustrates the fact that the labor purchases by old lenders are a function of $\tilde{\lambda}$, but also of K_{t+1}. However, the latter is inelastic and independent of $\tilde{\lambda}$, since it was chosen before the realization of the latter. Interestingly, $L_{t+1}\left(\tilde{\lambda}\right)$ is a key indicator of the welfare of old lenders, since it

[72] $L_{t+1}\left(\tilde{\lambda}\right)$ is undefined and discontinuous when $K_{t+1} = \alpha\cdot x$, since $\lim_{K_{t+1}\to(\alpha\cdot x)^-} = \infty$ and $\lim_{K_{t+1}\to(\alpha\cdot x)^+} = -\infty$.

represents their ability to produce their final good, for a given K_{t+1}. The following proposition illustrates the changes in the old lenders' welfare when facing an innovation in $\tilde{\lambda}$.

Proposition 1 An innovation in $\tilde{\lambda}$ reduces the ability of old lenders of purchasing labor by reducing their resources available. The latter also reduces their ability of producing their final good, thus reducing its supply in equilibrium.

Proof: After differentiating (20a) with respect to $\tilde{\lambda}$ and an extensive rearrangement of terms, one obtains

$$\frac{dL}{d\tilde{\lambda}} = -\frac{2 \cdot q_{t+1} \cdot L^{\alpha+1} \cdot \left(\lambda_H - \tilde{\lambda}\right)}{(1-\alpha) \cdot \left[2\theta \cdot \left(\lambda_H - \lambda_L\right) \cdot K^{\alpha} + q_{t+1} \cdot L^{\alpha}\right]} < 0, \qquad (20b)$$

which indicates that the lender's purchases of labor fall in the face of an innovation of $\tilde{\lambda}$.

Second, the total production of the final good $Y_{t+1} = K^{\alpha} \cdot L^{1-\alpha}$ depends on both capital and labor purchases. At the time when an innovation of $\tilde{\lambda}$ is observed, K is inelastic, and only L will respond to this change. Thus, differentiating Y_{t+1} with respect to $\tilde{\lambda}$ yields

$$\frac{dY_{t+1}}{d\tilde{\lambda}} = (1-\alpha) \cdot K^{\alpha} \cdot L^{-\alpha} \cdot \frac{dL}{d\tilde{\lambda}} < 0. \qquad (20c)$$

Of course, the latter implies a reduction of total output in the presence of an innovation of the cut-off value of the shock.

Q.E.D.

The Welfare of Old Borrowers

In a symmetric general equilibrium, the following basic conditions illustrate important properties that are consistent with the contingent plan of action that borrowers formulated when young:

$$\hat{c}_{2,t+1}\left(\tilde{\lambda}\right) = \breve{c}_{2,t+1} = \frac{w_{t+1}}{\theta_{t+1}} = \frac{(1-\alpha) \cdot K^{\alpha}}{L^{\alpha}} > 0, \qquad (21a)$$

$$\hat{c}_{2,t+1}\left(\lambda_{t+1}\right) - \hat{c}_{2,t+1}\left(\tilde{\lambda}\right) = \frac{q_{t+1} \cdot \left(\lambda_{t+1} - \tilde{\lambda}\right)}{\theta_{t+1}}, \qquad (21b)$$

$$\hat{c}_{2,t+1}\left(\lambda_{t+1}\right) = \frac{w_{t+1}}{\theta_{t+1}} + \frac{\left(\lambda_{t+1} - \tilde{\lambda}\right) \cdot q_{t+1}}{\theta_{t+1}} = \frac{(1-\alpha) \cdot K^{\alpha}}{L^{\alpha}} + \frac{\left(\lambda_{t+1} - \tilde{\lambda}\right) \cdot q_{t+1}}{\theta_{t+1}} > 0. \qquad (21c)$$

First, equation (21a) indicates that the marginal old borrower –who experiences the realization $\lambda_{t+1} = \tilde{\lambda}$ - is indifferent between defaulting and repaying her debt. Second, condition (21b) illustrates two facts. On the one hand, an old borrower who experiences realizations such that $\lambda_{t+1} \in \left[\lambda_L, \tilde{\lambda}\right)$ will be worse-off than defaulting if she chooses to repay her debt since $\hat{c}_{2,t+1}\left(\lambda_{t+1}\right) < \left(w_{t+1}/\theta_{t+1}\right)$. On the other hand, an old borrower who experiences realizations such that $\lambda_{t+1} \in \left(\tilde{\lambda}, \lambda_H\right]$ will find it in her best interest to repay her debt, since $\hat{c}_{2,t+1}\left(\lambda_{t+1}\right) > \left(w_{t+1}/\theta_{t+1}\right)$. Equation (21c) describes the consumption of an old borrower who chooses to repay her debt. It is thus apparent that when

$\lambda_{t+1} \in \left[\lambda_L, \tilde{\lambda}\right)$ obtains, the old borrower will choose to default on her debt and she will consume $\breve{c}_{2,t+1} = \left(w_{t+1}/\theta_{t+1}\right)$, while when $\lambda_{t+1} \in \left(\tilde{\lambda}, \lambda_H\right]$ obtains, the borrower will choose to repay her debt and she will consume $\hat{c}_{2,t+1}\left(\lambda_{t+1}\right) = \left(w_{t+1}/\theta_{t+1}\right) + \left[q_{t+1} \cdot \left(\lambda_{t+1} - \tilde{\lambda}\right)/\theta_{t+1}\right] > \breve{c}_{2,t+1}$.

I now turn to the analysis of the effects of an innovation of $\tilde{\lambda}$, which are described in the following proposition.

Proposition 2 An innovation in $\tilde{\lambda}$ affects the welfare of the two groups of old borrowers differently: it makes the borrowers who chose to default on their debt better-off, while the borrowers who chose to repay their debt worse-off. However, the group of old borrowers as a whole are made worse-off by the increase in $\tilde{\lambda}$. Moreover, the number of borrowers in the first group increases, while the number of borrowers in the second group decreases.

Proof: I start with the consumption of borrowers who choose to default in equilibrium. By differentiating (21a) with respect to $\tilde{\lambda}$, one obtains

$$\frac{d\breve{c}_{2,t+1}}{d\tilde{\lambda}} = -\frac{\alpha \cdot (1-\alpha) \cdot K^{\alpha}}{L^{\alpha+1}} \cdot \frac{dL}{d\tilde{\lambda}} > 0 . \tag{21d}$$

Equation (21d) indicates that old borrowers who choose to default are made better-off by an innovation in $\tilde{\lambda}$. The fraction of borrowers who default, given by $\left(\tilde{\lambda} - \lambda_L\right)/\left(\lambda_H - \lambda_L\right)$, increases with $\tilde{\lambda}$, which is consistent with the increase observed in $\breve{c}_{2,t+1}$.

Regarding the consumption of borrowers who choose to repay their debt, one must differentiate equation (21c) with respect to $\tilde{\lambda}$. By doing so, one obtains

$$\frac{d\hat{c}_{2,t+1}\left(\lambda_{t+1}\right)}{d\tilde{\lambda}} = \frac{2 \cdot \theta_{t+1} \cdot q_{t+1} \cdot K^{\alpha}\left[(\alpha-1)\cdot\lambda_H - \alpha \cdot \tilde{\lambda} + \lambda_L\right] - \left(q_{t+1}\right)^2 \cdot L^{\alpha}}{\theta_{t+1} \cdot \left[2 \cdot \theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right) \cdot K^{\alpha} + q_{t+1} \cdot L^{\alpha}\right]} < 0 . \tag{21e}$$

Interestingly, equation (21e) indicates that an innovation in $\tilde{\lambda}$ causes the old borrowers who chose to repay their debt to be worse-off. Finally, the fraction of borrowers who choose to repay their debt, given by $\left(\lambda_H - \tilde{\lambda}\right)/\left(\lambda_H - \lambda_L\right)$, decreases with $\tilde{\lambda}$. The latter is consistent with the reduction in $\hat{c}_{2,t+1}$. These two effects continue until $\hat{c}_{2,t+1}(\tilde{\lambda}') = \left(w_{t+1}/\theta_{t+1}\right)'$, where $\tilde{\lambda}'$ is the new cut-off value of the shock.

I now turn to analyze the aggregate welfare of borrowers, given by $\left(\hat{m}_{t+1}/\theta_{t+1}\right)$ in the equation below.

$$\frac{\hat{m}_{t+1}}{\theta_{t+1}} = \frac{w_{t+1}}{\theta_{t+1}} + \left(\frac{q_{t+1}}{\theta_{t+1}}\right) \cdot \int_{\tilde{\lambda}}^{\lambda_H} \frac{\left(\lambda_{t+1} - \tilde{\lambda}\right)}{\left(\lambda_H - \lambda_L\right)} \cdot d\lambda_{t+1} . \tag{21f}$$

After some non-trivial effort, I was able to show that

$$\frac{d\left(\hat{m}_{t+1}/\theta_{t+1}\right)}{d\tilde{\lambda}} = q_{t+1} \cdot \left(\lambda_H - \tilde{\lambda}\right)\left\{\frac{\left[2 \cdot \theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right) \cdot K^{\alpha}\right] \cdot (\alpha - 1) - q_{t+1} \cdot L^{\alpha}}{\theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right) \cdot \left[2 \cdot \theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right) \cdot K^{\alpha} + q_{t+1} \cdot L^{\alpha}\right]}\right\} < 0 , \tag{21g}$$

which indicates that the aggregate welfare of old borrowers is reduced by an innovation in $\tilde{\lambda}$.

Q.E.D.

Interestingly, Proposition 3 illustrates the fact that, in the time of a mortgage crisis, the borrowers who choose to default experience welfare gains, while the "honest" borrowers will be made worse-off. Moreover, the losses experienced by the "honest" borrowers dominate, making the borrowers as a group worse-off. This result is the more appealing when one confronts the conventional wisdom around the subprime mortgage crisis, where a case is being made to attempt to improve the conditions faced by the borrowers who have been forced to default on their mortgages. It would appear that, instead, the case shall be made to help those who are trying to repair their mortgages, not the ones who defaulted.

The Welfare of Old Lenders

Now I am in a position to analyze the welfare of the old lenders in equilibrium. A lender's welfare depends on her consumption of the final good she produces, $l_{2,t+1}$. In turn, following equation (3a), it transpires that the lenders' consumption depends on both the amount produced of the good and the consumption of the old borrowers. The proposition below presents some very provoking results that involve the lenders' welfare.

Proposition 3 The old lenders experience welfare gains, after a mortgage crisis in the form of an innovation in $\tilde{\lambda}$ has obtained.

Proof: After imposing the general equilibrium conditions in equation (3a) and differentiating with respect to $\tilde{\lambda}$, it transpires that

$$\frac{dl_{2,t+1}}{d\tilde{\lambda}} = \frac{2\cdot\theta_{t+1}\cdot\left(\lambda_H-\lambda_L\right)\cdot\left(\lambda_H-\tilde{\lambda}\right)\cdot K^{\alpha}\cdot\left(1-\alpha-L\right)+\left(q_{t+1}\right)^2\cdot\left(\lambda_H-\tilde{\lambda}\right)\cdot L^{\alpha}}{\theta_{t+1}\cdot\left(\lambda_H-\lambda_L\right)\cdot\left[2\cdot\theta_{t+1}\cdot\left(\lambda_H-\lambda_L\right)\cdot K^{\alpha}+q_{t+1}\cdot L^{\alpha}\right]}>0. \quad (22a)$$

The former equation indicates that an innovation in $\tilde{\lambda}$ increases the old lenders' consumption in equilibrium. The latter, of course, will result in a welfare gain for these agents.

Q.E.D.

The expression in (22a) indicates that, contrary to what one expect, the lenders' welfare improves in the time of a crisis originated by an innovations in the cut-off value of the shock. Given the circumstances, one would tend to look first at the reduction of output as one factor that would undermine the lenders' welfare. However, the reduction in the borrowers' welfare dominates this effect. Thus, the old lenders are made better-off at the expense of the old borrowers'.

The Welfare of the Bank

The welfare of the different individual banks is directly related to their profits, which take the form of purchases of labor when old, a_{t+1}. After imposing the general equilibrium conditions on (14c), one obtains the following expression

$$a_{t+1}=\kappa\left(\tilde{\lambda}\right)-\xi\left(\tilde{\lambda}\right), \quad (23a)$$

Where $\kappa\left(\tilde{\lambda}\right)$ represents the bank's income/revenue function and $\xi\left(\tilde{\lambda}\right)$ denotes the bank's cost function. I proceed now by analyzing each of these functions separately, at first.

Properties of the Bank's Revenue Function. The revenue function in a general equilibrium is given by

$$\kappa\left(\tilde{\lambda}\right)\equiv\frac{\tilde{\lambda}^2\cdot q_{t+1}}{2\cdot w_{t+1}}+\frac{\left(\lambda_H-\tilde{\lambda}\right)\cdot\hat{\rho}_{t+1}\cdot q_t\cdot\left(1-y\right)}{w_{t+1}}+\frac{\chi_t}{w_{t+1}}-\frac{\lambda_L^2\cdot q_{t+1}}{2\cdot w_{t+1}} \quad (23b)$$

The evaluation of the revenue function at the extreme values of the distribution of $\tilde{\lambda}$ provides interesting insight. I start by evaluating the revenue function at $\tilde{\lambda}=0\le\lambda_L$

$$\kappa(0)=\frac{\phi\cdot p_t\cdot(x-K)}{w_{t+1}}-\frac{\phi\cdot p_t\cdot\left(b_t/p_t\right)}{w_{t+1}}-\frac{\lambda_L^2\cdot q_{t+1}}{w_{t+1}}-\frac{\alpha\cdot(L/K)\cdot\left(b_t/p_t\right)\cdot\left(\lambda_H-\lambda_L\right)}{(1-\alpha)}<0\,.\quad(23c)$$

Thus, the bank would receive negative revenue for values of $\tilde{\lambda}$ that are sufficiently low. However, $\kappa(\tilde{\lambda})$ increases as $\tilde{\lambda}$ continues to increase, as one can deduct from the following expression

$$\frac{d\kappa(\tilde{\lambda})}{d\tilde{\lambda}}=\frac{\hat{R}_{t+1}\cdot p_{t+1}\cdot\left(b_t/p_t\right)}{w_{t+1}}\cdot\left[\frac{2\cdot\left(\lambda_H-\tilde{\lambda}\right)+1}{\left(\lambda_H-\tilde{\lambda}\right)}\right]-\frac{q_{t+1}}{w_{t+1}}>0\,.\qquad(23d)$$

It is easy to figure out from (23d) that $\left.\frac{d\kappa(\tilde{\lambda})}{d\tilde{\lambda}}\right|_{\tilde{\lambda}=0}>0$ although $\lim_{\tilde{\lambda}\to\lambda_H}\frac{d\kappa(\tilde{\lambda})}{d\tilde{\lambda}}=\infty$. In the limit, it transpires that $\kappa\left(\lambda_H\right)=\left[\left(\lambda_H^2-\lambda_L^2\right)\cdot q_{t+1}\right]\Big/2\cdot w_{t+1}+\chi_t/w_{t+1}>0$ holds. In summary, the bank's revenue function is an increasing and convex function of $\tilde{\lambda}$ which happens to be negative for low enough values of its argument.

Properties of the Bank's Cost Function. The bank's cost function in a general equilibrium has the following form

$$\xi(\tilde{\lambda})\equiv\frac{\hat{R}_{t+1}\cdot p_{t+1}\cdot d_t}{w_{t+1}}=\frac{\hat{R}_{t+1}\cdot p_{t+1}\cdot(x-K)}{w_{t+1}}-\frac{\hat{R}_{t+1}\cdot p_{t+1}\cdot\left(b_t/p_t\right)}{w_{t+1}}>0\,.\qquad(24a)$$

Equation (24a) indicates that there are two forces at play in the cost function: the effect that increases the interest rate on deposits has as $\tilde{\lambda}$ increases, and the effect that reduces the deposits under the same conditions. The latter is apparent from[73]

$$\frac{d\xi(\tilde{\lambda})}{d\tilde{\lambda}}=\left(\frac{d\hat{R}_{t+1}}{d\tilde{\lambda}}\right)\cdot p_{t+1}\cdot\left[x-K-\left(b_t/p_t\right)\right]-\left[\frac{d\left(b_t/p_t\right)}{d\tilde{\lambda}}\right]\cdot\hat{R}_{t+1}\cdot p_{t+1}\,.\qquad(24b)$$

It is the case that, in equilibrium, the costs are increasing in $\tilde{\lambda}$ for values of $\tilde{\lambda}$ that are low enough, while they decrease for higher values of $\tilde{\lambda}$. Moreover, $d^2\xi(\tilde{\lambda})\big/d\tilde{\lambda}^2<0$ holds, indicating that costs are a strictly concave function of the cut-off value of the shock.

Properties of the Bank's Profits. The diagram below will help us understand the behavior of a_{t+1} by illustrating together the behavior of revenue and costs as a function of $\tilde{\lambda}$.

[73] See the following sections for details on $\hat{R}_{t+1}$, $\hat{\rho}_{t+1}$ and $\left(b_t/p_t\right)$.

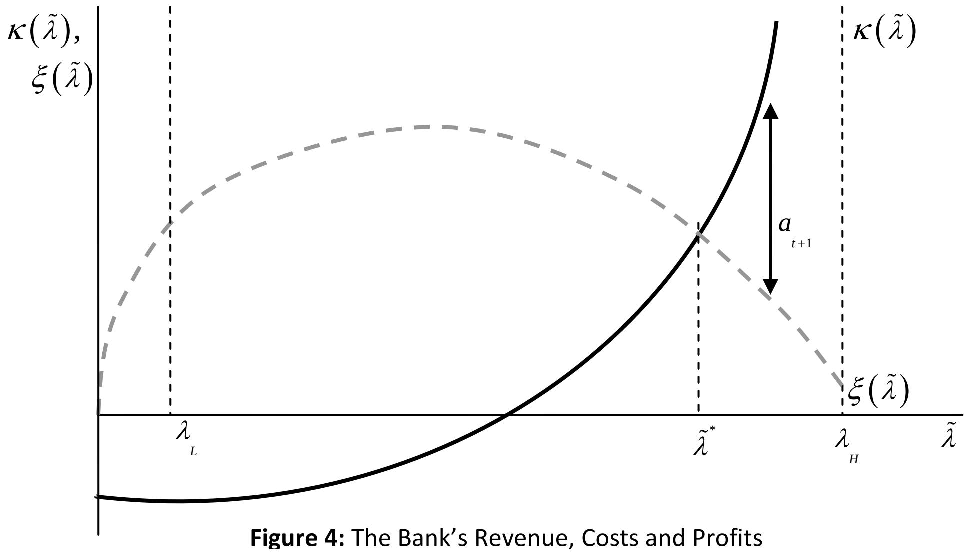


Figure 4: The Bank's Revenue, Costs and Profits

Now, I restrict my attention to the analysis of the equilibria that have economic meaning, as described in the following proposition.

Proposition 4 Equilibria can only exist in the range $\tilde{\lambda} \in \left[\tilde{\lambda}^*, \lambda_H\right]$, where profits are positive.

Proof: In this model, the bank uses her profits to purchase labor. Since one cannot purchase negative amounts of labor, and because the bank has no supply of labor to offer when old[74], we can rule out equilibria where $\tilde{\lambda} \in \left[\lambda_L, \tilde{\lambda}^*\right)$ and $a_{t+1} < 0$. Thus, equilibria will exist only for allocations where $\tilde{\lambda} \in \left[\tilde{\lambda}^*, \lambda_H\right]$ and $a_{t+1} \geq 0$ hold.

Q.E.D.

In this model, even though banks are local monopolies, they are vulnerable to the presence of allocations where operation is not economically viable. Contrary to conventional wisdom, the banks will prefer allocations associated with higher rates of default and risk, since only there can they expect to make a profit and remain open to the public. Allocations where $\tilde{\lambda} \in \left[\lambda_L, \tilde{\lambda}^*\right)$ can also be interpreted as equilibria in which financial autarky is present and the market for mortgages does not operate. In the latter case, though, the market for IOUs could still be operative. One can then say that, in this model economy, banks are slaves to risk and conscious risk-takers in equilibrium. However, there is a double edge to this statement, since in the limit, when $\tilde{\lambda} = \lambda_H$, there is universal default and banks could not subsist either since deposits would be close to zero.

The Amount Issued of IOUs

The real value of the IOUs issued and its properties will indicate the direction of the strategic interaction present in equilibrium in this economy. I proceed by first combining equations (11a), (16a) and (16b) and imposing an interior solution for both $\left(b_t / p_t\right)$ and $\tilde{\lambda}$. Then, one obtains the following expression, which defines the function $\frac{b_t}{p_t}(\tilde{\lambda})$ implicitly:

[74] One could think as well of the possibility of endowing old banks with labor when old, but this seems to complicate matters greatly without the corresponding gain in insight.

$$\left[\frac{b_t}{p_t}(\tilde{\lambda})\right]^{-1} = \frac{\hat{R}_{t+1}(\tilde{\lambda})\cdot p_{t+1}}{q_{t+1}}\cdot \ln\left\{w_{t+1} + \lambda_H \cdot q_{t+1} - \hat{\rho}_{t+1}(\tilde{\lambda})\cdot q_t \cdot (1-y) - \hat{R}_{t+1}(\tilde{\lambda})\cdot p_{t+1}\cdot\left[\frac{b_t}{p_t}(\tilde{\lambda})\right]\right\}$$
$$-\frac{\hat{R}_{t+1}(\tilde{\lambda})\cdot p_{t+1}}{q_{t+1}}\cdot \ln\left\{w_{t+1} + \tilde{\lambda}\cdot q_{t+1} - \hat{\rho}_{t+1}(\tilde{\lambda})\cdot q_t \cdot (1-y) - \hat{R}_{t+1}(\tilde{\lambda})\cdot p_{t+1}\cdot\left[\frac{b_t}{p_t}(\tilde{\lambda})\right]\right\}. \qquad (25a)$$

Equation (21) a second order, nonlinear equation in $\frac{b_t}{p_t}(\tilde{\lambda})$, in which $\tilde{\lambda}$ also appears in the denominator of $\hat{R}_{t+1}$. On the first hand, if, in equilibrium, it is observed that $\left(b_t/p_t\right)$ increases with $\tilde{\lambda}$, it will indicate the presence of strategic complementarities. On the other hand, if $\left(b_t/p_t\right)$ decreases with $\tilde{\lambda}$, strategic substitutability dominates in equilibrium. Given the complex nature of the expression in (25a), it is not clear at first sight whether the IOUs issued are increasing or decreasing in $\tilde{\lambda}$. The following proposition illustrates the results.

Proposition 5 $\left(b_t/p_t\right)$ is increasing in the cut-off value of the shock, $\tilde{\lambda}$, over the interval $\left(\lambda_L, \lambda_M\right)$, where λ_M is such that $\frac{b_t}{p_t}\left(\lambda_M\right) = \frac{w_{t+1}}{\theta_{t+1}}$.

Proof:

a) Boundaries for the domain of $\left(b_t/p_t\right)$. First, $\frac{b_t}{p_t}\left(\lambda_L\right)$ does not exist[75]. Second, $\left(b_t/p_t\right) = \left(w_{t+1}/\theta_{t+1}\right)$ does not obtain in equilibrium. Two reasons explain the latter. First, the lenders' investment into capital must be strictly positive, and second, lenders will realize that borrowers will not have any resources left to repay their mortgages, thus defaulting both on mortgages and the IOUs.

b) After excruciating pain and suffering, I was able to find that[76]

$$\frac{d\left(b_t/p_t\right)}{d\tilde{\lambda}} = \frac{A_1(\tilde{\lambda}) + A_2(\tilde{\lambda})}{A_3(\tilde{\lambda})} \qquad (25b)$$

where I have assumed that $dL/d\tilde{\lambda} = 0$, to fix ideas. Moreover, one observes that $d\left(b_t/p_t\right)/d\tilde{\lambda} > 0$ everywhere but on the interval $\left[\bar{\lambda}_1, \bar{\lambda}_2\right]$, where $1 < \bar{\lambda}_1, \bar{\lambda}_2 < \lambda_H$ and $d\left(b_t/p_t\right)/d\tilde{\lambda} < 0$. Thus, there is an interval $\left[\bar{\lambda}_1, \bar{\lambda}_3\right]$, for $\bar{\lambda}_2 < \bar{\lambda}_3 < \lambda_H$, where more than one equilibria can exist, in the sense that the same value $\overline{(b/p)} > 0$ transpires for more than one value of $\tilde{\lambda}$. Graphical analysis indicates that at most three equilibria can obtain.

Q.E.D.

The previous result is very interesting. It implies that, in general, when $\tilde{\lambda}$ is higher, the young borrowers know that there will be a higher rate of default in the following date, and thus choose to

[75] See details in section 2 of the Technical Appendix.
[76] See the Technical Appendix for details on this proof.

borrow more, since they will very likely choose not repay it. Thus, the equilibrium in this model economy has the potential for strategic complementarities, which could lead to the presence of multiple equilibria, as it is the case in the interval $\left[\bar{\lambda}_1, \bar{\lambda}_3\right]$. However, I must point out that there is a limit to this effect, since values of $\tilde{\lambda}$ such that $\tilde{\lambda} \geq \lambda_M$ do not obtain in equilibrium. I must also point out that the amount issued of IOUs increases at the expense of deposits, which are perfect substitutes from the standpoint of the lenders.

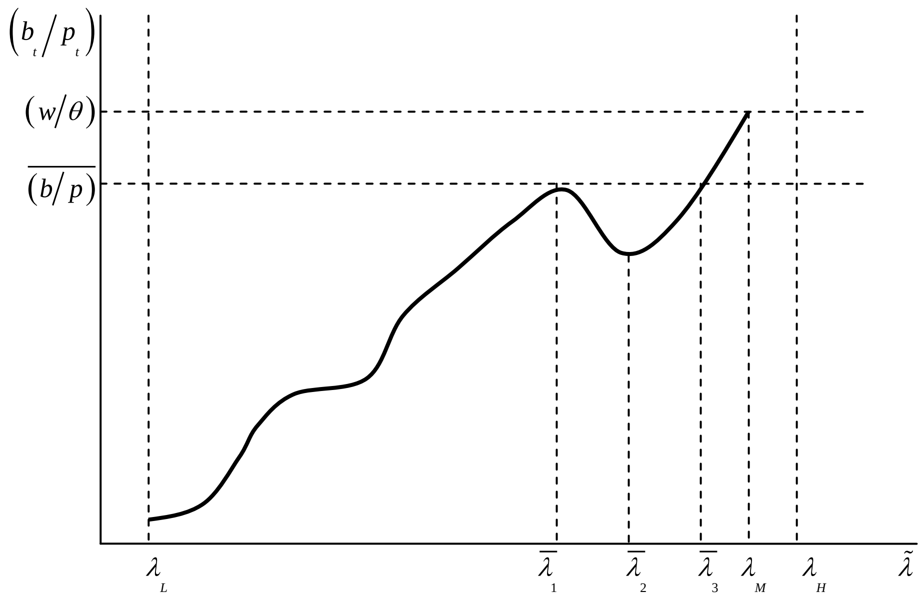


Figure 5: The Amount Issued of IOUs and Multiple Equilibria

The Interest Rate on Deposits and IOUs

The interest rates play a very important role in defining the properties of the equilibrium in this model. I start by discussing the properties of the interest rate on deposits and IOUs. These two interest rates are equal in equilibrium because there is no arbitrage in rates of return on assets that are perfect substitutes. In addition, the return on both deposits and IOUs represent a significant source of income to old lenders. After imposing equilibrium conditions on (16a), one obtains the following expression

$$\hat{R}_{t+1}\left(\tilde{\lambda}\right) = \hat{r}_{t+1}\left(\tilde{\lambda}\right) = \left[\frac{\alpha \cdot \theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right)}{p_{t+1} \cdot K^{1-\alpha}}\right] \cdot \frac{L^{1-\alpha}}{\left(\lambda_H - \tilde{\lambda}\right)} > 0 . \qquad (26a)$$

The reader may notice that, at date $t+1$, K_{t+1} is independent of $\tilde{\lambda}$, since it was determined before the shocks are realized. However, this is not the case for L_{t+1}, which is affected by the default of borrowers. One could alternatively regard the interest rate loan in terms of the return on capital, which is a perfect substitute to both deposits and IOUs. Let $k_{t+1} \equiv \left(K_{t+1} / L_{t+1}\right)$ denote the capital-labor ratio and $\Psi\left(k_{t+1}\right)$ denote the marginal product of capital. Then, the equilibrium interest rate on deposits can also be expressed as $\hat{R}\left(\tilde{\lambda}\right) = \theta_{t+1} \cdot \Psi\left(k_{t+1}\right) \cdot \left(1 - \pi_{t+1}\right)^{-1}$, implying that there is no arbitrage among the returns of all the assets available to young lenders.

Proposition 6 The equilibrium interest rate on deposits and IOUs $\hat{R}_{t+1}$ is nonlinear and convex function of $\tilde{\lambda}$. In particular, $\hat{R}_{t+1}$ is decreasing over the interval $\left[\lambda_L, \lambda_R\right)$ and decreasing when $\tilde{\lambda} \in \left(\lambda_R, \lambda_H\right]$, where λ_R is such that $d\hat{R}_{t+1} \big/ d\tilde{\lambda}\Big|_{\tilde{\lambda}=\lambda_R} = 0$. Moreover, this interest rate grows without upper bound as $\tilde{\lambda}$ grows closer to λ_H.

Proof: Differentiating (26a) with respect to $\tilde{\lambda}$ yields

$$\frac{d\hat{R}_{t+1}}{d\tilde{\lambda}} = \left[\frac{\alpha \cdot \theta_{t+1} \cdot \left(\lambda_H - \lambda_L\right)}{p_{t+1} \cdot K^{1-\alpha}}\right] \cdot \left\{\left[\frac{\left(1-\alpha\right) \cdot L^{-\alpha}}{\left(\lambda_H - \tilde{\lambda}\right)}\right]\frac{dL}{d\tilde{\lambda}} + \frac{L^{1-\alpha}}{\left(\lambda_H - \tilde{\lambda}\right)}\right\}. \quad (26b)$$

The first term inside of the curly brackets in (26b) is negative and decreasing in $\tilde{\lambda}$, while the second term is positive and increasing in $\tilde{\lambda}$. It is apparent, then, that there exists a value $\lambda^{**} \in \left(\lambda_L, \lambda_H\right)$, such that $d\hat{R}_{t+1}\left(\lambda^{**}\right) \big/ d\tilde{\lambda} = 0$ obtains. Moreover, it transpires that $d\hat{R}_{t+1}\left(\hat{\lambda}_1\right) \big/ d\tilde{\lambda} < 0$ for $\forall \hat{\lambda}_1 < \lambda^{**}$, while $d\hat{R}_{t+1}\left(\hat{\lambda}_2\right) \big/ d\tilde{\lambda} > 0$ for $\forall \hat{\lambda}_2 > \lambda^{**}$.

Secondly, after evaluating (20a) at $\tilde{\lambda} = \lambda_H$, one obtains

$$\lim_{\tilde{\lambda} \to \lambda_H} \hat{R}_{t+1}\left(\tilde{\lambda}\right) = \theta \cdot \Psi\left(k_{t+1}\right) \cdot \left(\frac{\lambda_H - \lambda_L}{\lambda_H - \tilde{\lambda}}\right) = \infty, \quad (26c)$$

which proves the second part of this proposition.

Q.E.D.

Proposition 6 indicates that there may be two equilibria for a given fixed value of the interest rate on deposits $\overline{R} > 0$, as illustrated in Figure 6 below. On the one hand, the value $\hat{\lambda}_1$ corresponds to an equilibrium where the interest rate is $\overline{R}$ and there is a low rate of default. On the other hand, the value $\hat{\lambda}_2$ is associated with an equilibrium where the interest rate is also equal to $\overline{R}$, but there is a high rate of default instead.

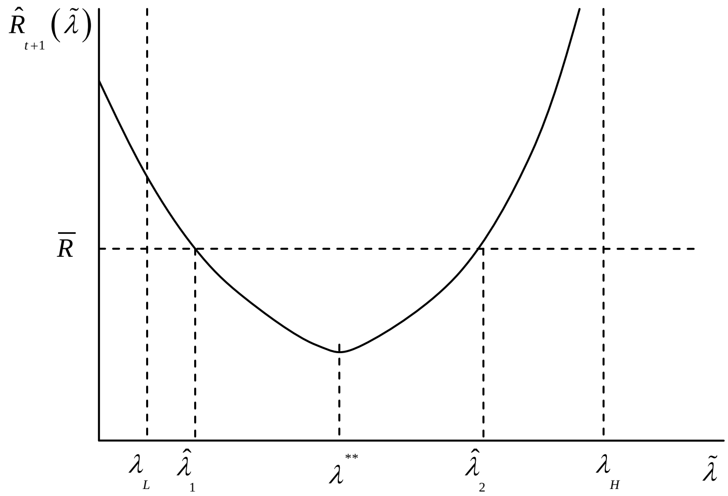


Figure 6: The Interest Rate on Deposits and Multiple Equilibria

The Interest Rate on Mortgages

The equilibrium interest rate on mortgages $\hat{\rho}_{t+1}$ plays a twofold role in this model. First, it is one of the key factors that affect the decision by borrowers of whether to default or not in equilibrium. Second, it represents the main source of income for the old bank, thus affecting her profit and the amount of labor services a_{t+1} that she will be able to purchase and consume. After imposing the equilibrium conditions on (16b), one obtains the following expression for the interest rate on mortgages:

$$\hat{\rho}_{t+1}(\tilde{\lambda}) = \frac{\tilde{\lambda} \cdot q_{t+1}}{q_t \cdot (1-y)} - \left[\frac{\alpha \cdot \theta_{t+1} \cdot (\lambda_H - \lambda_L)}{q_t \cdot (1-y) \cdot K^{1-\alpha}}\right] \frac{(b_t/p_t) \cdot L^{1-\alpha}}{(\lambda_H - \tilde{\lambda})}. \qquad (27a)$$

It is apparent from (27a) that $\hat{\rho}_{t+1}$ is a somewhat complex function of $\tilde{\lambda}$, making the analysis of its properties a bit complicated. Given the latter, I thus proceed by using a modified version of my fixed-point technique. I present the results in the following proposition.

Proposition 7 The following are properties of $\hat{\rho}_{t+1}$ in equilibrium:

1. $\hat{\rho}_{t+1}(\tilde{\lambda}) \geq 0$ is a smooth, nonlinear function of $\tilde{\lambda}$, such that $\left(d\hat{\rho}_{t+1}/d\tilde{\lambda}\right) > 0$ for $\tilde{\lambda} < \overline{\lambda}^*$, and $\left(d\hat{\rho}_{t+1}/d\tilde{\lambda}\right) < 0$ for $\tilde{\lambda} > \overline{\lambda}^*$. Moreover, $\left(d^2\hat{\rho}_{t+1}/d\tilde{\lambda}^2\right) < 0$, $\forall \tilde{\lambda} \in [\lambda_L, \lambda_H]$.
2. An interest rate on mortgages with meaningful economic content –i.e. $\hat{\rho}_{t+1}(\tilde{\lambda}) \geq 0$- in this model exists only for the range $[\tilde{\lambda}_1, \tilde{\lambda}_2]$ of its domain, where $\tilde{\lambda}_1 > \lambda_L$ and $\tilde{\lambda}_2 < \lambda_H$ are such that $\hat{\rho}_{t+1}(\tilde{\lambda}_1) = \hat{\rho}_{t+1}(\tilde{\lambda}_2) = 0$.

Proof:

Part 1: After differentiating equation (27a) with respect to $\tilde{\lambda}$, one obtains the expression

$$\frac{d\hat{\rho}_{t+1}}{d\tilde{\lambda}} = \left[\frac{q_{t+1}}{q_t \cdot (1-y)} - \frac{B \cdot (1-\alpha) \cdot (b_t/p_t) \cdot L^{-\alpha}}{(\lambda_H - \tilde{\lambda})^2} \cdot \frac{dL}{d\tilde{\lambda}}\right] - \frac{B \cdot L^{1-\alpha}}{(\lambda_H - \tilde{\lambda})^2}\left[(\lambda_H - \tilde{\lambda}) \cdot \frac{d(b_t/p_t)}{d\tilde{\lambda}} + (b_t/p_t)\right] \qquad (27b)$$

where $B \equiv \frac{\alpha \cdot \theta_{t+1} \cdot (\lambda_H - \lambda_L)}{q_t \cdot (1-y) \cdot K^{1-\alpha}} > 0$. It is apparent that the first term on the right hand side of (27b) is strictly positive and decreasing in $\tilde{\lambda}$, while the second term is strictly negative and nonlinear in $\tilde{\lambda}$. Thus, there exists a value of $\tilde{\lambda}$, that I will denote by $\overline{\lambda}^*$, such that $d\hat{\rho}\left(\overline{\lambda}^*\right)/d\tilde{\lambda} = 0$. Moreover, $\left(d\hat{\rho}_{t+1}/d\tilde{\lambda}\right) > 0$ holds for $\tilde{\lambda} < \overline{\lambda}^*$, while $\left(d\hat{\rho}_{t+1}/d\tilde{\lambda}\right) < 0$ for $\tilde{\lambda} > \overline{\lambda}^*$. All of the above implies that $\left(d^2\hat{\rho}_{t+1}/d\tilde{\lambda}^2\right) < 0$ holds[77], $\forall \tilde{\lambda} \in [\lambda_L, \lambda_H]$.

Part 2: There exist two values of $\tilde{\lambda}$ that I will denote by $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, such that $\hat{\rho}(\tilde{\lambda}_1) = \hat{\rho}(\tilde{\lambda}_2) = 0$, where $\lambda_L \leq \tilde{\lambda}_1 < \tilde{\lambda}_2 \leq \lambda_H$. Given the proof of part 1 of this proposition, it follows directly that $\hat{\rho}(\tilde{\lambda}) > 0$ for all $\tilde{\lambda} \in (\tilde{\lambda}_1, \tilde{\lambda}_2)$.

[77] This proof is available upon request.

Q.E.D.

It follows, from Proposition 7, that there exist two equilibria for a fixed real interest rate on loans $\overline{\rho}$, as indicated by Figure 7 below. The first equilibrium is associated with $\overline{\lambda}_1^*$, indicating the low rate of default $\pi_1^* = \overline{\lambda}_1^* \big/ \left(\lambda_H - \lambda_L\right)$. On the contrary, the second equilibrium has the cut-off value $\overline{\lambda}_2^*$, indicating the high rate of default $\pi_2^* = \overline{\lambda}_2^* \big/ \left(\lambda_H - \lambda_L\right)$.[78]

I must point out that the latter results are consistent with the equilibrium properties of $\hat{R}_{t+1}$ that I described in Proposition 6. Thus, this economy has the potential for multiple equilibria, which is explained mostly from the fact that strategic complementarities are present in this model economy. One, however, must still be careful when making statements about the number of equilibria in this economy due to the properties of the bank's profits in equilibrium. The latter rule out a set of low enough values of $\tilde{\lambda}$ for which the bank's profits are negative.

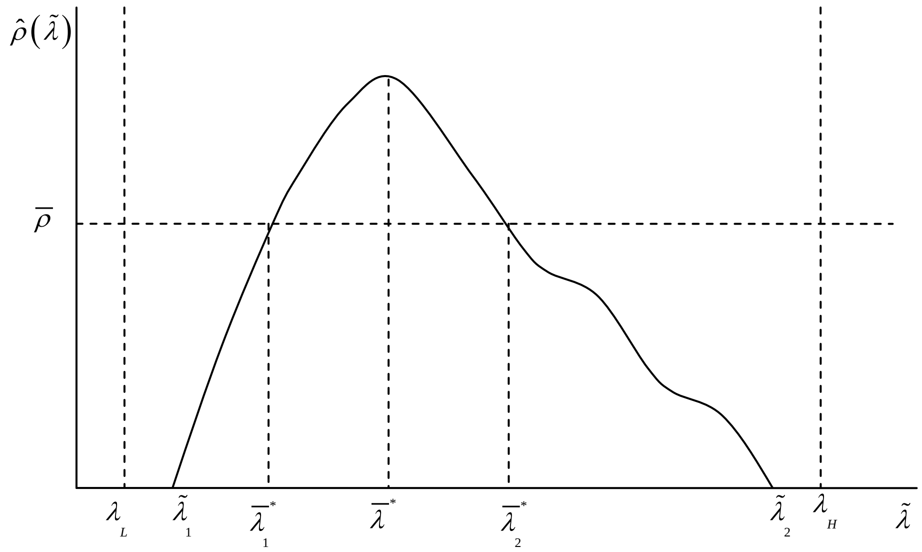


Figure 7: The Interest Rate on Loans and Multiple Equilibria

Preliminary Conclusions[79]

The main properties obtained from my model so far are:

1. There is always a positive rate of default in equilibrium, whether it is unique or not. Moreover, universal default and universal repayment cannot obtain in equilibrium due to the nature of the shock that continuously hits this economy and the signaling to lenders by borrowers, respectively.
2. The equilibrium interest rates on deposits, IOUs and loans display the potential for the existence of two equilibria: one with a low rate of default and the other with a high rate of default. However, the model so far has not produced any criteria for equilibrium selection yet.
3. Contrary to standard intuition, the monopolistic banks obtains positive profits only when the equilibrium rates of default are sufficiently high; my interpretation of this result is that banks are risk lovers in equilibrium.
4. In this model, young borrowers formulate a contingent plan in light of the realization of the shock that they will experience the following period. This works through the choice of a cut-off

[78] Of course, $\pi_1^* < \pi_2^*$ obtains.

[79] I apologize for the lack of flow in my writing, but it is mostly due to my time constraint.

value from the distribution of the shock, such that realizations below the cut-off will lead to defaulting while realizations above the cut-off will lead to repayment.

5. A crisis in this model takes the form of an innovation in the cut-off value of the shock.
6. A crisis reduces total output in this economy by reducing the resources available to purchase labor. Thus, one would expect the GDP to fall in a time of crisis.
7. A crisis also reduces the aggregate welfare of borrowers. However, this effect can be misleading, since the borrowers who default experience welfare gains, while the “honest” borrowers who choose to repay their debt are made worse-off. Thus, contrary to conventional wisdom, these result shows that the borrowers who default do not suffer from this shock and that the ones who are in the need of help, probably from government agencies, are the “honest” borrowers.
8. The lenders in this economy (equivalent to the general public who holds deposits on banks) experience significant welfare gains as a result of a crisis. This, then, is another sector that is not in need of government assistance.
9. A crisis increases the bank’s income but reduces its costs, thus increasing the bank’s profits in equilibrium. This result seems to go against conventional wisdom, but remember that there are no investment banks in this model nor mortgage backed securities. This is consistent with the fact that commercial banks seem to be doing reasonable well in light of the subprime mortgage crisis.

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