ESJ Natural/Life/Medical Sciences

The Philosophy of Probability Values Behaviour through Fractions and Composite Probability Function for Independent Events in the Discrete Case

Abdulaziz Jughaiman
Bachelor Degree of Science in the Field of Operations Research
King Saud University, Saudi Arabia

Doi:10.19044/esj.2023.v19n18p1

Submitted: 28 April 2023
Accepted: 22 June 2023
Published: 30 June 2023

Cite As:
Jughaiman A. (2023). *The Philosophy of Probability Values Behaviour through Fractions and Composite Probability Function for Independent Events in the Discrete Case*. European Scientific Journal, ESJ, 19 (18), 1. https://doi.org/10.19044/esj.2023.v19n18p1

Abstract

This paper focuses on dealing with probability theory, and it adopted an objectivism philosophic approach, as well as a mathematical approach. These approaches aim to study the probability values behaviour as discrete quantities in the case of discrete sample space for independent events through fractions and composite functions. This requires a discussion of the usage of probability statements, the causes behind the existence of probabilistic phenomenon, and an explanation that admits to measuring causality in probability theory through the concept of complement and fractions. The paper uses an experiment with a design that addresses the shortcoming of traditional experiments through the concept of fractions. This, in turn, reflects some aspects of the probabilistic behaviour, including some important consequences that follow through. This paper also uses the relative frequency of events and the probability axioms, which provides the sample space to include all possible events in the form of sequences and sub sequences. The paper further provides some definitions that define some elements of the fractions probabilities and the sample space, alongside some proven propositions, lemma, and corollaries that admit to calculating composite probability functions. This is in addition to a brief discussion of the continuous cases.

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

Keywords: Causality, Complement, Experiment, Sequence, Stochastic

Introduction

The theory of probability precedes Pascal and Fermat. However, the important period of the development of probability theory was from 1575 to 1825 (Todhunter, 1949). This is in addition to the periods of the nineteenth and twentieth centuries (Bingham, 2000). Here, the nature of probability theory has at least two main classes of concepts (Parzen, 1960). Concerning the concept of probability theory, if it had an ambiguous side, it would be an ambiguous use rather than an ambiguous sense of the probability statement. Therefore, this results to the following question: in what sense can the probability statement be used? A clear literary meaning of the scientific phrase sometimes helps to accept it more in the scientific approach. Also, when the statement "probability" carries a determination connotation in its use, without changing the meaning of the probability statement, it will be accepted more than mere speculation. Thus, if the need to use probability requires the statement of probability, then the statement of probability is true, otherwise, the statement is not necessarily true. Sémantique addresses the change of word meaning linguistically in detail (Guiraud, 1969). To discuss the question with some logic, there are two conditions. First, if the statement is applied to the occurrence of an absolute event, where the sample space would be unspecified. Therefore, if it means a kind of knowing, then it is not necessarily a probability statement but a non-numerical probability statement. Second, if the occurrence of an event is bounded by several options, such as winning or losing, which means that the sample space is known. Subsequently, if the statement means the unknowing, then it is also not necessarily a probability statement but a numerical probability statement. Therefore, when the prophet prophesies, he does that with feeling, and he does not depend on a sample space. This means that he saw without sample space, even if that has been taken as a kind of determinism. Such intuition mostly depends on the feeling that constitutes an aspect of its experience. If the statements in the previous case are considered as probability statements and if the probability statement is considered as a kind of knowing, with some conditions, rather than unknowing, then the knowing will refer numerically to finite or infinite and countability, and the unknowing will refer to uncountability. Thus, they can be studied with the aid of probability theory itself.

This paper uses the plural form of the probability word to deal with probability as a set of probabilities. This is because a fraction or part means some, while deterministic may mean, in some possible sense, the whole or the integer. Probability does not mean deterministic and thus refers to the parts, not the whole. For example, suppose an observer, two events A, B, and the following conditions: If both events had occurred before, and if the event B

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

only occurs when the event A does not occur, then the observer has some knowledge about the events A and B and their relations. But, if the event B had never occurred before, and the observer has known nothing about the event B, then the occurrence of event A is not always truly deterministic. Falsely, the occurrence of event B would not always be probable. Therefore, there is a fraction probability of B occurrence, which does not appear.

Probability Statement Usage

Probability statements are frequently used in political expressions to avoid determination. Therefore, a single determinism statement could be responded to by some probabilistic statements, such as a strategy of the game. Thus, it is important to (s^1) denote the statement: Probability of an event. When this statement is taken from the point of view of usage, some meanings such as truth meanings or psychological meanings could be derived from the need to use the statement. Originally, the probability statement is used to carry the meaning of the occurrence of an event, rather than non-occurrence. However, with the mathematical development of probability theory, the probability statement became abstract in mathematical formulas. As a result, the concept of probability has acquired the value of 0 to denote the impossible event and 1 to denote the sure event. Thus, in gambling games, each player enters with an incentive probability that favors winning, otherwise, the player would not have entered the gambling game. For example, if someone y used the statement to expect an event by x probability, then the statement means that y is likely getting x. Consequently, the probability will take a set of values in the interval $(\frac{1}{2}, 1]$ or $\frac{1}{2} < x \leq 1$. On the other hand, if y uses it in the opposite case, then the statement means the doubt and the probability will take a set of values in the interval $[0, \frac{1}{2})$ or $0 \leq x < \frac{1}{2}$. Nevertheless, when the word takes the meaning of possible, then its value falls at $\frac{1}{2}$. In either case, some observers $y_1, y_2, \dots, y_n$ will also put their probabilities $x_1, x_2, \dots, x_n$ to measure y expectation. Here, the probability of the probabilities statement will, in principle, stand out as statement (s^2). The probability degree can be evaluated by who is expecting and the observers. In addition, when y by subjective probability predicts an event by x, a probability of an inconstant nature is given that does not carry a determined value. Also, truth will not be stated without a determined degree (Interval). Hence, probability needs an aspect of determination and should be a science that provides truth. In addition, if expectation gives different values of probability for one event at the same time, then there is a mental function f_y that is wrong mathematically. Also, the observers have mental functions $f_{y_1}, f_{y_2}, \dots, f_{y_n}$ that are affected by different factors, which in turn give different values for one variable. In this case, if a

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

probability value is equal to φ, then the observers values will be in the range of zero to 1. Nonetheless, a probability value that does not exceed φ could be expressed by function mapping. In addition, the value of φ would be created as a discrete quantity.

On the other hand, the concept of probability and its interpretation are connected with epistemology by a function that investigates truth. For example, take into cognizance the mind (memory) of a blind man and the probability of knowing his friends subjectively or by their tones. On this note, if some one was writing the names after him, then the writer may write the wrong name, which may create a false statement or wrong science. Therefore, probability can be used to measure truth or certainty. This, however, may be affected by laws such as religions that reduce belief in probabilistic nature or state laws that adopt or do not adopt the probabilistic models. At the same time, if the intuition gives a deterministic statement about certain truth, and proves it theoretically with feasibility to adopt it, the general educational approach that needs a certain level with a goal that serves its need for different people has its effect to adopt or not adopt certain theory, as well as the intuition.

Conditional and Complement As A Causality

It can not be asserted that the laws of nature are based on probabilistic model, but it would be feasible to ascertain that they are built on the basis of scarcity and competition. However, it can be said that nature is built on deterministic laws, but there are causes that lead the results of these laws to have a probabilistic nature. This poses two questions: (a) why is there a probabilistic phenomenon? (b) what probability could measure the causes?
Part of the answer of these questions may evolve through this significant question: is the space able to admit all phenomena to be deterministic?
When tossing a coin, the speculation is the process of tossing. Subsequently, probability is the method of calculating or evaluating (Figure 1).

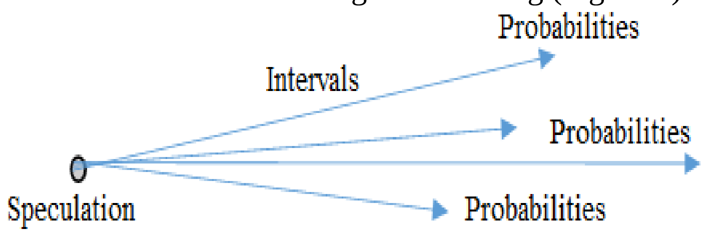

Figure 1. Causes and Results; Speculation-Time-Probability

However, when the process is done with a biased heavy weight, then the process is not speculation but a determinism process, relatively. The paradox here, that differs nature from speculation, shows that the deterministic nature can take two aspects: to determine a result with one description and one

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value or to determine the result with two descriptions or more, with the same one value that was distributed over the descriptions, which in turn gives nature the probabilistic aspect. Furthermore, when supposing space, supposing time is marked with an aspect of scarcity. This means that the case can be changed from deterministic phenomenon to probabilistic phenomenon. In the case of several micro particles with large speed, some factors may carry a kind of scarceness. Bohr has shown in detail, in a number of interesting thought experiments, how the finite value of the frequently recurring constant ħ in uncertainty relation makes the coexistence of wave and particle both possible and necessary. If the probability of finding a particle in some bounded region of space decreases as time goes on, the probability of finding it outside of this region must increase by the same amount (Merzbacher, 1970).

On the other hand, if the cause is constant, it may be considered as a constant law and the event may be dedicated from universal laws. Conversely, the event will be dedicated from initial conditions. Also, if the universal laws are considered as conditions, then they would be considered as events. These conditions take an event from space to space, such that for the conditional probability of an event B, given that an event A has occurred, the sample space reduced from S to A (Meyer, 1970). Mathematically speaking, any law that exists denotes a function with known or unknown fixed operations $(\pm, \times, \div, \ldots)$, including domain and variability with a range. In a probabilistic nature, events may take these paths: $Event_1$ (cause) gives an $event_2$ (result). $Event_1$ consists of $event_1$, $event_2$,..., $event_i$, while $event_2$ also consists of $event_1$, $event_2$,..., $event_j$. Thus, there would be a conditional so that the event may correspond to one or some events. Here, the concept of conditional can be used and expressed in terms of sub σ-field of events. In some elementary case, the initial probability space (Ω, ξ, P) is replaced by the probability space (Ω, ξ, P_B), where Ω is the sure event, ξ is σ-field of events, and P_B is the conditional probability (Loève, 1978). Nevertheless, the condition gives a prior information or a prior event to the last event. Hence, if it synchronizes to the last event, it will be a complement and there will be an occurrence and a non-occurrence. If the phenomenon is a probabilistic phenomenon, it could be due to a set of causes, which could be fractions that may be created randomly, thus making the cause unknown. This actually poses a question: What is the phenomenon and the cause? If one tosses a true coin and is only looking for the head when the existence of the other face (tail) is not known, the phenomenon reveals that in many thousands trials, the head is gotten in half of these thousands trials. The unknown face is the cause or the causes to get the phenomenon of half, and the two halves are in competition nature. Therefore, probability is a science of competition. In probability theory, if the conditional is considered as a causality for dependent events, the concept of complement could be considered as a causality for independent events.

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Conceptually, when probability measures an event's complement, it measures some causes. Also, competition has its causes, which is the scarcity (Figure 2)

Scarcity (Variable) $\longrightarrow$ Competition (Variable) $\longrightarrow$ Probabilistic Phenomenon

Figure 2. Variable Causality

If the constant of the resulted value is considered as a deterministic result, the random order will break this deterministic. In the case of throwing 10 true coins by one hand at the same time, the causality will be mixed and complicated than tossing 1 coin. Nonetheless, it will always give a constant frequency.

In general, and in addition to the human boundary knowledge, there are many factors that may lead to the probabilistic phenomenon, such as political factors, moral, religion, as well as human and nature necessity.

Methods

Until now, this paper has dealt with the concept of probability from a philosophical point of view. This is in addition to the linguistic and logical analysis, besides the examples. Also, since the experiment is possible, it will involve the direct method. This, in turn, needs mathematical techniques to provide mathematical results.

Experiment purpose: This experiment consists of observing the appearance of fraction heads or fraction tails probability and providing answers to the following questions: (a) What is the probability of $\wp_i$? (b) how does the probability value behaves?

Experiment steps: This paper will use a coin after shaping it into a spherical shape (true ball), with radius of r. Thereafter, the following steps will be taken:

Step **1**: Determining $\boldsymbol{m}$ and choosing $\boldsymbol{n}$.

Step **2**: Dividing curved surface area to find the fraction unit area, which will be: $A = \left(\frac{4\pi r^2}{mn+m}\right)$, and the area $4\pi r^2$ by some manner will constitute the sample space. However, this does not provide explanation until the ball is divided into n order closed circles with replacement for every half separately. On every circle, the value of a probability fraction is recorded before throwing the ball. Also, the peak of each upper half of the ball is considered as success (Figure 3).

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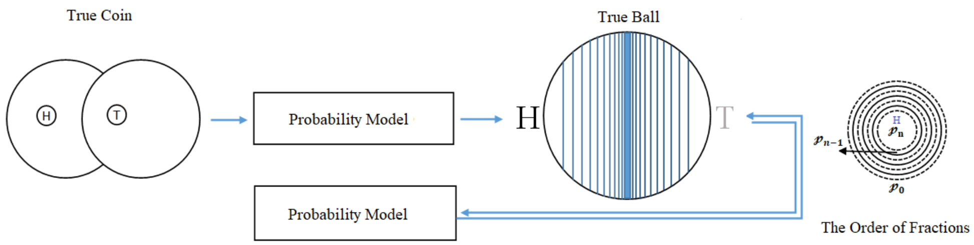

Figure 3. ($\mathcal{p}_i$) Values will be in An Ascending Order from the Middle to the Peak for each Half

Consequently, dividing 360° by some numbers will not always give an integer or finite number. Therefore, one can do this experiment with dice. Also, the difference between this ball and any celestial body moving in an orbit shows that this ball is moving less freely. As a result, it is either static or moving under the influence of unnatural force. However, it would have been possible for its details to appear at each point of time t with an approximated determined angle that makes the appearance known. In addition, it should be noted that the heavier the weight, the more stable the movement, and vice versa.

Step **3**: Finding the sample space and building the probability model to identify fractions events probabilities.

Sample description: This paper only considers the sample description of tossing a regular true coin. In addition, the zero fraction occurrence probability assumes ($\emptyset$). The nil event is the event of non-occurrence, and the zero fraction of the head event $\{\boldsymbol{H}\}$ is attached to the zero fraction of the tail event $\{\boldsymbol{T}\}$, which is one area or one fraction event. Accordingly, all events are considered independent.

Results

Figure 4 explains the mathematical procedure and the derived expressions.

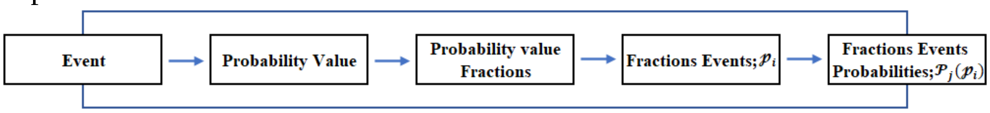

Figure 4. Steps that Define Fractions Events and Fractions Events Probabilities Expressions

Also consider the following:

The probability of head in tossing a regular true coin one time is as follows:

The sample space is $\boldsymbol{S} = \{\boldsymbol{H}, \boldsymbol{T}\}$, the probability model is $P_H = \frac{H}{S}$, and the probability to a head is $P_H = \frac{H}{S} = \frac{1}{2}$.

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Definition $\boldsymbol{I}$: The value $\boldsymbol{f_A} = \frac{\boldsymbol{m_A}}{\boldsymbol{m}}$ is called the relative frequency of event $\boldsymbol{A}$, and it has the following properties;
$\mathbf{1-0} \leq \boldsymbol{f_A} \leq \mathbf{1}$.
$\mathbf{2} - \boldsymbol{f_A} = \mathbf{0}$ if $\boldsymbol{A}$ never occurs, and $\boldsymbol{f_A} = \mathbf{1}$ if $\boldsymbol{A}$ occurs on every repetition.
$\mathbf{3}$ $-$ If $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually exclusive events, then $\boldsymbol{f_{A\cup B}} = \boldsymbol{f_A} + \boldsymbol{f_B}$ (Hines & Montgomery, 1990).

Definition II: For every event with a probability value, the fractions of its probability value are events, regardless of how small they are.

Definition III: The sample space of the experiment is the space that consists of all fractions events $\wp_{i=0,1,..n}$.

Let $\mathfrak{D}$ be the sample description or the possible descriptive outcomes of tossing a regular true coin and m be the quantity of possible descriptive outcomes. Then, $\mathfrak{D} = (H, T) = 2 = m$.

As a result, for throwing a true ball, there will be $mn + m$ events for each trial. Thus, for $\boldsymbol{n} = \mathbf{100}$, the number of divisions is $\frac{mn+m}{m} = (n+1) = 101$ circles, for every half of the ball (Table 1).

Table 1. Sample Space for $m \geq 2$

m	$mn + m$ Circles	$\frac{mn+m}{m}$ For Every Part
2	202	101
3	303	101
⋮	⋮	⋮
10	1010	101

Let $\mathfrak{F}$ be the sample space and $\mathcal{P}_H$ be the probability of an occurrence of the probability value fraction for head $\wp_i$ in throwing a true ball once.

Thus, $\mathfrak{F} = (P(H_0), P(H_1), \dots, P(H_{100}), P(T_0), P(T_1), \dots, P(T_{100}))$.
From probability definition, if $\wp_1 + \wp_2 + \cdots + \wp_n = 1$ and
$\wp_1 = \wp_2 = \cdots = \wp_n$, then $n\wp_i = 1$ and $\wp_i = \frac{1}{n}$ for $i = 1, \dots n$.
Also, , $\wp_1 = 1 - (\wp_2 + \cdots + \wp_n), \wp_2 = 1 - (\wp_1 + \wp_3 + \cdots + \wp_n)$, ... and so on. Then, $\wp_1 = \wp_2 = \cdots = \wp_n = 1 - [(n-1)\wp_i]$. .
For $\wp_i = \frac{1}{n}$, $\wp_1 = \wp_2 = \cdots = \wp_n = 1 - \left[(n-1)\frac{1}{n}\right] = 1 - \left[\frac{(n-1)}{n}\right]$. ... (Eq1)
Therefore, by replacing $(n-1)$ by $(n-i)$ for $i = 0,1\dots, n$, there will be a cumulative with replacement, and $\wp_i$ has the following probability values:
$\wp_0 = 1 - (\wp_1 + \cdots + \wp_n), \wp_1 = 1 - (\wp_2 + \cdots + \wp_n), \dots, \wp_n = 1$.
Or, $\wp_0 = 1 - \left[\frac{(n-0)}{n}\right], \wp_1 = 1 - \left[\frac{(n-1)}{n}\right], \dots, \wp_n = 1 - \left[\frac{(n-n)}{n}\right]$. ... (Eq 2).
Hence, the following sequence $\{\wp_n\} : \wp_0 < \wp_1 < \cdots < \wp_n$ has the following limit, $\lim_{n\to\infty} \wp_n = \lim_{n\to\infty} \left(1 - \left[\frac{(n-n)}{n}\right]\right) = 1$, and it is convergent.

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It is also included in each other $\wp_0 \subset \wp_1 \subset \cdots \subset \wp_n$.
Furthermore, let $H = 1$ and $T = 2$, then $\mathcal{P}_H(\wp_i) = \mathcal{P}_1(\wp_i)$ and $\mathcal{P}_T(\wp_i) = \mathcal{P}_2(\wp_i)$.

Here are some terms of $\mathcal{P}_1(\wp_i)$:
$\mathcal{P}_1(\wp_{25} = 0.25) = 2\left[\frac{0.25}{202}\right] = 0.002475248.$
$\mathcal{P}_1(\wp_{100} = 1) = 2\left[\frac{1}{202}\right] = 0.00990099.$
$\mathcal{P}_1(\wp_0 = 0) = 2\left[\frac{0}{202}\right] = 0.$
Here is the sum of each series:

$$\sum_{i=0}^{100} \mathcal{P}_1(\wp_i) = m\left[\frac{(1-1)}{mn+m} + \frac{(1-0.99)}{mn+m} + \cdots + \frac{\left(1-\left(\frac{1}{n}\right)\right)}{mn+m} + \frac{(1-0)}{mn+m}\right] = \frac{1}{2} = \frac{1}{m}.$$

$$\sum_{i=0}^{100} \mathcal{P}_2\,(\wp_i) = m\left[\frac{(1-1)}{mn+m} + \frac{(1-0.99)}{mn+m} + \cdots + \frac{\left(1-\left(\frac{1}{n}\right)\right)}{mn+m} + \frac{(1-0)}{mn+m}\right] = \frac{1}{2} = \frac{1}{m}.$$

As a result, the sequence associated with the last two finite series is not necessarily a random sequence until it satisfies some conditions, and it could be equidistributed (k −distributed) ($\infty -$ distributed). It also depends on a precise definition of random sequence, even if it behaves as a random. This depends on the experiment or phenomenon nature. Knuth (1998) provides more details about random sequence and its conditions.
Table **2** provides the order values of $\mathcal{P}_1(\wp_i)$. In this case, one may agree that the sample space consists of 101 points. Feller (1950) further expantiates on sample space and more applications.

Table 2. Result of 101 H Fractions

i	$\frac{n-i}{n}$	$1-\frac{n-i}{n}$	$m\left[\frac{\left(1-\frac{n-i}{n}\right)}{\mathfrak{F}}\right]$	i	$\frac{n-i}{n}$	$1-\frac{n-i}{n}$	$m\left[\frac{\left(1-\frac{n-i}{n}\right)}{\mathfrak{F}}\right]$	i	$\frac{n-i}{n}$	$1-\frac{n-i}{n}$	$m\left[\frac{\left(1-\frac{n-i}{n}\right)}{\mathfrak{F}}\right]$	i	$\frac{n-i}{n}$	$1-\frac{n-i}{n}$	$m\left[\frac{\left(1-\frac{n-i}{n}\right)}{\mathfrak{F}}\right]$
0	1	0	0	25	0.75	0.25	0.002475248	50	0.5	0.5	0.004950495	75	0.25	0.75	0.007425743
1	0.99	0.01	9.90099E-05	26	0.74	0.26	0.002574257	51	0.49	0.51	0.005049505	76	0.24	0.76	0.007524752
2	0.98	0.02	0.00019802	27	0.73	0.27	0.002673267	52	0.48	0.52	0.005148515	77	0.23	0.77	0.007623762
3	0.97	0.03	0.00029703	28	0.72	0.28	0.002772277	53	0.47	0.53	0.005247525	78	0.22	0.78	0.007722772
4	0.96	0.04	0.00039604	29	0.71	0.29	0.002871287	54	0.46	0.54	0.005346535	79	0.21	0.79	0.007821782
5	0.95	0.05	0.00049505	30	0.7	0.3	0.002970297	55	0.45	0.55	0.005445545	80	0.2	0.8	0.007920792
6	0.94	0.06	0.000594059	31	0.69	0.31	0.003069307	56	0.44	0.56	0.005544554	81	0.19	0.81	0.008019802
7	0.93	0.07	0.000693069	32	0.68	0.32	0.003168317	57	0.43	0.57	0.005643564	82	0.18	0.82	0.008118812
8	0.92	0.08	0.000792079	33	0.67	0.33	0.003267327	58	0.42	0.58	0.005742574	83	0.17	0.83	0.008217822
9	0.91	0.09	0.000891089	34	0.66	0.34	0.003366337	59	0.41	0.59	0.005841584	84	0.16	0.84	0.008316832
10	0.9	0.1	0.000990099	35	0.65	0.35	0.003465347	60	0.4	0.6	0.005940594	85	0.15	0.85	0.008415842
11	0.89	0.11	0.001089109	36	0.64	0.36	0.003564356	61	0.39	0.61	0.006039604	86	0.14	0.86	0.008514851
12	0.88	0.12	0.001188119	37	0.63	0.37	0.003663366	62	0.38	0.62	0.006138614	87	0.13	0.87	0.008613861
13	0.87	0.13	0.001287129	38	0.62	0.38	0.003762376	63	0.37	0.63	0.006237624	88	0.12	0.88	0.008712871
14	0.86	0.14	0.001386139	39	0.61	0.39	0.003861386	64	0.36	0.64	0.006336634	89	0.11	0.89	0.008811881
15	0.85	0.15	0.001485149	40	0.6	0.4	0.003960396	65	0.35	0.65	0.006435644	90	0.1	0.9	0.008910891
16	0.84	0.16	0.001584158	41	0.59	0.41	0.004059406	66	0.34	0.66	0.006534653	91	0.09	0.91	0.009009901
17	0.83	0.17	0.001683168	42	0.58	0.42	0.004158416	67	0.33	0.67	0.006633663	92	0.08	0.92	0.009108911
18	0.82	0.18	0.001782178	43	0.57	0.43	0.004257426	68	0.32	0.68	0.006732673	93	0.07	0.93	0.009207921
19	0.81	0.19	0.001881188	44	0.56	0.44	0.004356436	69	0.31	0.69	0.006831683	94	0.06	0.94	0.009306931
20	0.8	0.2	0.001980198	45	0.55	0.45	0.004455446	70	0.3	0.7	0.006930693	95	0.05	0.95	0.009405941
21	0.79	0.21	0.002079208	46	0.54	0.46	0.004554455	71	0.29	0.71	0.007029703	96	0.04	0.96	0.00950495
22	0.78	0.22	0.002178218	47	0.53	0.47	0.004653465	72	0.28	0.72	0.007128713	97	0.03	0.97	0.00960396
23	0.77	0.23	0.002277228	48	0.52	0.48	0.004752475	73	0.27	0.73	0.007227723	98	0.02	0.98	0.00970297
24	0.76	0.24	0.002376238	49	0.51	0.49	0.004851485	74	0.26	0.74	0.007326733	99	0.01	0.99	0.00980198
												100	0	1	0.00990099
												101	$\sum_{i=0}^{100} \mathcal{P}_H(P_{\wp_i})$		0.5000

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

To generalize, let $\mathcal{P}_j(\wp_i)$ for $m \geq 2$, $j = 1,2, \ldots m$. Where m is a positive integer number, i and n is initially a positive integer number, but it could be extended to take positive real number R. However, for i to take negative value, it should be in an absolute sign.

Also, $j > 1$, $\wp_i = 1 - \left(\frac{jn-ji}{jn}\right)$ and is considered as a distribution for $(\frac{1}{2}, \frac{1}{2})$ mass in some fractions n. This distribution thus provides a description of $\wp_i$ behaviour (Larson, 1973).

Therefore, $\mathcal{P}_j(\wp_i) = m\left[\frac{1-\left(\frac{jn-ji}{jn}\right)}{mn+m}\right]$.

$\{\mathcal{P}_j(\wp_i)\}$ is the sequence of the following probability values:
$\mathcal{P}_1(\wp_0, \ldots, \wp_n), \mathcal{P}_2(\wp_0, \ldots, \wp_n), \ldots, \mathcal{P}_m(\wp_0, \ldots, \wp_n)$.

It has the following limit, $\lim\limits_{\substack{m\to\infty \\ n\to\infty}} \mathcal{P}_m(\wp_n) = \lim\limits_{\substack{m\to\infty \\ n\to\infty}} \frac{m}{mn+m}\left[1 - \left(\frac{mn-mn}{mn}\right)\right] = \lim\limits_{n\to\infty} \frac{1}{n+1}\left[1 - \left(\frac{n-n}{n}\right)\right] = \lim\limits_{n\to\infty}\left(\frac{1}{n+1}\right) - \lim\limits_{n\to\infty}\left(\frac{n-n}{n^2+n}\right) = \lim\limits_{n\to\infty}\left(\frac{1}{n+1} \cdot \frac{n+1}{n+1}\right) - 0 = \lim\limits_{n\to\infty}\left(\frac{(n+1)}{(n+1)(n+1)}\right) = \lim\limits_{n\to\infty}\left(\frac{(\frac{n}{n}+\frac{1}{n})}{(\frac{n}{n}+\frac{1}{n})(\frac{n}{n}+\frac{1}{n})}\right) = \lim\limits_{n\to\infty}\left(\frac{(1+\frac{1}{n})}{(1+\frac{1}{n})(1+\frac{1}{n})}\right) = 1$, and it is convergent.

Hence, $\sum_{j=1}^{m}\sum_{i=1}^{n} \mathcal{P}_j(\wp_i) = [\mathcal{P}_1(\wp_0) + \mathcal{P}_1(\wp_1) + \cdots + \mathcal{P}_1(\wp_n)] + [\mathcal{P}_2(\wp_0) + \mathcal{P}_2(\wp_1) + \mathcal{P}_2(\wp_n)] + \cdots + [\mathcal{P}_m(\wp_0) + \mathcal{P}_m(\wp_1) + \cdots + \mathcal{P}_m(\wp_n)] = 1$.

Subsequently, the domain of $\mathcal{P}_j(\wp_i)$is the set of all real numbers in the closed interval $[0\,, 1]$, and the range of $\mathcal{P}_j(\wp_i)$ is the set of all real numbers in the closed interval $\left[0, \frac{m}{mn+m}\right]$, for all j, (Table 3).

Table 3. $\wp_i$Range is the Domain of $\mathcal{P}_j(\wp_i)$with Some Possible Intervals

$\wp_i$		$\mathcal{P}_j(\wp_i)$
Domain	Range	
$(-\infty, \infty)$ $And, n > 0$	Domain	Range
	$(0,1]$, $(0,1)$, $[0,1]$ or $[0,1)$	$\left(0, \frac{1}{mn+m}\right]$, $\left(0, \frac{1}{mn+m}\right)$, $\left[0, \frac{1}{mn+m}\right]$ or $\left[0, \frac{1}{mn+m}\right)$

As a result of dividing the ball into n order circles with replacement for every half separately, there will be unequal order divisions. Consequently, consider the following proposition:

Proposition I: The greater the fraction probability value, the greater the probability of its occurrence. This denotes that the occurrence of $\wp_n$ implies the occurrence of $\wp_{n-1}, \ldots, \wp_0$, but the occurrence of $\wp_{n-1}$ does not imply the occurrence of $\wp_n$, and so on.

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Proof: Suppose that,
$I_r := \left[0, \frac{1}{r}\right], I_{r-1} =: \left[0, \frac{1}{r-1}\right], \dots, I_{r-(r-1)} =: \left[0, \frac{1}{r-(r-1)}\right]$, $r > 1$ are some intervals on the real line and $l_1, l_2, \dots, l_k$ are the lengths of these intervals, where k and r are positive integers and $\mathfrak{p}_0, \mathfrak{p}_1, \dots, \mathfrak{p}_n$ are some sets belonging to these intervals, respectively. Thus, if $I_r \subset I_{r-1} \subset \dots \subset I_{r-r+1}$ and $l_1 < l_2 < \dots < l_k$, then $\mathfrak{p}_0 \subset \dots \subset \mathfrak{p}_n$. Also, if $\wp_0 \in \mathfrak{p}_0, \dots, \wp_n \in \mathfrak{p}_n$, then $\wp_0 < \wp_1 < \dots < \wp_n$. This proposes that if this is true for the sub sequence $\{\wp_i\}$ of the sequence $\{\mathcal{P}_j\}$, then it is true for the sequence $\{\mathcal{P}_j\}$. To clarify the theoretical idea behind this proposition, suppose the experiment of tossing true coin in Table 4.

Table 4. The Probability of H is Greater than the Probability of T in the Interval $(t_2, t_4]$

Toss	Result	Equality	Time			
1	T	T > H	t_1			
2	H	T = H		t_2		
3	H	T < H			t_3	
4	H	T < H				t_4

Definition IV: The probability of probability is a composite function. Let $\wp$ and $\mathcal{P}$ denote some functions, such that $\wp: D_1 \to R_1$, and $\mathcal{P}: R_1 \to R_2$ respectively, then $(\mathcal{P} \circ \wp)$ is a composite function. To illustrate that,
let $x = (n - i)$, $\wp(x) = 1 - \frac{x}{n}$, $\mathcal{P}(x) = \frac{x}{n+1}$, for $x \le n$, and $x, n \in R$.
Then, the composite function is: $\mathcal{P}(x) = \frac{1-\frac{x}{n}}{n+1} = \frac{n-x}{n(n+1)}$.
Generally, if x is a real number and X is a discrete random variable (real valued function), the distribution function of the random variable X is:
Let x denote the number of occurrences of $\wp_i$.
$F_X = P_X(X \le x)$, for all $x \in (-\infty, \infty)$ and the probability function is:
$P_X(x_i) = F_X(x_i) - F_X(x_{i-1})$.
$F_X(x_i) = P_X(X \le x_i) = \sum_{x \le x_i} P_X(x)$.
Therefore, the probability distribution of X is the collection of pairs:
$[(x_i, P_X(x_i)), i = 1,2, \dots]$ (Hines & Montgomery, 1990). Dudewicz (1988) illustrates the possible graphs of the distribution function and its characteristics. On the other hand, the density function could be divided into m parts. For example, in the case of continuous random variable, if $\mathcal{P}_1(\wp_0, \dots, \wp_n), \mathcal{P}_2(\wp_0, \dots, \wp_n), \dots, \mathcal{P}_m(\wp_0, \dots, \wp_n)$ are non-negative probability values that sum to 1, and if the interval $(0, t]$ is divided into m parts, then the density function will be divided into m parts and $F_1, F_2, \dots, F_m$ are some distributions. Knuth (1998) expantiates more on the techniques.

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Definition V: For every fraction event, there is a probability of its occurrence denoted by $\mathcal{P}_j(\wp_i)$.

Corollary I: Every probability consists of a finite or countable infinite set of probabilities and every independent event consists of fractions events $\wp_i = 1 - \left(\frac{n-i}{n}\right)$. Here, the index of $\wp$ could be extended to $\mathcal{P}_j(\wp_{i_{1,2,\dots,l}} \dots)$ (Borel, 1898; Khinchin, 1964).

In addition, the following proves that $\mathcal{P}_j(\wp_i)$ verifies the probability axioms:

Proof:

1- Since all values of $\mathcal{P}_j(\wp_i)$ are not negative, then $\mathcal{P}_j(\wp_i) \geq 0$.

2- If $\mathcal{P}_j(\wp_i) = \frac{f_{\wp_i}}{n}$, where $f_{\wp_i}$ is the frequency, and n is a positive integer number such that $f_{\wp_i} \leq n$ and $\frac{f_{\wp_i}}{n} \leq 1$, then $\mathcal{P}_j(\wp_i) \leq 1$.

3- To prove that $\mathcal{P}_j(\mathfrak{F}) = 1$. Suppose $\mathfrak{F} = \{H\}$ then, $P(\mathfrak{F}) = P(H) = 1$ and for $\mathcal{P}_j(\mathfrak{F})$ when $m = 1$, let $\sum_{i=0}^{n} \mathcal{P}_j(\wp_i) = \mathcal{P}(\mathfrak{F})$ and $\sum_{i=0}^{n} \mathcal{P}(\wp_i) = \frac{1}{(n+1)} \sum_{i=0}^{n} \wp_i = 1$then, $(n+1)\wp_{i=0,\dots,n} = (n+1) = 1 = \mathcal{P}(\mathfrak{F})$ For $\mathfrak{F} = \{H\}$, suppose tossing a coin with H on both sides once.

Also, the countable infinite property can be proved intuitively in Figure 5.

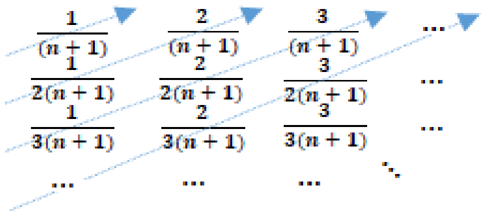

Figure 5. The Set of $\boldsymbol{\mathcal{P}_j(\wp_i)}$ Infinite Elements

Lemma I: For every independent event E, the complement event probability $P(\bar{E})$ is the fraction event of that event, if $P(E) = P(\bar{E})$ and if they are homogenous.

Proof: Let E_1, E_2 be the independent events and $\wp_i$ be the fraction event of $P(E_1)$. If $P(E_1) = P(E_2)$, then $P(E_1) = P(\bar{E}_1)$. Thus, the fraction of $P(E_1)$ is $\wp_i = \frac{P(E_1)}{n}$ and $\wp_i = \frac{P(\bar{E})}{n}$. If $n = 1$, then $\wp_i = P(\bar{E})$.

For $m > 2$, note that if $E \cap \bar{E} = \emptyset$, then $P(\bar{E}_1) = P(E_2) \cup \dots \cup P(E_m)$.

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Corollary II: For every fraction event $\wp_{j_i}$, a complement event $\bar{\wp}_{j_i}$ is $\bar{\wp}_{j_i} = \wp_{j_n} - \wp_{j_i} + \wp_{l_{i=1,\dots,n},\dots,m_{i=1,\dots,n}}$, $l \neq j$, if $\wp_{j_i}$ satisfies the proposition I. Also, every fraction event $\wp_i$ is a fraction of a fraction event $\wp_{i+1}$, if $\wp_i < 1$. Proof: Eqs 2.

Intuitively, set involves different elements with common features or certain source, and every set is constructed by ϕ, besides some axioms. This means that for some sets A and B with only one element $a \in A$, $b \in B$ and without any common elements $(a \neq b)$. Also, if $(a = a$ or $a = \emptyset)$ and $(b = b$ or $b = \emptyset)$, then $A \cap B = \emptyset$. At the same time, if $(a \neq a$ and $a \neq \emptyset)$ or $(b \neq b$ and $b \neq \emptyset)$, then one of the two sets does not exist. Therefore, for every set to exist, it should consist of at least $\emptyset$. Thus, there exists an empty set with $\emptyset$ of some features (or description) and without any element. From the set theory, a power of set could be 2^N and if a sample space is a set of events, then there are 2^N subsets, where N is a finite size for a sample description space (Parzen, 1960). In the present case where $m = 2$, and for throwing the ball twice, there will be $2^{m^2n^2}$ subsets after excluding $\mathcal{P}_1\wp_0$ and $\mathcal{P}_2\wp_0$, which is represented by ϕ in $\mathfrak{F}_{\mathfrak{F}}$.

Subsequently, $((P(H_0), P(T_0)): \wp_{H_0}, \wp_{T_0}: \emptyset)$.

$((\wp_{H_1}, \wp_{T_1}), \dots, (\wp_{H_{100}}, \wp_{T_{100}}), (\wp_{T_1}, \wp_{H_1}), \dots, (\wp_{T_{100}}, \wp_{H_{100}}),$
$(\wp_{H_1}, \wp_{H_1}, \dots, (\wp_{H_{100}}, \wp_{H_{100}}), (\wp_{T_1}, \wp_{T_1}, \dots, (\wp_{T_{100}}, \wp_{T_{100}})$.

Also, if $\mathfrak{F}_1 = (\wp_{H_1}, \wp_{T_1}), \mathfrak{F}_2 = (\wp_{H_1}, \wp_{T_2}), \mathfrak{F}_3 = (\wp_{H_1}, \wp_{T_3}), \dots$, then $\mathfrak{F}_{\mathfrak{F}} = \{\{\mathfrak{F}_1\}, \{\mathfrak{F}_2\}, \{\mathfrak{F}_3\}, \{\mathfrak{F}_4\}, \dots, \{\mathfrak{F}_1, \mathfrak{F}_2\}, \dots, \mathfrak{F}, \phi\}$

Thus, if $\mathfrak{F}$ is the fundamental set of events, let A and B represent any two events of $\mathfrak{B}$ $(A \in \mathfrak{B}, B \in \mathfrak{B})$. Then, let $A \cup B \in \mathfrak{B}$, complement $\bar{A} \in \mathfrak{B}$ and $A \cap B \in \mathfrak{B}$. Consequently, a non-empty collection of subsets $\mathfrak{B}$ of a set $\mathfrak{F}$ is a σ-field of subsets of $\mathfrak{F}$.

Definition VI: A probability measure $P(\cdot)$ on a σ-field of subsets $\mathfrak{B}$ of a set $\mathfrak{F}$ is a real valued function having a domain that satisfies the following properties:

$1 - \mathcal{P}_j(\wp_i) \geq 0$ for all $\wp_i \in \mathfrak{F}$..

$2 - \mathcal{P}_j(\mathfrak{F}) = 1$..

$3 -$ If $\wp_i, i = 1,2,3$ ... are mutually disjoint sets in $\mathfrak{F}$ then,

$\mathcal{P}_j(\cup_i^\infty \wp_i) = \sum_j^m \sum_i^\infty \mathcal{P}_j(\wp_i)$.

Also, the probability space is $(\mathfrak{F}, \mathfrak{B}, \mathcal{P}(\cdot))$ (Heol et al., 1971).

Here, $P(\cdot)$ could be defined by different functions, which may give different values. Nonetheless, this will depend on the explanation of experiment, sample space, and the description of event (Parzen, 1960). The function behaviour could also reflect some aspects of probability values behaviour.

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Corollary III: The impossible occurrence happens if the occurrence probability of all events are $\wp_n$ simultaneously. (Here the peak point of every event is supposed to be $\wp_n$).

Proof: If the occurrences of $\mathcal{P}_{j=1,\dots,m}\wp_{i,\dots,n}$ happens only when $\mathcal{P}_{\substack{l=1,\dots,m\\ l\neq j}}\wp_{i,\dots,n}$ is not happen, then $\mathcal{P}_{\substack{l=1,\dots,m\\ l\neq j}}\wp_{i,\dots,n} = 0$. Thus, if $\mathcal{P}_{j=1\dots,m}\wp_{i,\dots,n} = \mathcal{P}_{\substack{l=1,\dots,m\\ l\neq j}}\wp_{i,\dots,n}$ simultaneously, then $\mathcal{P}_{j=1,\dots m}\wp_{i,\dots,n} = 0$. Therefore, there is no occurrence.

Corollary IV: The nil (equally fractions) occurrence happen if the occurrence probabilities of fractions events of all or some events are $\wp_0$ simultaneously.

Proof: It is analogous to the previous proof and has the same result. However, the difference here reveals that the peak point of all or some events will be zero or $\wp_0$ simultaneously, with a representation of $\emptyset$.

Corollary V: For more than one continuous function of independent event, if one of them expresses non-decreasing function at time t in the open interval $\left(\frac{1}{m}, 1\right)$, then the other one indicates decreasing function at time t, in the open interval $\left(0, \frac{1}{m}\right)$.

Proof: Take $m = 2$ and suppose that $\mathcal{P}_1(\wp_i) > \mathcal{P}_2(\wp_i)$ and $\frac{1}{2} < \mathcal{P}_1(\wp_i) < 1$. From $\mathcal{P}_1(\wp_i) + \mathcal{P}_2(\wp_i) = 1$ then $\mathcal{P}_2(\wp_i) = 1 - \mathcal{P}_1(\wp_i)$.

Also, from $\mathcal{P}_1(\wp_i) > \mathcal{P}_2(\wp_i)$ then $\mathcal{P}_1(\wp_i) > 1 - \mathcal{P}_1(\wp_i)$. It is clear that every increase in $\mathcal{P}_1(\wp_i)$ gives a decrease in $\mathcal{P}_2(\wp_i)$, if $\mathcal{P}_1(\wp_i)$ and $\mathcal{P}_2(\wp_i)$ are continuous functions in the open intervals $\left(\frac{1}{2}, 1\right)$ and $\left(0, \frac{1}{2}\right)$, respectively. Thus, the function behaviour reflects the probability values behaviour of these two events.

The construction of $\mathcal{P}_j(\wp_i)$ is not restricted to the coin experiment. However, it could be constructed for any random experiment, such as Brownian motion for two independent continuous parameter random variables $X_d(t)$ and $X_l(t)$, where $\{X(t), 0 \le t < \infty\}$ (Parzen, 1962).

Discussion

What are the deterministic aspects that probability theory can carry? This includes the description that the probability theory provides by specific sample space, including the finite and infinite countable aspects. Furthermore, the statistical probability may carry more determination characteristics than the inductive probability.

How can the statement “probability of probability” be carried out? A probability of probability statement (s^2) will take an analogous concept of the

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

mathematical statements of $log\ log\ x)$ or $\sqrt{\sqrt{x}}$, and employ the statement of s^2 in same function of the statement s^1. Similarly, the statement s^2 derived from statement s^1 does not merely measure the statement s^1, it further provides a necessary meaning of the statement s^1. Also, it is not necessarily a mathematical necessity, but a conceptional necessity.
Why are the fractions unequal in the experiment design? It is necessary for the fractions to have an order probability value to be unequal, and this happens because every fraction had been taken with replacement. Nonetheless, the two faces are likely equal. Also, this unequal order can provide a description for the moments of $\mathcal{P}_H(\wp_i)$ and $\mathcal{P}_T(\wp_i)$ tendency to the value of $\frac{m}{mn+m}$ (Proposition I).

Is it possible to distribute fractions events randomly on the ball? And what is the reasonable criteria to choose $\boldsymbol{n}$ divisions? It is possible and useful in some cases to distribute the fractions events with a random topology that extends in directions and curvatures. In this case, the uniformity of head or tail will be impossible. Also, the main criteria to choose $\boldsymbol{n}$ divisions is to determine the required fraction quantity that can change event occurrence prior to the last step, regardless of how small it is. At the same time, it is possible to represent a pairwise occurrence $(\mathcal{P}_1(\wp_i)\,,\mathcal{P}_2(\wp_i))$ if the upper half of the ball (instead of the peak) is considered as success. Hence, there would be occurrence and a fraction of an occurrence, which is affected by the replacement law even for a large m and a large n. This provides an explanation for the case when the peak is considered as success. As a result, an event will not appear on the peak, and it will not acquire its probability of occurrence, until more than $\frac{1}{2m}$ of its fractions appear on the half of the ball.

How to explain zero fraction

Since there is no physical fraction that has zero value, the probability of zero is zero or $\emptyset$. However, zero occurrence means that there is no occurrence and every event does not occur, simultaneously. At the same time, when every event occurs simultaneously, the same result will be produced. However, a mixed process alongside some events with different values may result in an occurrence or fraction. Accordingly, in the case of true coin, if the coin lands on the edge, the probability will not be $P(H) = P(T) = \frac{1}{3}$, unless the weight is considered (Taylor & Karlin, 1998).

Is there a natural necessity in the experiment of tossing a true coin?
Here, both events should be considered as one event and the natural necessity, if it exists, will be a common one. At the same time, there will be missing events, and this does not satisfy the natural necessity condition (for example, the edge in the true coin experiment will be the missing one). Therefore, the fall should be less restrained, as nature tends to be distributed towards optimal

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

fair shape. Furthermore, it could be assumed in this experiment that the physical body fractions is represented theoretically by a probability value. In other words, the physical body exists as a probability that takes values from $\mathbf{0}$ to $\mathbf{1}$, which are also mathematical values. Heisenberg (1930) expounds more on physical experiment, while Rényi (1970) proffers the definition of experiment.

How to explain Corollary V in reality

For instance, in the case of dice, face 1 and face 2 could be in an inverse relation with at least two faces of (3,4,5,6), in the prior step to the last step, where the intervals will be critical in determining the showed up face and how many faces are in relation. In the probability theory, the functions behavior will depend on the dimensions, rather than the number of events. Therefore, the behavior in two dimensions will be different from the behavior in the case of more than two dimensions such as in the case of random diffusion, where there would be a function of velocity. As a result of this experiment, the fractions will reflect this fact.

Mathematically, how does the result use the probability theory?

The result used the probability axioms through a composite function in several sequences for discrete sample space. The concepts of distribution, moment (implicitly), complement, impossibility, and stochastic (implicitly) are also used. Nevertheless, if the ball continues to move randomly, by throwing it once on an unlevelled surface and observing $\wp_i$ on the peak of the ball at every time t until the ball stops completely, then the function will be continuous at time t. In addition, all neighborhood fractions to the peak could take different measures and have the sense of distance from the peak or the distance between two events. In some physical problems, constructing the probability space will depend on the following questions: (a) Is there a σ-field $\mathfrak{B}$ that contains all intervals as members? (b) Is there a probability measure defined on $\mathfrak{B}$ that assigns the desired probability to the interval? Here, these intervals could represent coherent quantities, regardless of how small they are (Heol et al., 1971; Feller, 1971; Tucker, 1967; Loève, 1977; Lebesgue, 1904). Li & Shao (2001) throw more light on processes such as centered Gaussian processes known as “fractional brownian motion” with advanced treatment. On the other hand, for theoretical studies of randomness regarding the infinite sequence that is $\infty -$ distributed, the following defintion can be an appropriate basis: “A [0,1) sequence is defined to be random if it is an $\infty -$ distributed sequence” (Knuth, 1998).

Does the result repeat the concept of distribution function or the concept of the probability sum? The distribution function outlines how the values of the random variable are distributed and gives the distribution of values in cumulative form (Mood et al., 1974). However, the resulted function is a non-decreasing function, and only the $\{\wp_i\}$ sub sequence behaves as a

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

distribution function. At the same time, $\wp_i$ depends on $\boldsymbol{n}$ and not on the trials number . Also, the result does not interpret the fractions of the probability into a total probability, but rather seeks to interpret the probability as a set of fractions probabilities. Subsequently, it is not partitioning set into a subset, even if it uses partitioning techniques.

How does the result differ from the frequency theory of Von Mises? The result differs in the experiment aspect and it gives value to the probability of probabilities, which does not proceed as each event has a certain character (Popper, 2002; Mises, 1957).

Conclusion

This paper concludes that the probability statement sense depends on the necessity to use it, and this necessity also depends on the level of knowledge. Furthermore, probability theory is connected with epistemology as an approach to investigating truth or as a science to provide truth. Thus, probability values can be expressed in different methods. As a result, this necessity may add or omit some conditions that the probability values behavior may be subject to.

In addition, the most important cause behind the existence of the probability phenomenon is the cause of scarceness. This scarceness creates conditions that probability values depend on or are independent of. Since the fractions appearance is necessary (inevitable) and the complement event is considered as a cause, it could be treated as fractions event. Hence, the probability theory can measure causes by measuring the fractions of events. One of its results shows that the greater the fraction probability value, the greater the probability of its occurrence.

Furthermore, the experiment designing should be significant in order to achieve the natural logic of the occurrence. This admits to searching for all possible outcomes. Also, the design of this experiment eliminates some inevitability in the case of the experiment of the true coin. In addition, the notion of one-to-one correspondence is employed as a natural necessity condition. This condition would not be unique for each event without the fraction element. Similarly, this experiment is able to represent a unique ϕ. This is in addition to the concept of sample space extended to be $mn + m$ outcomes.

The composite function $\mathcal{P}_j(\wp_i)$, can be expressed in terms of sequence $\{\mathcal{P}_j(\wp_i)\}$ and sub sequence $\{\wp_i\}$, with probability values. In addition, the outcome of the experiment is described by a discrete sample space $\mathfrak{F}$ with a probability function $P(\cdot)$ of a discrete random variable and the corresponding distribution function.

At least for two events, the concept of impossibility can be explained by the Impossible value and the nil value, where events behave towards $\wp_n$

 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

inversely. At the same time, they behave towards $\wp_0$ directly, depending on the number of events and their directions. Conversely, for more than one continuous function of independent events, these functions behave inversely in some open intervals. This depends on the intervals and the dimensions, rather than the events number.

Therefore, every independent event probability can be expressed in a finite or a countable infinite set of probabilities, and the composite probability function can be used as a technique to study the probability values behavior. In addition, the probability values behavior can be interpreted as a discrete quantity that consists of coherent quantities. As a result, this can reflect the dynamic part of the probability theory. This, in turn, can describe the probability values behavior as a stochastic process.

Conflicts of Interests: To my knowledge, there is no conflicts of interest.

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 ISSN: 1857-7881 (Print) e - ISSN 1857-7431

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