

RECTANGULAR OPEN CHANNELS OF COMBINED ROUGHNESS

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Abstract:

In this experimental study the Manning roughness coefficient n is examined, through a considerable number of laboratory measurements with artificially roughened walls of a rectangular open channel-which always has a smooth bed. The roughness elements are vertical rubber strips, which are systematically varying as far as their projections into the flowing water and their distances among them are concerned. The case of the completely smooth rectangular channel is also examined. Since n has the disadvantage of being in the metric system of units, some suitable dimensionless groups of parameters are used. n appears not to be constant at any present roughness condition and this probably is due to the nature of any flow, where even at the same Run its behavior is not constant. Apart from this, n is also varying in the completely smooth channel case. The results are strictly holding for uniform-subcritical-turbulent-steady water flows in rectilinear rectangular open channels of small longitudinal slope.

Key Words: Rectangular Open Channels, Combined Roughness

Introduction

Fig. 1 shows the characteristics of the uniform-subcritical-turbulent-steady water flow within a rectangular open channel (various water depths, z_i , reference depths $z_{ri} < z_i$, width $b=24$ cm and longitudinal slope $J_o=0.0012$), where for depth z_i the Manning roughness coefficients is n_i (or n) while for z_{ri} depths corresponding roughness coefficients are n_{ri} , both determined from the Manning formula, that is $n_i=(1/V_i) \cdot R_i^{2/3} \cdot J_o^{1/2}$, with n_i in the metric system ($s \cdot m^{-1/3}$), and R_i =hydraulic radius, V_i =cross sectional velocity ($=Q_i/(z_i \cdot b)$), all in the same units' system.

In this investigation both channel walls were systematically supplied with adhered rubber strips ($\kappa \times h$, where always $\kappa=4$ mm and $h=8-4-0$ mm) at various distances $\lambda=50-100-200-400$ mm, forming a number of λ/h ratios, $\lambda/h=12.5-25-50-100$ or $\lambda/h=\infty$ (for $h=0$ -smooth walls). The channel bed was always smooth ($h=0$, $\lambda/h=\infty$), and rectilinear, while the various λ/h were alternating on the walls, creating some artificial combined roughnesses with considerable differences among them and to the channel bed (especially, for $h=8$ mm). The roughness is not only h dependent but also λ/h ratio dependent: Any small λ/h ratio (e.g. 12.5) corresponds to rougher wall (e.g. from $\lambda/h=25$).

Apart from the first part of this study, further on an effort is made to correlate various dimensionless quantities (including the Manning's n) in order to receive systematic results for all present laboratory measurements, i.e. overcoming the metric units dependence.

In this paper, as most interesting previous books or papers the following texts are considered, by Chow, (1959), Ramesh et al, (2000), Pyle, (1981), Robertson et al, (1973), Rosso et al, (1950), Ghosh, (1978), Demetriou, (2003), Demetriou et al, (1999), Demetriou, (2000), and finally, Demetriou, (2001).

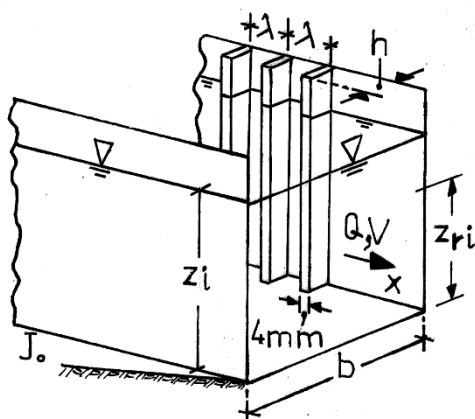


Figure 1. Flow and roughness characteristics.

The Experiments

All measurements were performed in the hydraulics Lab. of the School of Civil Engineering of the Nat. Technical Univ. of Athens (Greece), in a channel of 12 m long mainly with glass boundaries and steel bed.

8 Series with a number of $i=15 \div 17$ Runs (for each Series) were organized. z_i , R_i , Q_i , V_i , Re_i , Fr_i , were measured or calculated, while corresponding roughness coefficients (for each Run) were determined through the Manning formula. Table 1 shows a summary of all laboratory measurements and all aspect ratios (A.R.= b/z_i), where max n_i or min n_i do not always correspond to max or min A.R.

Table 1. Laboratory measurements.

Series N ^o	Runs (i)	z_i (cm)	Q_i (l/sec)	V_i (cm/sec)	h (mm)	λ (mm)	λ/h	n_i ($s \cdot m^{-1/3}$)	A.R.
1	i=1 to i=17	4.97 to 35.28	4.71 to 41.83	39.50 to 49.40	4	50	12.5	0.0106 to 0.0163	4.83 to 0.68
2	i=1 to i=17	4.70 to 35.53	3.84 to 41.83	34.07 to 49.05	4	100	25	0.01062 to 0.0160	5.11 to 0.68
3	i=1 to i=17	3.13 to 35.45	3.84 to 39.56	34.07 to 49.05	4	200	50	0.00577 to 0.0159	7.67 to 0.68
4	i=1 to i=17	4.92 to 34.30	5.43 to 45.79	46.05 to 55.68	4	400	100	0.00803 to 0.01524	4.88 to 0.70
5	i=1 to i=16	4.10 to 36.53	3.84 to 32.61	39.05 to 37.19	8	100	12.5	0.00867 to 0.01890	5.85 to 0.67
6	i=1 to i=15	5.25 to 36.93	5.43 to 38.71	43.13 to 43.67	8	200	25	0.00884 to 0.01870	4.57 to 0.65
7	i=1 to i=15	5.68 to 36.17	5.08 to 42.00	37.27 to 48.39	8	400	50	0.01061 to 0.0152	4.23 to 0.66
8	i=1 to i=16	4.62 to 36.25	3.84 to 52.08	34.68 to 59.87	0	-	∞	0.0100 to 0.0133	5.19 to 0.66

In any Run the smallest n_i value corresponds to the biggest A.R., but the biggest n_i value may be found at intermediate A.R. ratios. For the smooth channel $0.010 \leq n_i \leq 0.013$, as is referred in various books, e.g. by Chow, (1959). Finally, some of the initial results were rejected, while some other were smoothed out.

Results And Discussion

The method of interrelations

In order to elaborate the experimental results some reference depths, z_{ri} , with corresponding roughness coefficients, n_{ri} , were selected in any one Series, and so the ratios z_i/z_{ri} and n_i/n_{ri} were calculated. The typical selected z_{ri} depths were the closest to a scale of $z_{ri} \approx 10-15-20-25-30-35$ cm, where the finally chosen z_{ri} values were the actual depths in each Run. This method, i.e. of self-compared n_i/n_{ri} to z_i/z_{ri} in any Series, is named here as the method of interrelations.

Table 2 presents all results based on the above method, while Figs. 2 graphically show corresponding results, where n_i and n_{ri} are indirectly measured through the Manning formula.

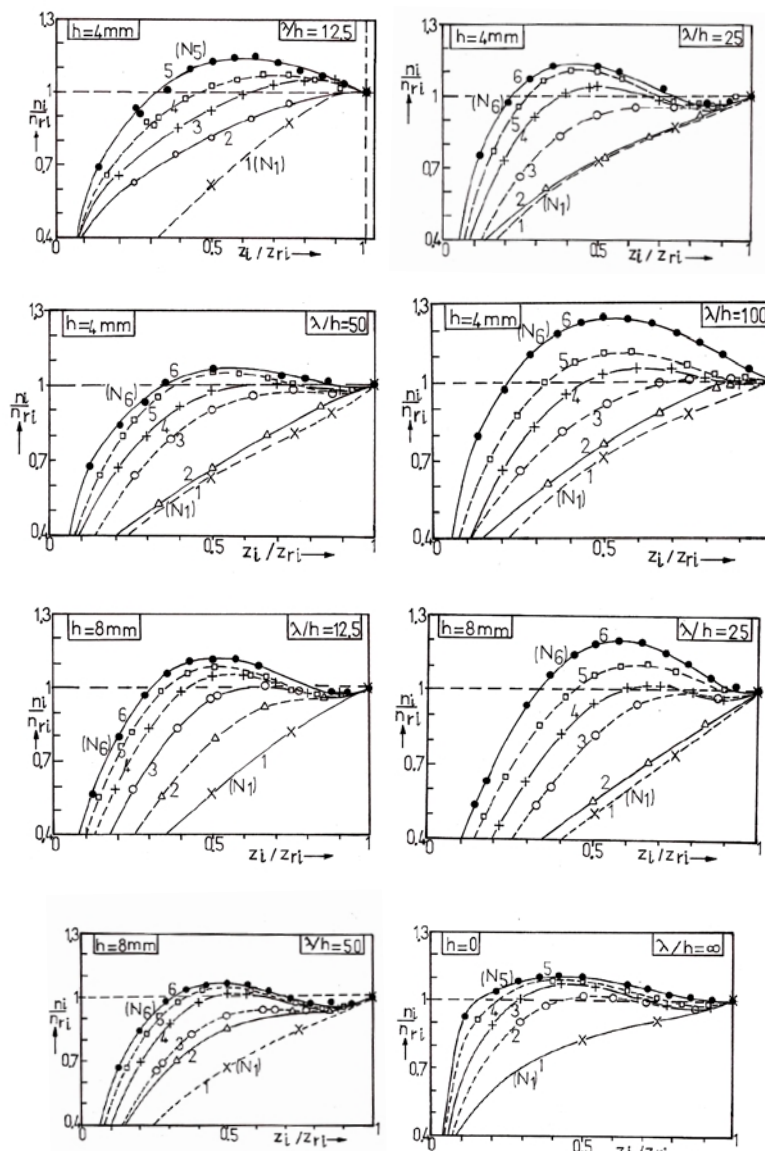
Table 2. Experimental measurements' elaboration.

Eqs. $N_i = n_i/n_{ri}$ vs $Z_i = z_i/z_{ri}$, in the form of $N_i = a \cdot Z_i^b \cdot [1 - Z_i]^c + Z_i$							
h=4 mm, $z_{ri} \approx 10-(15)-20-25-30-35$ cm				h=8 mm, $z_{ri} \approx 10-(15)-20-25-30-35$ cm			
$\lambda/h=12.5$	a	b	c	$\lambda/h=12.5$	a	b	C
N_1	0.465	1.383	0.663	N_1	0.652	2.017	1.182
N_2	0.712	0.274	0.877	N_2	3.464	1.933	1.676
N_3	0.889	0.310	0.740	N_3	4.564	1.620	1.812
N_4	1.489	0.510	1.909	N_4	4.962	1.296	1.892
N_5	1.930	0.550	1.090	N_5	4.583	1.105	1.862
$\lambda/h=25$	a	b	c	N_6	3.789	0.920	1.682
N_1	0.934	0.674	1.361	$\lambda/h=25$	a	b	C
N_2	0.680	0.412	1.133	N_1	1.193	0.470	12.055
N_3	2.520	0.957	1.617	N_2	0.160	0.621	0.855
N_4	3.770	0.949	1.856	N_3	3.893	2.059	1.562
N_5	4.234	0.881	1.929	N_4	4.978	1.720	1.754
N_6	3.227	0.665	1.702	N_5	3.676	1.237	1.470
$\lambda/h=50$	a	b	c	N_6	4.022	1.087	1.457
N_1	0.894	0.886	1.806	$\lambda/h=50$	a	b	C
N_2	0.290	0.159	0.667	N_1	0.846	0.926	1.369
N_3	1.991	0.946	1.364	N_2	2.156	1.003	1.613
N_4	2.034	0.758	1.352	N_3	3.217	1.143	1.821
N_5	2.920	0.829	1.580	N_4	3.685	1.011	1.835
N_6	2.015	0.550	1.285	N_5	3.576	0.883	1.826
$\lambda/h=100$	a	b	c	N_6	3.000	0.725	1.689
N_1	0.921	0.869	1.243	$\lambda/h=\infty$	a	b	C
N_2	0.482	0.296	0.615	N_1	1.059	0.430	1.274
N_3	1.277	0.632	1.012	N_2	2.574	0.724	1.592
N_4	2.829	0.945	1.468	N_3	2.751	0.643	1.631
N_5	2.749	0.772	1.407	N_4	2.110	0.428	1.425
N_6	-	-	-	N_5	1.678	0.273	1.210

The above curves are very smooth and for any one of them the ratio $N_i = n_i/n_{ri}$ may be compared to $Z_i = z_i/z_{ri}$, in the form of a typical equation

$$N_i = a \cdot Z_i^b \cdot (1 - Z_i)^c + Z_i \quad (1)$$

where the coefficients a, b, c, are also shown in Table 2. In this way to any Series a number of N_i equations (in total 45 equations) are produced, where each one of them corresponds to a respective curve of Figs. 2.



Figures 2. Numbered curves of n_i/n_{ri} vs z_i/z_{ri} for various h , λ/h , and corresponding equations N_i .

Other methods

To simplify the results, the roughness coefficients' symbol ($n=n_i$) corresponding to any depth z_i of each experimental Run is used again, while the index (i) is used only for z_i .

a) Apart from the previous description (Figs. 2 and Table 2), some more essential results are further presented, based on the following dimensionless groups among the Manning roughness coefficient n and g , R (=hydraulic radius), J_o , or λ/h ,

$$A=n \cdot g^{1/2} \cdot R^{-1/6}, A'=n \cdot g^{1/2} \cdot h^{-1/6},$$

$$B=(g \cdot R \cdot J_o)^{1/2} \cdot V^{-1}, A'=(g \cdot R \cdot J_o)^{1/2} \cdot (\lambda/h)^{1/2} \cdot V^{-1}.$$

A and A' mainly include Manning's n , while B and B' include $(g, R, J_o)^{1/2}$ and V , where for $\lambda/h \rightarrow 1$ it is $B' \rightarrow B$. n is in the metric system, but the above groups are dimensionless. A (or A') may combined to B (or B'), in order that some suitable descriptive curves are properly determined. In some cases the term $C=n \cdot (g \cdot z_n^{-1/3})^{0.5}$ is also used.

b) Smooth Channels

For the completely smooth ($\lambda/h=\infty$ on bed/walls), Fig. 3 shows all present experimental results (open circles) in terms of B vs C , where the corresponding (single) curve (for all present n, R, V, J_o) has the approximate descriptive equation

$$n \cdot (g \cdot z_n^{-1/3})^{0.5} = -0.0661 + 0.4855 \cdot [(g \cdot R \cdot J_o)^{0.25} \cdot V^{-0.5}], \tag{2}$$

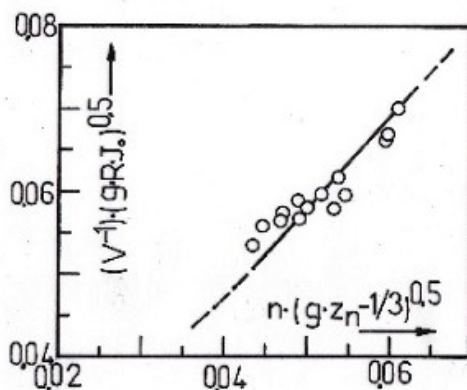


Figure 3. B vs C for completely smooth channel.

holding for $0.054 \leq B \leq 0.070$, $0.043 \leq C \leq 0.061$, and with a correlation coefficient $r^2 \approx 0.998$.

Other smooth channel cases are for $\lambda \rightarrow \infty$ (with $\kappa \neq 0$), or $\lambda \rightarrow \kappa$ (full contact of all roughness elements).

c) Rough walls and smooth bed.

Figure 4 compares A to B' (based on the present measurements) for $\lambda/h=12.5-25-50-100$

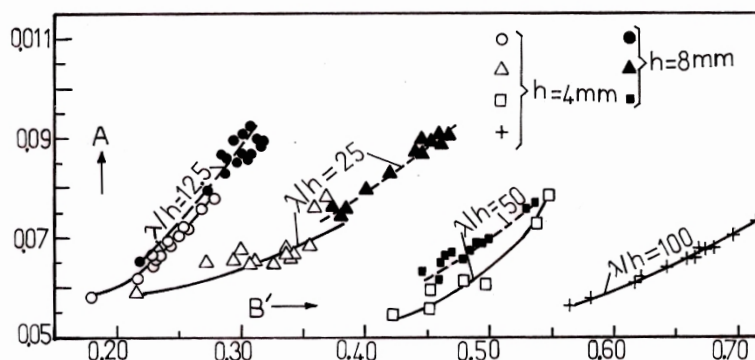


Figure 4. A vs B' for $\lambda/h=12.5-25-50-100$ ($h=4$ mm) and $\lambda/h=12.5-25-50$ ($h=8$ mm).

($h=4$ mm, open experimental symbols) and $\lambda/h=12.5-25-50$ ($h=8$ mm, black experimental symbols). All corresponding curves, through the experimental points, show that the $h=8$ mm results (dashed curves) are over the $h=4$ mm results (continuous curves), a fact which is considered as reasonable because of the increased wall roughness. Other facts are that all results produce separate curves, i.e. not a single equation can be given for all experimental results, and that when B' are increasing A are also increasing along each curve.

Among the (continuous) curves of $h=4$ mm a large space is offered for interpolations, and the same holds for the (dashed) curves ($h=8$ mm).

Contrary to the above results in Figs. 5, 6, 7, the same (as in Fig. 4) experimental results, are treated in terms of A' vs B. In Fig. 5 all results (for $h=4$ mm and all $\lambda/h=12.5-25-50-100$) are well concentrated around a single curve. This curve has a simple equation (through the experimental points-open circles), and since it holds for $\lambda/h=12.5$ to $\lambda/h=100$, it also holds in the entire field $12.5 \leq \lambda/h \leq 100$ (for $\kappa=4$ mm) and is described by the following equation,

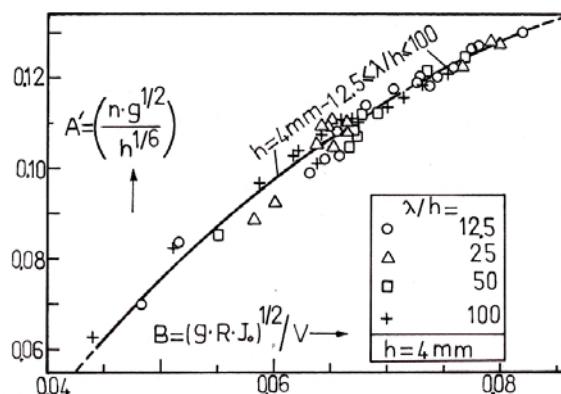


Figure 5. A' vs B , for all $h=4$ mm and $12.5 \leq \lambda/h \leq 100$ results.
 $A' \approx 0.3224 - 0.055 \cdot B^{-0.5}$, (3)

holding for $0.045 \leq B \leq 0.08$, $7 \leq A' \leq 12$ and $4.83 \leq A.R. \leq 7.67$. n is a function of h , R , J_0 and V , while when B are increasing n are also increasing, i.e. n is not constant throughout.

Fig. 6 also presents A' vs B for all $h=8$ mm and $\lambda/h=12.5-25-50$, experimental results (black symbols) which are also well concentrated around another single curve, holding for $h=8$ mm, the entire $12.5 \leq \lambda/h \leq 50$ range and $0.05 \leq B \leq 0.093$, $0.06 \leq A' \leq 0.113$, $0.65 \leq A.R. \leq 5.85$.

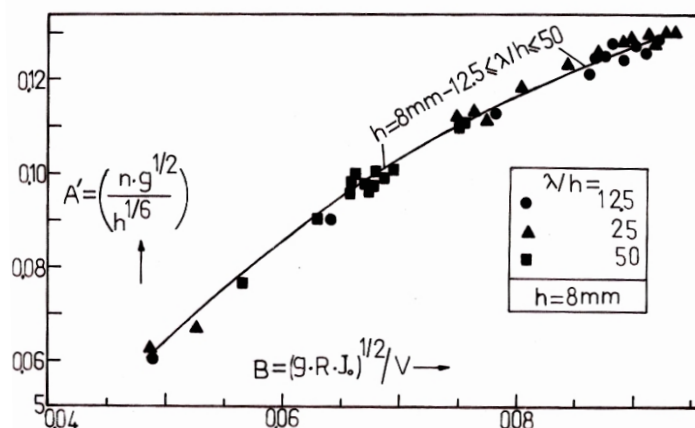


Figure 6. A' vs B , for all $h=8$ mm and $12.5 \leq \lambda/h \leq 50$ results.

Finally, Fig. 7 shows both curves (of Figs. 5, 6), in order to compare them. The curve for all $h=8$ mm experimental results is quite similar to the curve of $h=4$ mm results, and lies under the last curve at a constant vertical distance of $1.35 \cdot A'$. This fact may give a similar equation to the $h=8$ mm curve, since at same B value the lower curve lies at the above constant distance under the upper curve.

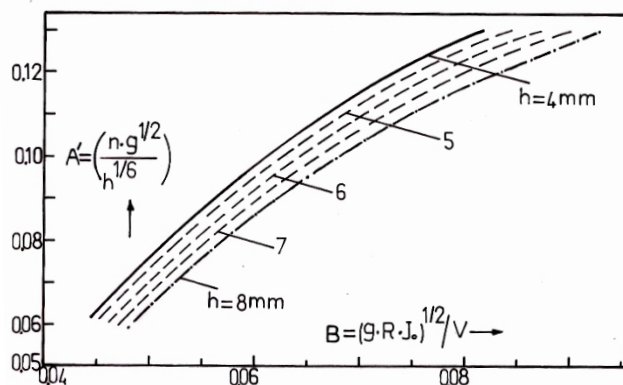


Figure 7. A' vs B , for all present rough walls' results and interpolations for $h=5-6$ and 7 mm.

If one wishes to explain why the $h=8$ mm curve is under the $h=4$ mm curve, this may be due to the combination of all quantities participating in the comparison, since n is proportional to $R^{2/3}$, $J_o^{1/2}$, and proportional to V^{-1} : When $h=8$ mm strips are used the flow (at $Q=\text{const.}$) has in general, larger velocities (in comparison to $h=4$ mm), since the channel's width which is used is constant for both cases and the flow is more intermixed.

As an important note on Fig. 7, since both curves, for $h=4$ and 8 mm are very similar among them, a family of intermediate (dashed) curves can be approximately traced by suitable interpolations, corresponding to $h=5-6$ and 7 mm, or even to more dense curves. More general, the entire group of curves of Fig. 7 holds for $4\text{mm} \leq h \leq 8\text{mm}$ and at least for $12.5 \leq \lambda/h \leq 50$.

It also may be noticed that for $h > 8$ mm corresponding curves are expected to be placed under the 8 mm curve, while for $h < 4$ mm (and even for $h=0$, smooth curve at $\lambda/h=\infty$) corresponding curves are expected to be placed over the 4 mm curve, i.e. apart from interpolations there is also a wide space for extrapolations.

The presentation of the results of Fig. 7 gives a rather broad possibility for a wide generalization of the results over the 4 mm curve, between $4 \leq h \leq 8$ mm, and below the $h=8$ mm curve, equally corresponding to a large λ/h range.

Finally, Chow, (1959), p. 98-99, referring to the Manning n establishment and its dimensions, is noting that "this was not a problem of concern to the forefathers of hydraulics" and "it is unreasonable to suppose that the roughness coefficient would contain time". Apart from this, the Manning formula has a factor of 1.49 for the English measuring system (in ft.), while this factor is 1 for the usual metric system. These problems could be solved if a dimensionless roughness coefficient is used. Although this has been already suggested in the past, it is not very clear who has made this suggestion, which actually, consists of the present terms A and B and their equation. If to A is given a new symbol $n^*(=A)$ then the dimensionless roughness coefficient is $n^*=A=(n \cdot g^{1/2} \cdot R^{-1/6})$, where-of course-the initial Manning equation (with n) is modified when n^* is used, $n^*=(g \cdot R \cdot J_o)^{1/2} \cdot (V^{-1})=B$.

Conclusions

In this experimental study the Manning n roughness coefficient is investigated for water uniform-subcritical-turbulent-steady flows, within rectilinear rectangular open channels of combined roughness, i.e. rough walls and smooth bed. The walls are artificially roughened through vertical strips (projecting into the flow at $h=4$ mm or $h=8$ mm) at regular distances λ , with $\lambda/h=12.5-25-50-100$ or $\lambda/h=\infty$ ($h=0$, smooth walls), while the bed constantly remains smooth ($h=0$, $\lambda/h=\infty$). The main conclusions are: 1) The application of the so-called method of interrelations, gives a large number of equations among the dimensionless roughness coefficients' ratios vs corresponding dimensionless flow depths. 2) These equations are giving some good relative results. 3) Since a big problem is coming from the fact that Manning roughness coefficients are in the metric system ($s \cdot m^{-1/3}$), four groups (A, A', B, B') of parameters are proposed in various combinations among n, g, R, h, J_o, V and λ/h , in order that some more general results may arise. 4) For the complete smooth channel ($h=0$, $\lambda/h=\infty$, everywhere) eq. (2) is determined based on the experiments, after the elaboration of them in terms of A vs B , this equation and Fig. 3 are concluded. 5) For all present experimental results with all various roughness conditions, Fig. 4, in terms of A vs B' , gives a number of systematic curves, for $h=4$ and 8 mm and all present λ/h combinations, where for $\lambda/h=\text{const.}$ all $h=8$ mm curves lie over $h=4$ mm (less rough) results. 6) When all present n results are elaborated in terms of A' vs B , then all experiments for $h=4$ mm (at all λ/h ratios) give (Fig. 5) a single curve, while a similar curve is produced for all $h=8$ mm results (Fig. 6 for all λ/h ratios). 7) Eq. 3 is proposed for the previous curve, concerning $h=4$ mm and all relevant λ/h ratios. 8) A method is also suggested for the corresponding curve for $h=8$ mm and all relevant λ/h ratios. 9) An extrapolation group of curves are also proposed, for $h=5-6-7$ mm and all associated λ/h ratios. The present study mainly presents an effort to rationalize the disadvantage of the Manning's roughness coefficient which is given in the metric system, by proposing combinations of all parameters in such a way that some dimensionless groups of expressions may be used. The application of such dimensionless groups of expressions may give in some cases, more general results, e.g. in the form of suitable curves described by proper equations. This application may also give, by systematic extrapolations, results for intermediate h values, while an overall result appears to be that Manning's n is not a constant throughout, i.e. it depends on various parameters and not only on roughness factors, especially in channels of combined roughnesses.

References:

- Chow V. T. *Open Channel Hydraulics*, Mc Graw-Hill, 1959.
- Demetriou J., Manassakis E. and Dimitriou D. Resistance Coefficient Change in Open Channels, 4th Nat. Congress of the Greek Committee for the Water Management, Proceedings, Volos, Greece, June Proceedings, Vol. B, pp. 91-97, 1999.
- Demetriou J. Irrigation Channels With Differential Roughness, 2nd Nat. Congress of Agricultural Mechanics, Volos, Greece, Sept., Proceedings, pp. 42-49, 2000.
- Demetriou J., Pourliotis K. and Sarantos P. Flow in Open Channels Lined With Different Materials, 6th Greek Congress on Mechanics, Thessaloniki, Greece, July, pp. 108-113, 2001.
- Demetriou J. Differential Roughness in Open Channels With Large Roughness Elements, EYE Congress, April, Thessaloniki, Greece, 7 pages, 2003.
- Ghosh S. Boundary Shear Distribution in Channels With Varying Wall Roughness, Institution of Civil Engineers, Proceedings, London, UK, Vol. 53, Part 2, Dec., pp. 529-543, 1972.
- Pyle R. and Novak P. Coefficient of Friction in Conduits With Large Roughness, *Journal of Hydraulic Research*, 19, N^o 2, pp. 119-140, 1981.
- Ramesh R., Data B., Bhallamudi M. and Narayana A. Optimal Estimation of Roughness in Open-Channel Flows, *Journal of Hydraulic Engineering*, ASCE, Vol. 126, N^o 4, April, pp. 299-303, 2000.
- Robertson J., Bajwa M and Wright S. A General Theory for Flow in Rough Conduits, *Journal of Hydraulic Research* 12, N^o 2, pp. 223-240, 1973.
- Rosso M., Shiara M. and Berlamont J. Flow Stability and Friction Factor in Rough Channels, ASCE, *Journal of Hydraulic Engineering*, Vol. 116, N^o 9, Sept., pp. 1109-1119, 1990.