# Combinatorial Mathematical Tasks in the Education in Mathematics for Grades 1.- 4.

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**Abstract**

The choice of strategies and their correct combination over the course of training of the students to solve tasks from the area of combinatorics for composing combinatorial compounds from permutation type is one of the important factors for performance of efficient education in mathematics during the classes for extended and additional training in the Primary school.
Object of the study is the influence of the applied strategies on the process of acquiring of knowledge, development of skills and competencies in the students for solving tasks from the area of combinatorics in the education in mathematics during the classes for optional education in Grades 1. – 4.
The research work was performed in qualitative and quantitative aspects. The following methods were applied: experiment, observation, test, analysis of the content, mathematical-statistical method for data processing. As a result of the study it was found out that due to the applied methodology system of work where different strategies were combined, the competences and competencies for solving tasks from the area of combinatorics were developed in the students from Grades 1.- 4.

**Keywords:** Strategies, combinatorics, permutations, education in mathematics

## Introduction

Nowadays, at the present level of the education in mathematics in the Primary school the development of skills, knowledge and competencies in the students for solving tasks from the area of combinatorics is of significant importance for the purposes of achieving high level of general knowledge in mathematics.

## I.

**Goal** of the present study is through application of suitable strategies of work to facilitate the development of knowledge, skills and competencies

in the students for solving tasks from the area of combinatorics in and through the education in mathematics during the classes for Optional education in Grades 1.-4.

**Object** of the study is the process of education in mathematics during the classes for Optional education in Grades 1.-4.

**Subject** of the study is the influence of the applied strategies of work on the level of acquiring of knowledge and development in the students of skills, strategies and competencies for solving tasks from the area of combinatorics during the classes for Optional education in Grades 1.-4.

To achieve the goal of the study the researchers completed the following **tasks**:

1. Research and theoretical analysis of: Research works of foreign and Bulgarian authors regarding concepts related to: strategies, pedagogy technologies, approaches, methods and combinatorial compounds from permutation type.

2. Study of the main characteristics of some strategies and their application for development of a technology, a methodology system of work. Their use in the education in mathematics during the Optional mathematics classes in Grades 1.-4. with the idea to facilitate development of mathematical competencies in the students for solving tasks from the area of combinatorics directed to composing of combinatorial compounds from permutation type.

3. Study the efficiency of the applied strategies and methodology system of work during the Optional mathematical classes in Grades 1.-4. Comparative analysis and assessment of the results from the empiric study, summary. Presentation of conclusions.

For the purposes of the empiric study following criteria were offered: knowledge and skills to compose combinatorial compounds from permutation type with two elements – numbers; knowledge and skills to compose combinatorial compounds with three and more elements – numbers; knowledge and skills to compose combinatorial compounds with two elements of other type (different from numbers); knowledge and skills to compose combinatorial compounds with three and more elements of other type (different from numbers).

The following methods were used for the purposes of the research work: didactical experiment, analysis of content (content – analysis), observation, written work, mathematical-statistical methods for data processing.

In the process of analyzing Bulgarian and foreign literature on the topic it was found out that there are multiple studies on the problems related to the strategies for education. Some of the authors whose works were analyzed are Alexander (1991), Andreev (2001), Bruner (1995), Kostova –

Chavdarova and others, (2012), Merdzhanova (2005), Radev (2007), Woolfolk (1992) and etc.

In didactics the concept of strategy is used to describe procedures and activities related to teaching and studying, with organization of the educational process. Historically, this terminology is used as a synonym of "method" or "procedure" and contains the meaning for "system of knowledge", "skills, ability to manage", different means, tools and methods for achieving a goal. (The International Encyclopedia of education, 1987)

After summarizing the results from the performed theoretical analysis it was found out that there are different classifications depending on the basis on which different strategies have been formed. The following strategies were used for the purposes of the study: direct strategy (reproductive), indirect strategy (problem-productive, situational strategy) and a strategy for joint studying and cooperation (strategy for work on a project, topic).

The direct strategy puts an accent on teaching and its main characteristic is the clear purposefulness, the exact quantification of the performed activities and their structuring.

The problem-productive (indirect) strategy of education is based and organized on the familiarization and inclusion of the student in an active transforming activity. Studying through research, studying through solving problems and making decisions, studying through discovery– these are the solutions that normally are related to this strategy.

Typically, the strategy for joint studying and cooperation, provides that the goals and the interests of the group have got the priority. The following features are included: interdependence between members of the group; cooperation of "face-to-face" type, individual responsibility towards the common goal; development of skills for work in a small group; reflexive discussion of the work done. (Chavdarova – Kostova and others, 2012)

For the purposes of the empiric study and based on the performed analysis and the proposed theoretical concepts, a poli-didactical technology was developed and applied during the Optional mathematics classes for Grades 1.-4. The strategy by nature is purpose oriented – mathematical, developing, informational (acquiring of knowledge, skills and competencies) and heuristic (the creative skills of the students are getting developed), educational, makes the young students more active and finally, according to the prevailing method of education is problem oriented, creative.

In the course of the research work the author developed a methodology system of work that was systematically applied during the Optional mathematics classes in Grades 1.-4. The new system combined the strategy for joint studying and cooperation (strategy for work on a project, topic), problem – productive strategy (indirect, situational strategy) and a reproductive (direct) strategy.

Mathematical tasks assigned as combinatorial were known to people from ancient times. In XVII century the results from multiple works of Tartalia, Erigon, Pascal and Ferma on this topic were known, but the scientific foundation of the theory were laid in 1666 by the twenty years old Leibnitz in his work „Dissertatio de arte combinatoria“,where this area in mathematics received its name from.

There are multiple publications related to the combinatorics and the Probability Theory (Balabanova, Dineva, 1995), (Sougarev, Kamenarov, 1974) and etc.

According to Balabanova and Dineva combinatorial compound means “group of objects, numbers, letters, etc. amongst the elements of a given multiplicity which are created in accordance with a certain rule and differentiate from each other by the elements themselves or by their location in the group.” Depending on the ways of their creation, the compounds can be different. There are two types of combinatorial compounds: compounds without repetition and compounds with repetition. “When the elements in a given compound are different from each other, then this compound is called compound without repetition. In the opposite case the compound is called compound with repetition.” (Balabanova, Dineva, 1995)

Sougarev and Kamenarov described the permutations without repetition: “It is easy to understand the pattern for creation of permutations – to each element of the compound add the permutations of the rest of the elements. The number of these permutations is equal to the number of all the elements minus one.”

The multiplication of the successive natural numbers from 1 to n, i.e. 1 . 2 .3 . … . n can be defined as n! and shall be read “n factoriel”.

Consequesntly, the number of the possible permutations from n elements will be:

$P_n$ = 1 . 2 . 3. … . ( n – 1) . n = n!

Other words, the number of possible permutations Pn from n elements is equal to the result from the multiplication of the natural numbers from 1 to n”.

There are four areas (Clusters) of competency specified in the educational programs in mathematics approved by the Ministry of Education and Science of republic of Bulgaria. for Grades 1.-4.: “Numbers”, “Measuring”, “Geometry figures and bodies” and “Modelling”. The knowledge, the skills, the relations and the competences developed in the students as a result of solving combinatorial tasks belong to the competency Cluster “Numbers”.

Over the course of analyzing the educational documentation – textbooks and notebooks related to the mandatory educational content, it was found out that they content certain tasks from the area of combinatorics

without. However, the textbooks and the notebooks in question do not offer systematic approach for work with them during the mandatory mathematics classes in Grades 1.-4.

The methodology system of work with tasks from the area of combinatorics during the Optional mathematics classes for Grades 1.-4. was created to facilitate the development of: student's thinking; the skill to find non-standard solutions of mathematical tasks.

The new system includes series of mathematical tasks whose creation was based on the understanding that they are the "main tool" for development of mathematical competences. There are tasks which form separate elements of mathematical competences related to solving combinatorial tasks from competency Cluster "Numbers" as well as tasks that form the overall mathematical competency of the students. These tasks complete each other and create "complex repeating situation". (Petrov, Temnikova, 2016). Some of the tasks included in the series as well as their specifics are presented below.

In the course of education during the Optional mathematics classes for Grade 1 the work with combinatorial tasks starts even during the so called pre-number period. The number of elements in the compound from permutation type without repetition is two and these are colors used by the students to color objects sketched in advance – small houses, boats, ships, cars, airplanes, balls, flags, umbrellas, flowers and geometry figures (square, circle, triangle, and rectangle). When studying the numbers up to 10 the number of elements in the combinatorial compound shall be increased to three. To facilitate the students during solving the first tasks for composing compounds from permutation type with three elements, the total number of these compounds is given in advance and also a colored example of two of the compounds is given. The first graders have to color the other 4 compounds making analogy in their minds.

After studying the numbers up to 20 during learning addition and subtraction the following type of mathematical tasks were included: The numbers 1 and 0 are given. Wright with them one-digit numbers and two-digit numbers. Find the difference between the biggest and the smallest of them.

The alphabet letters learned during the Bulgarian language and literature classes can be used as elements in the combinatorial compounds. The following task can be used as an example: The word LAK is given. Compose and wright down all possible combinations with these three letters. Underline those who make sense.

The tasks related to construction of geometry figures are interesting for the children. The work with such tasks starts with a task where a square is divided in 4 parts – triangles. A figure of a human is composed by these

triangle which need to be colored by the students. On the next stage of the works the children work with the Chinese game "Tangram". This is a square which is divided into 7 parts. The students use them to compose new figures using given example – racket, cat, fish, ostrich, etc. The students work with individual didactic material.

In Grade II the work with combinatorial tasks continues with composing permutation compounds without repetition with three elements. These elements could be numbers or to be of other type. The students compose the compounds and perform additional activities like coloring fishes, three-color bands, toys, etc. The work related to construction of geometry figures continues with the game "Tangram" but the examples get more complicated in comparison with those used in Grade I.

In Grade III the students compose combinatorial compound of permutation type with three elements that might be numbers, geometry figures or other type. The methodology system of work includes tasks for composing three-digit numbers. Again, the additional requirements in the tasks after composing the combinatorial compounds are related to acquiring knowledge and development of skills and competencies from competency Cluster "Numbers" as the methodology system provides work with numbers within the range between 100 and 1000. The students do transfer of the competencies acquired during the activities performed in Grade II thus developing their thinking through convergent and divergent behavior over the process of applying the acquired knowledge as well as during performance of analysis, synthesis and analogy for solving combinatorial tasks. The conditions for transfer are created thanks to the combined higher class strategies used by the teacher – problem-productive (indirect) strategy and the strategy for joint studying and cooperation (strategy for work on a project or topic). The combinatorial compounds with three elements from other type which the students compose shall be colored by rows and columns (the rows and columns are formed by different objects) without repeating the colors. Also, there are tasks which require composition of combinatorial compounds (sentences) with three words (elements). For example: Compose all possible sentences with the following words: rain, outside, it.

The tasks related to modelling of different figures using example composed of sections of a square from the "Tangram" game get more complicated. The methodology system of work offers combinatorial tasks requiring modelling with the parts of the "Egg of Columbus". There is increasing difficulty in the example figures which have to be used by the students for the purposes of the task solving. The series of tasks include such tasks which require the students to create their own figures without following an example figure.

In Grade IV the students compose combinatorial compound of permutation type with three elements (numbers). Additionally, the tasks require the students to: put the composed numbers in a row starting from the smallest number or with the biggest number, present the combinations as sum of ordinal units. The students start composing permutations with four, five and six elements and consequently start composing four-digit, five-digit and six-digit numbers. These tasks require the students not to compose all possible permutations differently from the combinatorial tasks for composing permutations with two and three elements where all possible permutations need to be composed. For example the task: "Without repeating the numbers 2, 0, 6, 7 and 4, wright down at least 4 six-digit numbers. Put in a circle the biggest and the smallest number." As it can be seen, there is an additional requirement after composing the compounds and namely to determine the biggest and the smallest one. There are conditions offering the students different options for solving the task.

Some of the combinatorial tasks included in the methodology system of work offer a Table where the students wright down the combinatorial compounds of permutation type which they composed. The purpose of the Table is to help the students to easier solve the task.

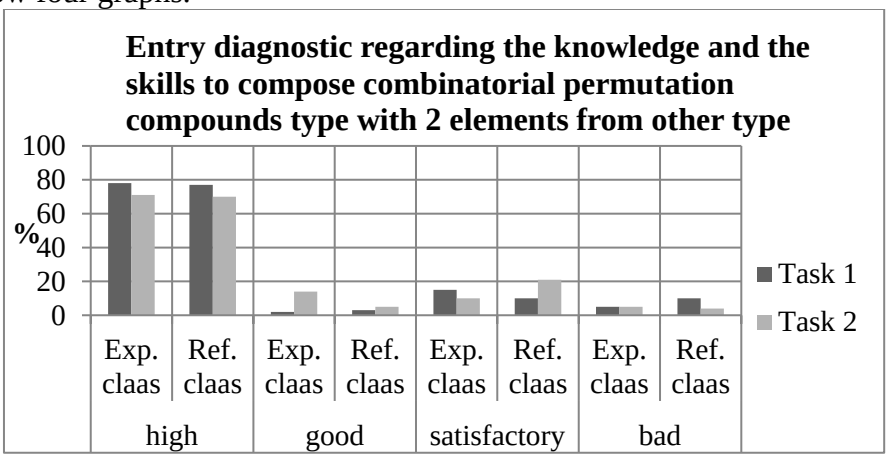
The teacher may allow team work and decide which task to be solved by separate teams or groups.

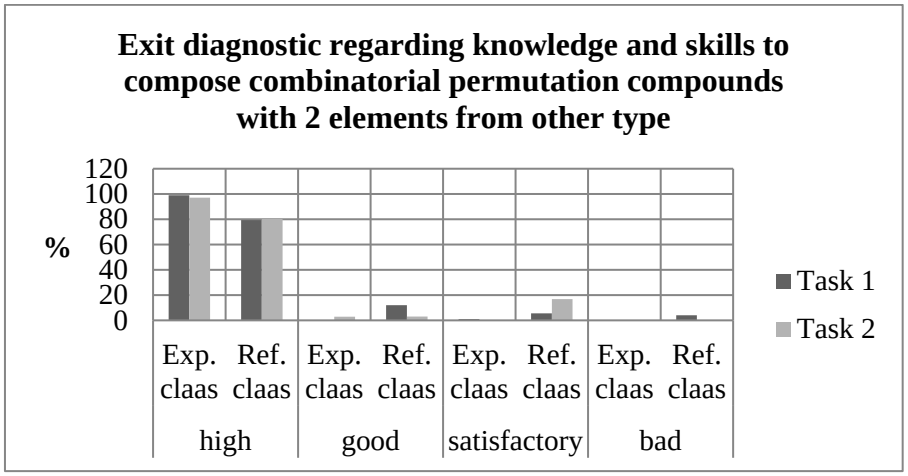
Over the period between 2012 and 2017 the researchers performed a study regarding the knowledge, the skills and the competencies of the students to compose combinatorial compounds without repetition from permutation type. The study concentrated on the education in mathematics for Grades 1.-4. during Optional mathematics classes. Two classes of students with statistically equal levels of educational achievements were subjected to the study. One of the classes was the experimental class where the newly developed system of work was applied and the second class was the referent class where the traditional system of work was applied.

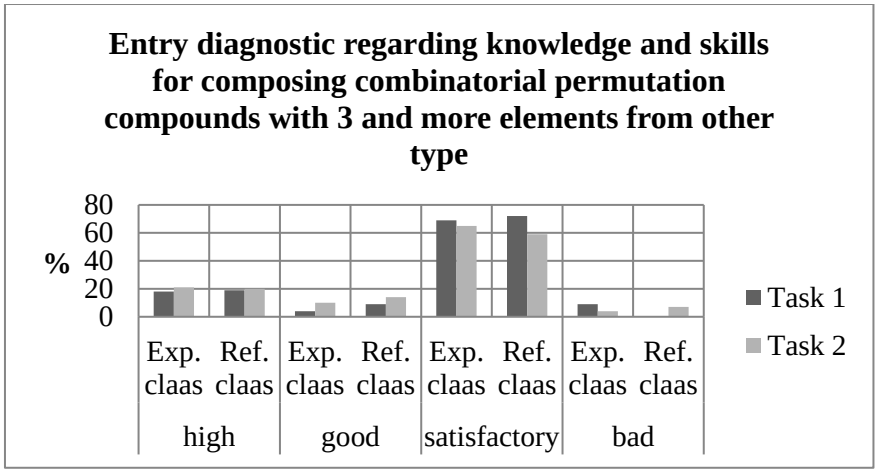
Two tests were used in the empiric study: one for determining the entry diagnostic and the other one for determining the exit diagnostic of knowledge, skills and competencies of the students to solve combinatorial tasks for composing permutations. The objectivity, the validity and the reliability of the tests were studied as well as the separating power of the tasks included in them.

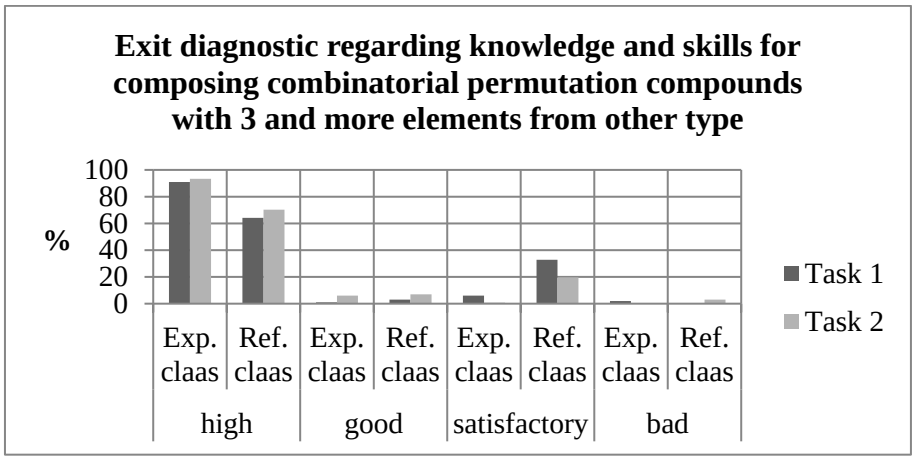
The results were analyzed including performance of comparative analysis. The data from the comparative analysis of the results from the entry and exit diagnostics regarding the knowledge and the skills of the students to compose combinatorial compounds from permutation type with two elements (numbers); regarding the knowledge and the skills to compose combinatorial compounds from permutation type with three and more

elements (numbers); knowledge and skills to compose combinatorial compounds with two elements from other type (different from numbers); knowledge and skills to compose combinatorial compounds with three and more elements from other type (different from numbers) are presented in the below four graphs.






The data from the entry diagnostic show that there is no statistically significant difference between the results from the respective lots of students subjected to the study from the experimental and the referent class in respect of knowledge, skills and competences. After the experimental work it was found out that in the exit diagnostic there is significant difference between the results achieved by the students from the experimental and the referent class. The percentage of students who correctly composed combinatorial compounds from permutation type with two elements (numbers) is within the limits of 97,1% and 99% for the experimental class and between 79,5% and 80,2% for the referent class. No students from the experimental class failed to compose combinatorial permutation compound with two elements from other type while 4% of the students from the referent class failed to do this. The level of knowledge, skills and mathematical competences of the students who correctly composed combinatorial compounds of permutation type with 3 and more elements (numbers) was increased from 15% to 90,1% for the

students from the experimental class and from 22,7% to 75,4% for the students from the referent class. 93,3% of the students from the experimental class correctly composed combinatorial permutation compounds with 3 and more elements of other type while only 70,3% of the students from the referent class managed to do this. The difference in the results was proved to be statistically significant.

**Conclusion:**

Based on the received results the following conclusions can be made: due to the applied new methodology system of work the students from the experimental class developed higher level of knowledge, skills and competencies for composing combinatorial compound of permutation type in comparison with the students from the referent class.

The systematic application and combination of strategies for joint studying and cooperation (strategy for work on a project, topic), the problem-productive (direct, situational) and reproductive strategies in the education during Optional mathematics classes in Grades 1.-4. for solving combinatorial tasks facilitates for both acquiring of knowledge, development of skills and competencies from competency Cluster "Numbers" and for development general mathematical competency in the Primary school students.

The applied methodology system of work helps to develop students' thinking and their ability to look for non-standard solutions of mathematical tasks. The combinatorial tasks from permutation type increase students' interest to mathematics as a whole and stimulate the creativity, research ambition in every child, widen his mathematical horizon.

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