MATHEMATIC-PHYSICAL MODEL OF DIMENSIONING SYSTEM IN THE PROPAGATION OF MICROWAVE "WAVEGUIDE-SLUDGE“ FROM WASTEWATER TREATMENT PLANTS

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Abstract:

This paper presents a physico-mathematical model sizing system microwave field propagation through a waveguide of the waveguide composed of a rectangular section and a pyramidal funnel section rectangular variable distribution. This model was adapted to the conditions of "waveguide-pond sludge bed" treaty. The system is to define parameters and propagation equations for adaptation.

Key Words: Physical and mathematical model, microwave, sludge settling, waveguide

Introduction

Making the physical and mathematical model was developed in order to create a mathematical tool for calculating the size of the wave guide stand for laboratory research on the field of microwave thermal processing of sludge from wastewater treatment pond.

The model is based on the general theory of microwave propagation presented in the literature [1], [2], [3], [4], [5], adapting to the specific mathematical relations of "waveguide-bed mud pond“. It takes into account phenomena occurring in electromagnetic wave propagation through a rectangular guide and mathematical relationships that define these phenomena.

Determination of model parameters

For the specific case of the plant designed to establish physical and mathematical model of microwave propagation system we took a structural analysis with the following composition: all the magnetron, wave guide, funnel distribution and resonant cavity, shown schematically in Figure 1.

Microwave transmission system, rectangular section; work on the fundamental propagation mode TE_{01}.

Determination of physical-mathematical model of microwave propagation in the system involves defining the parameters of the rectangular guide propagation.

Critical frequency is defined as the frequency below which no electromagnetic wave propagation occurs.

This frequency is calculated with:

$$f_c = \frac{c_0}{2}\sqrt{\left(\frac{m}{b}\right)^2 + \left(\frac{n}{a}\right)^2} \tag{1}$$

Critical wavelength is the wavelength corresponding critical frequency is determined with:

$$\lambda_c = \frac{c_0}{f_c} \tag{2}$$

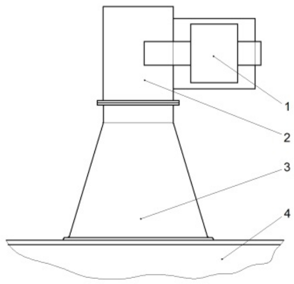

Figure 1. - The propagation of microwaves
1 - magnetron assembly, 2 - waveguide, 3 - Funnels distribution, 4 - cavity resonant

Guide wavelength is calculated with:

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}} \tag{3}$$

Vacuum wavelength λ_0, appropriate working frequency is calculated with:

$$\lambda_0 = \frac{c_0}{f} \tag{4}$$

Dephasing constant is calculated with:

$$\beta_{0,1} = \frac{2\pi \cdot f}{c_0}\sqrt{1-(f_c/f)^2} \tag{5}$$

Dephasing constant change relative to 2π (i.e. $\beta_0/2\pi$), the frequency relative to the speed of propagation in free space c_0 (i.e. f/c_0), is represented in Figure 2.

In this case the dephasing constant does not vary in proportion to frequency, so the user is a dispersive wave.

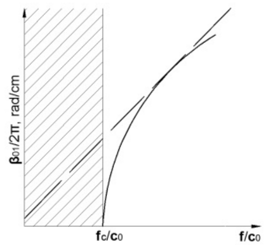

Figure 2. - Variation of constant lag in the reported frequency c_0 [1]

Group velocity is the actual speed of propagation of microwave energy is calculated with:

$$v_g = c_0 \cdot \sqrt{1-(f_c/f)^2 \cdot 10^{-2}} \tag{6}$$

Group velocity is less than the speed of light and by the asymptotic (figure 3.).

Wave impedance

If the wave magnetic field H_{01} (for mode propagation TE_{01}), the wave impedance is calculated with:

$$Z = Z_0 \cdot \frac{1}{\sqrt{1-(f_c/f)^2}} \tag{7}$$

The:

$Z_0 = \sqrt{\mu_0/\varepsilon_0} = 120 \cdot \pi = 377$ is the vacuum wave impedance.

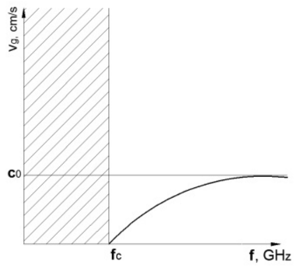

Figure 3.- Group-velocity variation with frequency [1]

In figure 4 is presented the wave impedance variation with frequency.

The expression for calculating the wave impedance (7) is valid for uniform waveguide (lossless) and air inside the propagation environment. Thus the frequency band $f > f_c$ wave impedance is purely resistive.

Distribution of electromagnetic field components in the guide is given by relations (8)...(10):

$$H_z = H_0 \cdot \cos\frac{\pi}{a} x \cdot \cos(\omega t - \beta_{01} \cdot z) \tag{8}$$

$$H_x = -\frac{\lambda_c}{\lambda_0} \cdot H_0 \cdot \sin\frac{\pi}{a} x \cdot \sin(\omega t - \beta_{01} \cdot z) \tag{9}$$

$$E_y = -\frac{\lambda_c}{\lambda_0} Z_0 \cdot H_0 \cdot \sin\frac{\pi}{a} x \cdot \sin(\omega t - \beta_{01} \cdot z) \tag{10}$$

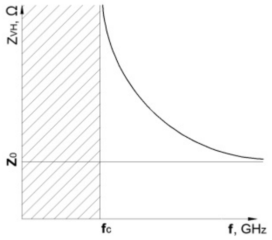

Figure 4. - Variation of impedance with frequency [1]

Curves of variation of the amplitude components x are presented according to figure 5, where it is shown as lines of electric and magnetic field in the waveguide where the wave H_{01}

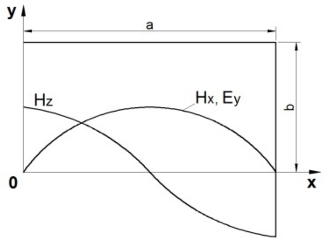

Figure 5. - Variation of intensity components H_z, H_x, and E_y depending on x [1]

From this figure two conclusions can be drawn: the electric field tangential to the wall is zero and shows many signs m and n of $H_{m,n}$ the semi-sine wave is present on the base a and the height b. If H_{01} for $x = a$ is a semi-sinusoid; and b the field is constant, i.e. not depending on y.

Attenuation constant determines the power losses in the waveguide. This constant is calculated from the dimensions of the waveguide section. For the wave H_{01}, the expression for calculating the attenuation constant is:

$$\alpha_{H_{01}} = \frac{\frac{2}{a} + \frac{2}{b}\left[1 + \left(\frac{\lambda_c}{\lambda_g}\right)^2\right]}{\delta \cdot \psi \cdot Z_0 \cdot \lambda_c{}^2 / \lambda_0 \cdot \lambda_g} \tag{11}$$

Penetration depth δ is determined by the:

$$d = \frac{5}{\sqrt{f \cdot \psi \cdot 10^7}} \cdot 10^{-6} \tag{12}$$

From this relation results the penetration depth [m], if the frequency is measured in [GHz] and constant wall material forming the guide, in [s/m];

The dissipated power in the walls of the guidance system is determined by the total attenuation of the microwave route guidance system, and is given by:

$$\alpha = \alpha_{H_{01}} \cdot l \tag{13}$$

Where:

l - is the length of the trail and microwave to the material is composed of: magnetron waveguide length, and the length of the distribution funnel. For the distribution funnel was taken a coverage factor of 2, so that the total equivalent length is:

$$l = l_1 + 2 \cdot l_2 \tag{14}$$

Since,

$\alpha = \log X$ and $X = \frac{P_i}{P_e}$

Where:

X - is the ratio of powers of entry into the route P_i and exit P_e.

results:

$$X = 10^{\frac{\alpha}{10}} \tag{15}$$

The dissipated power (which is converted into heat in the metal walls of the guidance system by the Joule-Lentz effect) is given by:

$$P_d = \frac{X-1}{X} \cdot P_i \tag{16}$$

The maximum allowable power is limited, depending on the guide size and accepted default losses. The maximum permissible power, transmitted through the waveguide for the wave $H_{0,1}$, in conditions of adaptation:

$$P_1 = 2500 \cdot \sqrt{1-\eta^2} \cdot a \cdot b \tag{17}$$

In case of inadaptation, power in load is the difference between the corresponding power of the incident wave and power of the reflected wave, i.e.:

$$P_s = \frac{a \cdot b}{4 \cdot Z} \cdot (E_i + E_r) \cdot (E_i - E_r) \tag{18}$$

Because

$E_i + E_r = E_{max}$; $E_i - E_r = E_{min}$ and $\frac{E_{max}}{E_{min}} = \sigma$ (Defined as standing wave factor), we obtain:

$$P_s = \frac{a \cdot b \cdot E_{max}{}^2}{4 \cdot Z \cdot \sigma} \tag{19}$$

That transmitted power is σ times less, than, for the same amount of electric field strength E_{max}.

The power variation (or the squared electric field strength, E^2) in the case of adaptation is shown in figure 6.

Basically, for the modeling of microwave sizing guide must always consider the condition of adaptation, the condition is satisfied if the standing wave factor is $\sigma = 1{,}1 \div 1{,}2$.

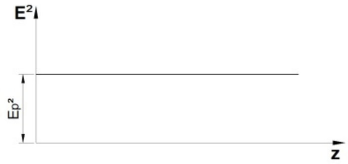

Figure 6. - Variation of the square intensity of the electric field, in case of adaptation [1]

For the maximum value of the standing wave factor $\sigma = 1{,}2$, reflection coefficient becomes $|\Gamma| = \frac{\sigma - 1}{\sigma + 1} = \frac{0{,}2}{2{,}2} = 0{,}09$ and so the reflected power (reported to the incident power) represent 1%, which is practically acceptable.

Determination of the specific equation for the primary physical model of adaptation

In the case of pyramidal scheme distribution funnel (figure 1), adopted when designing mobile applicator laboratory stand, we can define the physical model of primary adaptation, whose principle scheme is shown in figure 7.

This model have of three areas of propagation: propagation in space guided, which is composed of the constant section guide and the variable cross-section guide, the pyramidal funnel; the propagation in free space from the output of funnel and spread to the surface of sludge AA' and in the interior of the sludge bed, which is characterized as a mixture of two components, water and solid part.

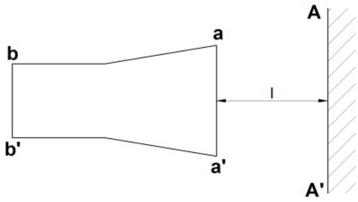

Figure 7. - Pyramidal funnel propagation system [1]

This propagation system can be equivalent with an electric circuit similar to that in figure 8.

Sludge bed is a loss medium characterized in terms of dielectric constant and conductivity electromagnetic times exactly tangent of loss angle:

$$tg\delta_p = \frac{\sigma}{\omega \cdot \varepsilon_p} \tag{20}$$

As the angle is small, it is considered to be equal to the value of angle tangent of loss angle in radians: $tg\delta_p \cong \delta_p$.

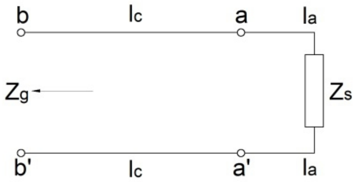

Figure 8. - Equivalent circuit of the adaptation system

Adaptation is the elimination of reflected wave (reflected power that is proportional to the square reflected wave shown). Elimination of reflected wave occurs when the transition from one environment to another is eliminated discontinuities, impedance i.e. at the left and right of the surfaces AA', aa' and bb' is conveniently turns on the same as shown.

It is necessary therefore to calculate the input impedance at the surface AA'.

Establishing the impedance calculation relationships of the at the entrance in the sludge bed:

Field distribution in the sludge bed, at the right of the surface AA' is exponential and there is no reflected wave, due to the high attenuation in the treated sludge. In this case the surface impedance is the impedance of semi-infinite space with the dielectric constant ε_p and the loss angle δ_p.
Impedance of free space is $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ [Ω] and in the case of dielectric environment with loss, the impedance becomes:

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0(\varepsilon_p - j\delta_p)}} \tag{21}$$

where the angle of loss is given by:

$$\delta_p = \frac{\sigma}{j \cdot \omega \cdot \varepsilon_0} \tag{22}$$

In this case – at 2,45 GHz frequency for the water $\varepsilon_p = 77$ and $\delta_p = 0{,}15$, and the solid part $\varepsilon_p = 4$ şi $\delta_p = 0{,}01$.

Equations for calculating the average electromagnetic constants and input impedance of sludge bed:

Sludge can be treated as a mixture of two components: water and the solid, so the average dielectric constant (ε_{pm}) is calculated with:

$$\varepsilon_{pm} = \varepsilon_{p1} \times p_1 + \varepsilon_{p2} \times p_2 \tag{23}$$

Impedance at the entrance in the sludge bed is determined by the relationship:

$$Z_{AA'} = \frac{Z_0}{\sqrt{\varepsilon_{pm}}} \tag{24}$$

The equations of variation of microwave reflection:

Input impedance in the sludge bed has different values depending on the water content, during drying process the water content is variable and the average dielectric constant is variable so that reflection of microwaves will occur.

Generally the reflection coefficient is given by:

$$\Gamma = \frac{Z_s - Z_{01}}{Z_s + Z_{01}} \tag{25}$$

Where:

Z_s - is the load impedance (for this case, the input impedance in the sludge bed);

Z_{01} - is the reference impedance, where the reflection coefficient is zero (in this case, the impedance of the water content for which is the adaptation).

The adjustment is made for the reference water content, calculating the reflectivity for the minimum and maximum water content in sludge and with (26) is determined the reflected power:

$$P_r = |\Gamma|^2 \cdot P_i \tag{26}$$

With relation (27) is determined the relationship between the maximum electric field and minimum electric field in the wave guide for the two water contents.

$$k = \frac{1+|\Gamma|}{1-|\Gamma|} \tag{27}$$

Permissible values of the water content in the sludge are given by the allowable amount of reflected power (P_r). In the general case of drying materials with high humidity, it can accept power reflected a permissible value of 10% of rated output (P_i) of the magnetron, in this case, reflectivity coefficient is:

$|\Gamma|^2 = \frac{P_r}{P_i} = 0{,}1$ i.e. $\Gamma = 0{,}316$ and $k = \frac{1{,}316}{0{,}684} = 1{,}923 \cong 2$

Establishing the line equivalent relations sludge-funnel:

The space between the sludge and funnel distribution mouth (figure 7) the distance l, between the surfaces AA' and aa', has the equivalent circuit two-wire line with tha length l_a, between the load Z_s (input impedance of the sludge bed) and terminals aa' (figure. 8).

Putting on that, the portion provided that propagation is lossless, the relationship for calculating the input impedance of a lossless line is:

$$Z_1 = Z_{01} \cdot \frac{Z_s + j \cdot Z_{01} \cdot tg\beta_0 \cdot l}{Z_{01} + j \cdot Z_s \cdot tg\beta_0 \cdot l} \tag{28}$$

Where:

Z_{01} - is the characteristic impedance of the line;

Z_s - is the load impedance;

β_0 - is the dephasing constant, with: $\beta_0 = \frac{2\pi}{\lambda_0}$;

l - is the length of the line.

Should be provided $tg\beta_0 \cdot l = 0$ and when $Z_s = Z_i$, i.e. the load impedance is equal to the distribution at the mouth of the funnel. The distance between the shape and surface area at the mouth of the distribution funnel is derived from the relation:

$$\beta_0 \cdot l = N_\pi,\ N = 0,1,2,3... \tag{29}$$

And results:

$$l = N \cdot \frac{\lambda_0}{2} \tag{30}$$

So the distance between the distribution funnel mouth area and surface of the sludge is an integer value equal to half the size of the wavelength.

Funnel size optimization equations:

To establish the equations for optimization of the funnel waveguide dimensions must consider the variation of the module of the reflectance coefficient (Γ), according to the ratio l/λ_g, shown in figure 9.

Thus, the analysis of figure 9, results that (Γ) is small when the ratio l/λ_g is around $l = N \cdot \frac{\lambda_g}{2}$.

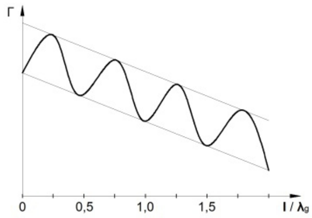

Figure 9. - The variation of the module (Γ) according to the ratio l/λ_g [1]

If the funnel section is quoted $a(x)$ in any point of its axis x (Figure 10), that can express this value by the relation:

$$a(x) = \frac{a_1 - a_0}{l_1} \cdot x + a_0 \tag{31}$$

Generally:

$$\int_0^{l_1} \frac{2\pi}{\lambda_g(x)} dx = (N-2)\pi \tag{32}$$

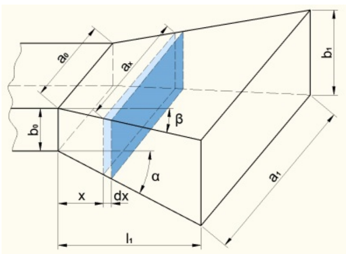

Figure 10. - The distribution funnel dimensions

And the guide wavelength at a distance x, we can write the relationship:

$$\lambda_g(x) = \lambda_0 / \sqrt{1 - \left(\frac{\lambda_0}{2a(x)}\right)^2} \tag{32}$$

So results:

$$\int_0^{l_1} \frac{2\pi}{\lambda_g(x)} dx = \int_0^{l_1} \frac{2\pi}{\lambda_0} \sqrt{1 - \left(\frac{\lambda_0}{2a(x)}\right)^2} dx = \frac{2\pi}{\lambda_0} \int_0^{l_1} \sqrt{1 - \left(\frac{\lambda_0}{2a(x)}\right)^2} dx \tag{33}$$

Noting:

$$T = \int_0^{l_1} \sqrt{1 - \left(\frac{\lambda_0}{2a(x)}\right)^2} dx \tag{34}$$

Results:

$$T = \frac{\lambda_0 \cdot l_1}{2(a_1 - a_0)} \left[\sqrt{\left(\frac{2a_1}{\lambda_0}\right)^2 - 1} - \sqrt{\left(\frac{2a_0}{\lambda_0}\right)^2 - 1} - \arccos\frac{\lambda_0}{2a_1} + \arccos\frac{\lambda_0}{2a_0} \right] \tag{35}$$

but:

$$\frac{2\pi \cdot T}{\lambda_0} = (N-2) \cdot \pi \tag{36}$$

It follows the relationship for calculating the length of the distribution funnel:

$$l_1 = \frac{(N-2) \cdot \pi \cdot \lambda_0}{T \cdot 2\pi} \ [cm] \tag{37}$$

Calculation of tilt angles of the distribution funnel:

- the long side of the section:

$$tg\alpha = \frac{b_1 - b_0}{l_1} \tag{38}$$

- the short side of the section:

$$tg\beta = \frac{a_1 - a_0}{l_1} \tag{39}$$

The calculation relationships for the reflectance coefficient:

The reflection coefficient in the distribution funnel is:

$$|\Gamma| = \frac{\lambda_0}{l}\left[\frac{K_0^2 + K_1^2}{64\pi^2} - \frac{K_0 \cdot K_1}{32\pi^2}\right]^{1/2} \tag{40}$$

Where:

$$K_0 = \frac{\frac{b_1 - b_0}{b_0} - \frac{a_1 - a_0}{a_0}\left[\frac{1}{1-(\lambda_0/2a_0)^2}\right]}{\left[1-(\lambda_0/2a_0)^2\right]^{1/2}} \tag{41}$$

$$K_1 = \frac{\frac{b_0 - b_1}{b_1} - \frac{a_0 - a_1}{a_1}\left[\frac{1}{1-(\lambda_0/2a_1)^2}\right]}{\left[1-(\lambda_0/2a_1)^2\right]^{1/2}} \tag{42}$$

Notations:

TE_{01} - fundamental transverse mode of propagation of microwave power;

f_c - critical frequency, in [Hz];

f - operating frequency, in [Hz].

c_0 - speed of light, $c_0 = 3\times10^{10}$ in [m/s];

a, b - the base dimensions of the rectangular guide, in [m];

λ_c - critical wavelength, in [m];

λ_g - guide wavelength, in [m];

λ_0 - corresponding vacuum wavelength for the work frequency , in [m];

$\beta_{0,1}$ - dephasing constant, in [rad/m];

v_g - group velocity, in [m/s];

Z - impedance microwave propagation in various materials, in [Ω];

Z_0 - impedance to the propagation of microwaves in a vacuum space, in [Ω];

Z_m - average wave impedance, in [Ω];

$E_{m,n}$, $H_{m,n}$ - electric and magnetic intensity of the propagation mode indices m and n of electromagnetic waves;

H_{01} - magnetic intensity for the fundamental wave, $m=0$ and $n=1$;

$\alpha_{H_{01}}$ - attenuation constant, in [dB/m];

ψ - constant guide wall forming material, in [s/m];

d - depth of penetration of waves into the material, in [μm];

l - length of the route guidance system of the microwave, in [m];

α - total attenuation of the microwave, in [dB];

X - ratio between the power at the entrance and the power at the exit of the guide route;

P_d - power dissipated in the walls of the guidance system, in [W];

P_1 - limit power (maximum allowable by the guide), in, [W];

$P_{r70\%}$ - reflected power, in [W];

m, n - values of electromagnetic waves for example in the case of the transverse electric propagation mode TE_{01}, we have $m=0$ and $n=1$;

η - Field efficiency of transmission defined as: $\eta = \frac{f_c}{f}$;

σ - stationary wave factor;
Z_s - load impedance, in [Ω];
$Z_{0,1}$ - reference impedance, in [Ω];
$|\Gamma|$ - reflection coefficient module;
ε_{pm} - average dielectric constant of a mixture;
ε_{p1}, ε_{p2} - dielectric constant of mixture components;
δ_p - angle of loss, in [rad];
p_1, p_2 - proportions of components in mixture, in [$\%$];
N - wave number of the rank;
a_0, b_0 - funnel entrance dimensions, in [m];
a_1, b_1 - funnel exit dimensions, in [m];
l_1 - funnel length, in [m];
K_0; K_1 - reflection parameters calculated according to the funnel section size at the entrance and exit;

Conclusion

This physic-mathematical model can be used to guide system sizing microwave applicator for mobile installations in the field of microwave processing of sludge from the wastewater treatment plants.

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