WEAK SEPARATION AXIOMS VIA $D_\omega, D_{\alpha-\omega}, D_{pre-\omega}, D_{b-\omega}$, AND $D_{\beta-\omega}$ -SETS

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Abstract:
In this paper we define new types of sets we call them $D_\omega, D_{\alpha-\omega}, D_{pre-\omega}, D_{pre-\omega}, D_{b-\omega}$, and $D_{\beta-\omega}$ −sets and use them to define some associative separation axioms. Some theorems about the relation between them and the weak separation axioms introduced by M. H. Hadi in [1] are proved, with some other simple theorems. Throughout this paper , (X,T) stands for topological space. Let (X,T) be a topological space and A a subset of X. A point x in X is called ***condensation point*** of A if for each U in T with x in U, the set $U \cap A$ is uncountable [10]. In 1982 the ω −closed set was first introduced by H. Z. Hdeib in [10], and he defined it as: A is ***ω −closed*** if it contains all its condensation points and the ***ω −open*** set is the complement of the ω −closed set. Equivalently. A sub set W of a space (X,T), is ω −open if and only if for each $x \in W$, there exists $U \in T$ such that $x \in U$and $U \backslash W$ is countable. The collection of all ω −open sets of (X,T) denoted T_ω form topology on X and it is finer than T. Several characterizations of ω −closed sets were provided in [3, 4, 5, 6].

Key Words: Axioms, weak separation

In [7,8,9] some authors introduced α −open , pre –open, b –open , and β −open sets. On the other hand in [2] T. Noiri, A. Al-Omari, M. S. M. Noorani introduced the notions $\alpha - \omega$ −open, $pre - \omega$ −open, $\beta - \omega$ −open, and $b - \omega$ −open sets in topological spaces. They defined them as follows: A subset A of a space X is called: ***$\alpha - \omega$ −open*** [2] if $A \subseteq int_\omega\left(cl\left(int_\omega(A)\right)\right)$ and the complement of the $\alpha - \omega$ −open set is called ***$\alpha - \omega$ −closed*** set, ***pre −ω −open*** [2] if $A \subseteq int_\omega\left(cl(A)\right)$ and the complement of the $pre - \omega$ −open set is called ***$pre - \omega$ −closed*** set, ***b −ω −open*** [2] if $A \subseteq int_\omega\left(cl(A)\right) \cup cl\left(int_\omega(A)\right)$ and the complement of the $b - \omega$ −open set is called ***$b - \omega$ −closed*** set, ***$\beta - \omega$ −open*** [2] if $A \subseteq cl\left(int_\omega\left(cl(A)\right)\right)$ and the complement of the $\beta - \omega$ −open set is called ***$\beta - \omega$ −closed*** set.

Now let recall some condition introduced by M. H. Hadiin [1]: Let (X,T) be topological space. It said to be satisfy**:** The ***ω −condition*** if every ω −open set is $\omega - t$ −set**. 2.** The ***$\omega - B_\alpha$ −condition*** if every $\alpha - \omega$ −open set is $\omega - B_\alpha$ −set. The ***$\omega - B$ −condition*** if every $pre - \omega$ −open is $\omega - B$ −set.

In Our paper we **firstly** introduce our dominion and some results related to it.

Definition 1. A subset A of a topological space (X,T) is called ***D −set*** [11] (resp.***D_ω −set , $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, $D_{\beta-\omega}$ −set***). If there are two open (resp. ω −open, $\alpha - \omega$ −open, $pre - \omega$ −open, $\beta - \omega$ −open, and $b - \omega$ −open) sets U and V with $U \neq X$ and $A = U \backslash V$.

Proposition 2. In any topological space satisfies ω −condition. Any D_ω −set is D −set.

Proposition 3. In any topological space satisfies $\omega - B_\alpha$ −condition. Any $D_{\alpha-\omega}$ −set is D −set.

Proposition 4. In any topological space satisfies $\omega - B$ −condition. Any $D_{pre-\omega}$ −set is D −set.

Proposition 5. In any topological space. Any $D_{b-\omega}$ −set with empty ω −interior is $D_{pre-\omega}$ −set .

Secondly now utilizing the weak D_ω sets we can define our separation axioms and a rather simple theorem related to it as follows:

Definition 6. Let X be a topological space. If $x \neq y \in X$, either there exists a set U, such that $x \in U$, $y \notin U$,or there exists a set U such that $x \notin U, y \in U$. Then X called

1. $\boldsymbol{\omega - D_0}$ ***space***, whenever U is D_ω −set in X.
2. $\boldsymbol{\alpha - \omega - D_0}$ ***space,*** whenever U is $D_{\alpha-\omega}$ −set in X.
3. $\boldsymbol{pre{-}\omega - D_0}$ ***space***, whenever U is $D_{pre-\omega}$ −set in X.
4. $\boldsymbol{b - \omega - D_0}$ ***space***, whenever U is $D_{b-\omega}$ −set in X.
5. $\boldsymbol{\beta - \omega - D_0}$ ***space***, whenever U is $D_{\beta-\omega}$ −set in X.

Definition 7. We can define the spaces $\omega - D_i$, $\alpha - \omega - D_i$, $pre - \omega - D_i$, $b - \omega - D_i$, $\beta - \omega - D_i$, for $i = 0,1,2$. And $\omega^\star - D_i$, $\alpha - \omega^\star - D_i$, $\alpha - \omega^{\star\star} - D_i$, $pre - \omega^\star - D_i$, $\alpha - pre - \omega - D_i$, $pre - \omega^{\star\star} - D_i$, $b - \omega^\star - D_i$, $pre - b - \omega - D_i$, $\alpha - b - \omega - D_i$, $b - \omega^{\star\star} - D_i$, $\beta - \omega^\star - D_i$, $\alpha - \beta - \omega - D_i$, $pre - \beta - \omega - D_i$, $\beta - \omega^{\star\star} - D_i$, and $b - \beta - \omega - D_i$, for $i = 1,2$, by replacing the sets: open, ω −open, $\alpha - \omega$ −open, $pre - \omega$-open, $b - \omega$-open, $\beta - \omega$-open, by the D −set, D_ω −set ,$D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, and $D_{\beta-\omega}$ −set**.**

Theorem 8. Let (X, T) be a topological space. Then X is $\omega - D_1$,(resp. $\alpha - \omega - D_1$, $\omega^\star - D_1$, $\alpha - \omega^\star - D_1$, $\alpha - \omega^{\star\star} - D_1$, $pre - \omega - D_1$, $pre - \omega^\star - D_1$, $\alpha - pre - \omega - D_1$, $b - \omega - D_1$, $pre - \omega^{\star\star} - D_1$, $b - \omega - D_1$, $b - \omega^\star - D_1$, $pre - b - \omega - D_1$, $\alpha - b - \omega - D_1$, $pre - b - \omega - D_1$, $b - \omega^{\star\star} - D_1$, $\beta - \omega - D_1$, $\beta - \omega^\star - D_1$, $\alpha - \beta - \omega - D_1$, $pre - \beta - \omega - D_1$, $\beta - \omega^{\star\star} - D_1$, $b - \beta - \omega - D_1$) if and only if it is $\omega - D_2$,(resp. $\alpha - \omega - D_2$, $\omega^\star - D_2$, $\alpha - \omega^\star - D_2$, $\alpha - \omega^{\star\star} - D_2$, $pre - \omega - D_2$, $pre - \omega^\star - D_2$, $\alpha - pre - \omega - D_2$, $b - \omega - D_2$, $pre - \omega^{\star\star} - D_2$, $b - \omega - D_2$, $b - \omega^\star - D_2$, $pre - b - \omega - D_2$, $\alpha - b - \omega - D_2$, $pre - b - \omega - D_2$, $b - \omega^{\star\star} - D_2$, $\beta - \omega - D_2$, $\beta - \omega^\star - D_2$, $\alpha - \beta - \omega - D_2$, $pre - \beta - \omega - D_2$, $\beta - \omega^{\star\star} - D_2$, $b - \beta - \omega - D_2$).

Thirdly we introduce the so called ***ω −net point*** and a rather theorems related to it.

Definition 9. A point $x \in X$ which has only X as ω −neighbourhood is called an ***ω −net point***.

Proposition 10. Let (X, T) be a topological space If X is $\omega - D_1$ space, then it has no ω −net point.

Theorem 11. If $f: (X, \tau) \to (Y, \sigma)$ is ω −continuous (resp. $\alpha - \omega$-continuous , pre $-\omega$-continuous, $\beta - \omega$ −continuous, b $-\omega$ −continuous) onto function and A is D_ω −set (resp. $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, $D_{\beta-\omega}$ −set) in Y, then $f^{-1}(A)$ is also D_ω −set (resp. $D_{\alpha-\omega}$ −set, $D_{pre-\omega}$ −set, $D_{b-\omega}$ −set, $D_{\beta-\omega}$ −set) in X.

Theorem 12. For any two topological spaces (X, τ) and (Y, σ).

1. If (Y, σ) be an $\omega^\star - D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is an ω −continuous bijection, then (X, τ) is $\omega^\star - D_1$.

2. If (Y, σ) be an , $\alpha - \omega^{\star\star} - D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is an $\alpha - \omega$ −continuous bijection, then (X, τ) is, $\alpha - \omega^{\star\star} - D_1$.

3. If (Y, σ) be a , pre $-\omega^{\star\star} - D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a $pre - \omega$-continuous bijection, then (X, τ) is $pre - \omega^{\star\star} - D_1$.

4. If (Y, σ) be a , b $-\omega^{\star\star} - D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a $b - \omega$ −continuous bijection, then (X, τ) is $b - \omega^{\star\star} - D_1$.

5. If (Y, σ) be a , $\beta - \omega^{\star\star} - D_1$ and $f: (X, \tau) \to (Y, \sigma)$ is a $\beta - \omega$ −continuous bijection, then (X, τ) is $\beta - \omega^{\star\star} - D_1$.

Theorem 13. A topological space (X, T) is $\omega^\star - D_1$ (resp. $\alpha - \omega^{\star\star} - D_1$, $pre - \omega^{\star\star} - D_1$, , $b - \omega^{\star\star} - D_1$, $\beta - \omega^{\star\star} - D_1$) if and only if for each pair of distinct points $x, y \in X$, there exists an ω −continuous (resp. α − ω −continuous, pre − ω-continuous, b − ω-continuous, β − ω-continuous) onto function $f: (X, \tau) \to (Y, \sigma)$ such that $f(x)$ and $f(y)$ are distinct, where (Y, σ) is $\omega^\star - D_1$ (resp. $\alpha - \omega^{\star\star} - D_1$, $pre - \omega^{\star\star} - D_1$, $b - \omega^{\star\star} - D_1$, $\beta - \omega^{\star\star} - D_1$) space.

References:

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