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The Philosophy of Probability Value Behavior: Fractions and Composite Probability Functions in the Continuous Case

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Abstract

This paper focuses on probability value behavior in the case of continuous sample space by employing fractions intervals and composite functions. The study evaluates value behavior rather than finding values directly, which involves utilization of some concepts from continuity, geometric probability, and measure theory. This paper primarily uses an experiment that contains two major events, head H and tail T, in all their occurrence phases. This spread in infinite and uncountable fractions by a continuous motion within intervals and in the predominant circumstances where events are probabilistic values. As a result, every circumstance reflects many important characteristics of probability theory. Among the main results, this paper provides proven propositions that help design experiments upon understanding the case nature, with some explanations to the existing relation between probability value and the case nature. Also, this paper provides a proven corollary that allows visualizing negative probability values as a particular trial. This in turn proposes necessary uses for the composite probability function $\mathcal{P}_j(\wp_i)$. Moreover, this paper provides numerical explanations of limits, which can demonstrate the nature of $\mathcal{P}_j(\wp_i)$, alongside some techniques. Also, this paper considered conditional probability through some corollaries and the possibility of using the non-negative function of the interval i, alongside many important results in form of discussions.

Keywords: Geometric Probabilities, Negative Probabilities, Dependent

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Introduction

The difference between haphazard, chance or accident, and the act is a thin thread. The same applies to the difference between finding a dumped coin and the chance between two players. Notably, in the game of chance, every player has some information about the other and has some choices. However, in the first case, no one has an idea about who may find the coin, in addition to the independence factor. “Randomness is to be understood as a special case of the epistemic concept of the unpredictability of a process” (Eagle, 2005). In randomness, events happen without any intentional action, and it is difficult to predict their occurrence. Also, they are rarely repeated, in addition to the factor of homogeneity. According to Laplace, “Les événements actuels ont avec les précédents une liaison fondée sur le principe évident, qu’une chose ne peut pas commencer d’être, sans une cause qui la produise. Cet axiome, connu sous le nom de *principe de la raison suffisante*, s’étend aux actions mêmes que l’on juge indifférentes. La volonté la plus libre ne peut, sans un motif déterminant, leur donner naissance ; car si, toutes les circonstances des deux positions étant exactement semblables, elle agissait dans l’une et s’abstenait d’agir dans l’autre, son choix serait un effet sans cause : elle serait alors, dit Leibnitz, le hasard aveugle des épicuriens. L’opinion contraire est une illusion de l’esprit qui, perdant de vue les raisons fugitives du choix de la volonté dans les choses indifférentes, se persuade qu’elle s’est déterminée d’elle-même et sans motifs” (Laplace, 1995).

On the other hand, the concept of continuity plays a significant role in randomness. This means that the discrete process is usually a segment of a continuous process, and it appears if the continuous process is interrupted. Moreover, when human consciousness or technology are unable to recognize the entire process or the uncountable instant such as in the case of absolute speed, there is need to consider the process as a continuous process. Therefore, the unseen indicates events which happened as discrete events from a continuous process or as unrepeated events in discrete processes, where the latter may be considered as deterministic events. Furthermore, information may be regarded somewhat as events.

In the process of tossing a true coin, there will be a continuous process as long as the coin fluctuates. Subsequently, this is considered as a kind of unknown because of the infinite and uncountable points, thus resulting to an infinite and an uncountable sample space. The continuous process is almost uncontrollable since the change is permanent and one can evaluate its limit rather than its exact value. Therefore, there is less determination and more approximation. Moreover, science cannot visualize the discrete case precisely, except through its continuous case. As a result, the previous paper (discrete case) has provided some description of the continuous process (see Corollary

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V) (Jughaiman, 2023). For instance, if a true coin is monitored during its fluctuation, at specific points and specific time intervals, to get the result of head and tail such that $(H,T)\colon(\frac{49}{100},\frac{51}{100})$, how could this result be repeated? Hence, if this question is necessary, then the answer is also necessary.

The notion of continuity dates back to Leonhard Euler (1707-1783). However, the more modern version of continuity is credited to Bolzano (1817) and Cauchy (1821). Interestingly, Bolzano and Cauchy were concerned with continuity on an interval, rather than continuity at a point (Stoll, 1997).

In reality, however, the epistemological value of the theory of probability is revealed only by limit theorems. Moreover, without limit theorems, it is impossible to understand the real content of the primary concept of all the sciences connected to the concept of probability. Historically, there are five considerable limit theorems and the first four deals with a sequence of independent events (Gnedenko & Kolmogorov, 1968).

Methods

The experiment focuses on observing the appearance of probability fractions (intervals) of head or probability fractions (intervals) of tail in a continuous process. However, it is important to answer the following question: How does the continuous value of probability behave?

This paper utilizes a true coin, shaping it into a spherical form (true ball), with a radius of r. Thereafter, the ball is theoretically divided into n ordered closed circles with replacement for every half separately, where n is unknown. At every point up to the peak, a cumulative value of probability fractions (circles) is supposed in an ordered manner, starting from the event $\wp_0 = 0$ as the lower fraction (minimum and lower bound) up to the event $\wp_n = 1$ as the upper fraction (maximum and upper bound) of the closed interval $[0,1]$. However, since the points are infinite and uncountable, $\wp_i$ is unknown, where $\wp_i$ is approaching $\pm\infty$. If $\wp_0$ and $\wp_n$ are expressed as points, they can also be expressed as intervals $\wp_{i=[0,0]}$, $\wp_{i=[1,1]}$, or inside an interval $\wp_{i=[0,1]}$. The composite function $\mathcal{P}_j(\wp_i)$ can be expressed as a sequence $\{\mathcal{P}_{j=1,..,m}(\wp_{i=0,\dots,n})\}$. In addition, the peak of the upper half of the ball represents success as shown in Figure 1.

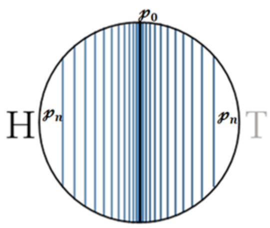

Figure 1. Every half is divided into infinite uncountable fractions

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It should be noted that in the discrete case, i is used to denote the number of fraction that can possibly happen by every throw separately (Jughaiman, 2023). However, in the continuous case, i is used to denote the fractions intervals at some arbitrary time intervals. In addition, this present paper utilizes the procedures and expressions illustrated in Figure 2.

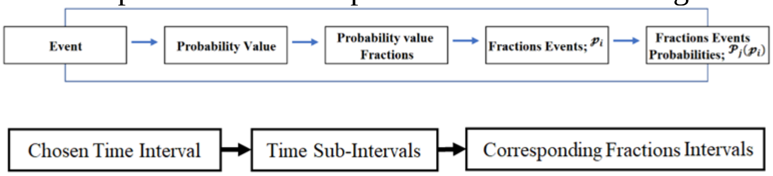

Figure 2. Used procedures and expressions

Where interval, unless specified otherwise, denotes the fractions interval. Regarding the time sub-interval (a, b), this paper considers the longest time sub-interval as the required time interval for the ball to traverse a distance from zero to $\frac{\pi}{2}$, at its fastest motion. The shortest one occurs when the ball remains motionless. In this experiment, two circumstances arise: if the ball moves along a straight line, then there is one direction; secondly, if the ball moves on an uneven surface, then there are different directions (Here, the physical circular motion is disregarded).

Results

If "two arbitrary points are selected on the surface of a sphere of radius R", then "the probability that an arc of a great circle passing through these points will make an angle less than α, where $\alpha < \pi$", is the area of the half surface of sphere minus this area multiplied by cosine α. This is further divided by the total area of the sphere's surface.
Thus,

$$"p = "\frac{2\pi R^2 - 2\pi R^2 \cos\ \alpha}{4\pi R^2} = "\frac{2\pi R^2(1-\cos\ \alpha)}{4\pi R^2}" = \frac{(1-\cos\ \alpha)}{2}$$
$$= "sin^2\frac{\alpha}{2}"$$

This is shown in Figure 3 (Sveshnikov, 1968).

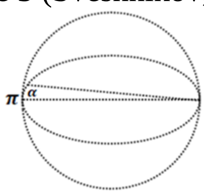

Figure 3. The probability that an arc $\mathbf{0}\frown\boldsymbol{\alpha}$**,** passing through two points is less than the probability that an arc, $\mathbf{0}\frown\boldsymbol{\pi}$, passing through these points by $\mathbf{2\pi R^2 \cos\ \alpha\ /4\pi R^2}$.

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Therefore, the probability that an arc of a great circle passing through two arbitrary points makes an angle equal to α, where $\alpha < \pi$, and if $\alpha = \frac{\pi}{2}$, is given as:

$$p = \frac{2\pi r^2}{4\pi r^2} = \frac{1}{2}, \text{ which equals to } p = \frac{p(H)}{S} = \frac{1}{2}.$$

Circumstance 1:One Direction

1. If the ball moves in a straight line, starting from zero with constant velocity, consider the following: Every sub-interval of time will correspond to the fractions of interval i in a motion of rotation of 90°. Thus, for an infinite number of sub-intervals of time, the sample space will encompass all points on the great circle (Figure 4).

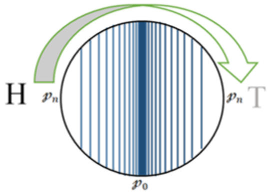

Figure 4. *H* is appearing completely when $\boldsymbol{p_0}$ coincides with the x-axis, $\mathbf{180°}$clockwise

Also, there will be ordered head fractions and ordered tail fractions, but no randomness. Nevertheless, as values are subjected to probability axioms, then these values can reflect probability characteristics. Also, if one applies probability theory, then the outcomes for $\{(H), (T)\}$ will be the ordered triplets: $\{(0,1,0), (0,1,0), \ldots + \infty\}$. This interprets why probability value takes the values of 0 and 1, and why the area under the density curve should be equal 1 in the case of continuous random variable.

2. If the ball moves in a straight line, starting from zero with variant velocity, then there is only one direction. However, as the rotation degree will be random for inconstant velocity, the corresponding fractions intervals on the curve will be unequal for equal time sub-intervals (See Figure 5).

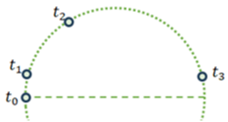

Figure 5. Equal and constant time intervals with unequal fractions intervals

This can result in drawing the normal distribution form (Gaussian distribution form). In addition, if the velocity of the ball increases arbitrarily, then the intervals become narrower and narrower (see Figure 6). The probability distribution of a sum of independent random variables tends to become gaussian as the number of random variables being summed increases without limit, such as, the shot noise generated in a thermionic vacuum tube and the voltage fluctuation produced by thermal agitation of electrons in a

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resistor (Davenport, 1958). As soon as the ball stops, the continuous case will breakup at a discontinuity point, and it is called the discrete case.

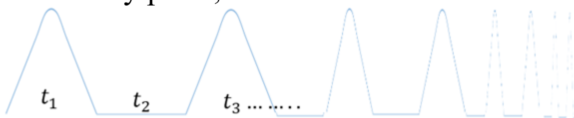

Figure 6. The intervals where H or T probability fractions are increasing and decreasing

Proposition I: For every random process, the number of trials is at least equal to the square number of the fundamental events, m^2.

While this proposition may seem trivial, it is also quite useful as it can reflect various sampling and counting techniques. For an alternative proposition and evaluation, refer to Uspensky (1937).

Proof: In principle, for the case of a true coin in a discrete process, the number of trials should be at least four to obtain either head or tail (Table 1).

Table 1. The chances of all events $\boldsymbol{m^2}$, not the permutation $\boldsymbol{m}!$.

H	T	1	2	3	1	2	3	4	$\cdots$
T	H	2	3	1	2	3	4	1	$\cdots$
		3	1	2	3	4	1	2	$\cdots$
					4	1	2	3	$\cdots$
					$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

In the case of throwing a ball, if one records the fraction that the ball moves through at every interval's end, then at every instant of time, there will be a new fraction, such that, for an infinite number of uncountable events (points) $\wp_i$, inside the closed interval $[0,1]$, where $\sum_{i=0}^{\infty} P(\wp_i) = 1$. If this quantity is less than 1, then there are missing events. In other words, there is no certain event. Since this is true and because of the uncountable aspect of real numbers, there is always a need for enough intervals that reflect the main characteristics in any experiment, besides avoiding errors. Furthermore, while this could be used to prove the uncountable aspect of real numbers, continuity, on the other hand, it helps to reduce errors that cannot be recognized in the discrete process. Consequently, if every event has besides its increasing path, a decreasing path towards all complement events, then this will clearly support the proposition I (Figure 10) to the left.

In addition, in the case of discrete processes, the outcome has values with a large difference, such as tossing a true coin twice with outcomes of 0 and 1. In contrast, in the continuous case, there is a value at every instant, and this value is very close to the previous value or to the next value. Functions that are continuous at every number in a given interval are sometimes thought of as functions whose graphs can be sketched without lifting the pencil from the paper. Also, "a small change in x produces only a small change in the function value $f(x)$. These are not accurate descriptions, but rather devices to help develop an intuitive feeling for continuous functions (Swokowski, 1988).

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For a sequence A_n, the set of all those points which belong to almost all A_n (all but any finite number) is called the *inferior limit* of A_n and $\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$. Also, the set of all those points which belong to infinitely many A_n is called the *superior limit* of A_n and $\limsup A_n = (\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} {A_k}^c)^c = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$. This implies that $\liminf A_n \subset \limsup A_n$. Thus, if the reverse inclusion is true, $\liminf A_n$ and $\limsup A_n$ are equal to the same set A. Therefore, A is called the *limit* of A_n. Also, a sequence A_n is said to be *monotone* if it is either *nondecreasing:* $A_1 \subset A_2 \subset \cdots$, and it is written as $A_n \uparrow$; or if it is *nonincreasing:* $A_1 \supset A_2 \supset \cdots$, and it is written as $A_n \downarrow$. Hence, "*every monotone sequence is convergent, and* $\lim\, A_n = \bigcup A_n$ *or* $\bigcap A_n$ *according as* $A_n \uparrow$ *or* $A_n \downarrow$" (Loève, 1977).

Corollary I: For fundamental events m, with probabilities of occurrence that equal to zero in a continuous process, the probabilities of fractions take negative probability values in a sub-sequence $\{\mathcal{P}_{j=1,\dots,m\downarrow}(\wp_{i=0,\dots,t})\}$ of a sequence $\{\mathcal{P}_{j=1,\dots,m}(\wp_{i=0,\dots,t})\}$ that sum to zero, where the composite functions $\mathcal{P}_{j\uparrow}(\wp_i) = -\mathcal{P}_{j\downarrow}(\wp_i)$ for $j = 1, \dots m$, $i = 0, \dots t$ and $\{\mathcal{P}_{j=1,\dots,m\uparrow}(\wp_{i=0,\dots,t})\}$ is also a sub-sequence of the sequence $\{\mathcal{P}_{j=1,\dots,m}(\wp_{i=0,\dots,t})\}$.

Proof: From probability axioms, $0 \leq \wp_i \leq 1$, for $i = 0,1,2, \dots \infty$. And if $\wp_i$, $i = 0,1,2, \dots \infty$ are mutually disjoint sets in $\mathfrak{F}$, then $\bigcup_i^{\infty} \wp_i = \sum_i^{\infty} \wp_i$. Also, if $\sum_{i=0}^{n} \mathcal{P}_H(\wp_i) + \sum_{i=0}^{n} \mathcal{P}_T(\wp_i) = 1$ and if $\sum_{i=0}^{n} \mathcal{P}_H(\wp_i) = \sum_{i=0}^{n} \mathcal{P}_T(\wp_i)$, then $\sum_{i=0}^{n} \mathcal{P}_H(\wp_i) = \sum_{i=0}^{n} \mathcal{P}_T(\wp_i) = \frac{1}{2}$. Also, it is well known from the discrete case that, $\sum_{0\leq\, i\, \leq n} \mathcal{P}_H(\wp_i) = \sum_{0\leq\, i\, \leq \frac{n}{2}} \mathcal{P}_H(\wp_i) + \sum_{\frac{n}{2}<\, i\, \leq n} \mathcal{P}_H(\wp_i) = \frac{1}{2}$.

Therefore, for a continuous motion, if $n = \pi$ and if,

$$\sum_{0\leq\, i\, \leq \frac{\pi}{2}} \mathcal{P}_H(\wp_i) = \sum_{\frac{\pi}{2}<\, i\, \leq \pi} \mathcal{P}_H(\wp_i) = \frac{1}{2}. \text{ Then,}$$

$$\sum_{0\leq\, i\, \leq \frac{\pi}{2}} \mathcal{P}_H(\wp_i) + \sum_{\frac{\pi}{2}<\, i\, \leq \pi} \mathcal{P}_H(\wp_i) > \frac{1}{2}.$$

This is contrary to the probability axioms. Therefore, one of the two series should be negative. Hence,

$$0 \leq \sum_{0\leq\, i\, \leq \frac{\pi}{2}} \mathcal{P}_H(\wp_i) + \sum_{\frac{\pi}{2}<\, i\, \leq \pi} \mathcal{P}_H(\wp_i) \leq \frac{1}{2}.$$

By multiplying both sides of every inequality by -1, we will arrive at:

$$-\frac{1}{2} \leq -\sum_{0\leq\, i\, \leq \frac{\pi}{2}} \mathcal{P}_H(\wp_i) - \sum_{\frac{\pi}{2}<\, i\, \leq \pi} \mathcal{P}_H(\wp_i) \leq 0.$$

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Then, by taking $\sum_{0\leq\, i\, \leq\frac{\pi}{2}} \mathcal{P}_H(\wp_i) = \frac{1}{2}$ and adding it to the both sides of every inequality, then $0 \leq -\sum_{\frac{\pi}{2}<\, i\, \leq\pi} \mathcal{P}_H(\wp_i) \leq \frac{1}{2}$, or $-\frac{1}{2} \leq \sum_{\frac{\pi}{2}<\, i\, \leq\pi} \mathcal{P}_H(\wp_i) \leq 0$.

As a result, the negative value here is necessary.

Now, suppose that the ball moves in one direction at constant velocity $\mathcal{v}$ through the time interval $[t_0, t]$, which is corresponding to the fraction interval [0,1]. And if $t_0 = 0$, then the displacement $\theta = \mathcal{v}\Delta t$, where Δt is the change of t. However, if $\mathcal{v} = 1$, then $\theta = \Delta t$ where $0 \leq \theta \leq t$. Also, if $t = n$, then $n - 1 \leq t \leq n + 1$. Consequently, $t \leq n + 1$, where t includes all possible values, recall $\mathcal{P}_J(\wp_i)$ from the discrete case, where $\sum_j^m \sum_i^n \mathcal{P}_J(\wp_i) = \sum_j^m \sum_i^n \frac{1}{n+1}\left[1 - \frac{J(n-i)}{Jn}\right]$.

The infinite r rounds in the alternating infinite sequence $(-1)^{r+1}$ is given as:
$(-1)^{r+1}\sum_i^t \mathcal{P}_H(\wp_i) + (-1)^{r+1}\sum_i^t \mathcal{P}_H(\wp_i) + \cdots$ for $r = 1,2, \ldots$ So, for $t > 0$,

$$(-1)^2\sum_i^t \mathcal{P}_H(\wp_i) = (-1)^2\mathcal{P}_H(t_0, t) = \frac{(-1)^2}{t}\int_{t_0}^t [\mathcal{P}_H(\theta)]\, d\theta = \frac{1}{t}\int_{t_0}^t [1 - \frac{(t-(t-t_0))}{t}]\, d\theta = \frac{1}{t}\int_{t_0}^t [\frac{t-t+(t-t_0)}{t}]\, d\theta = \frac{1}{t}\int_{t_0}^t [\frac{(t-t_0)}{t}]\, d\theta = \frac{1}{t^2}\int_{t_0}^t [\theta]\ d\theta = \frac{1}{t^2}\left[\frac{\theta^2}{2}\right]_{t_0}^t = \frac{(t)^2-(t_0)^2}{2t^2} = \frac{(t)^2-0}{2t^2} = \frac{t^2}{2t^2} = \frac{1}{2}.\ Result\ 1.$$

Also, for the same interval (t_0, t), $(-1)^3\mathcal{P}_H(t_0, t) = \frac{(-1)^3}{t^2}\int_{t_0}^t [(t - t_0)]\, d\theta =$

$$\frac{-1}{t^2}\int_{t_0}^t [\theta]\, d\theta = \frac{-1}{t^2}\left[\frac{\theta^2}{2}\right]_{t_0}^t = -\frac{(t)^2-(t_0)^2}{2t^2} = -\frac{(t)^2}{2t^2} = \frac{-t^2}{2t^2} = -\frac{1}{2}\ Result\ 2.$$

Also, from the integration $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$ (Lebesgue, 1989), if the theoretical part of the ball is restricted on the arc $0 \frown \frac{\pi}{2}$, then $results$ 1 and 2 can be obtained directly by:

$$\frac{1}{(\frac{\pi}{2})^2}\int_0^{\frac{\pi}{2}} [\theta]\, d\theta = \frac{1}{(\frac{\pi}{2})^2}\ [\frac{\theta^2}{2}]_0^{\frac{\pi}{2}} = \frac{(\frac{\pi}{2})^2-(0)^2}{2(\frac{\pi}{2})^2} = \frac{\left(\frac{\pi}{2}\right)^2}{2\left(\frac{\pi}{2}\right)^2} = \frac{1}{2}.$$

$$\frac{1}{(\frac{\pi}{2})^2}\int_{\frac{\pi}{2}}^0 [\theta]\, d\theta = \frac{1}{(\frac{\pi}{2})^2}\ [\frac{\theta^2}{2}]_{\frac{\pi}{2}}^0 = \frac{(0)^2-(\frac{\pi}{2})^2}{2(\frac{\pi}{2})^2} = -\frac{\left(\frac{\pi}{2}\right)^2}{2\left(\frac{\pi}{2}\right)^2} = -\frac{1}{2}.$$

$$\text{So, } \frac{1}{(\frac{\pi}{2})^2}\int_0^{\frac{\pi}{2}} [\theta] d\theta + \frac{1}{(\frac{\pi}{2})^2}\int_{\frac{\pi}{2}}^0 [\theta] d\theta = \frac{1}{2} + (-\frac{1}{2}).$$

$$\{\mathcal{P}_H(\wp_{i=0,\ldots,t})\} = \{\mathcal{P}_{H\uparrow}(\wp_{i=0,\ldots,t})\} + \{\mathcal{P}_{H\downarrow}(\wp_{i=0,\ldots,t})\} = 0\ Result\ 3.$$

Consequently, this process is a continuous process on $[0, 1]$, where the composite function $\mathcal{P}_H(\wp_i)$ does not cease at any point. Therefore, these results reflect the fact that $\wp_{(+\infty)} = 1$ and $\wp_{(-\infty)} = 0$. Also, these results

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reflect that nil value does not only mean that the impossible event is happening, but it also means that there is no discrete process. At the same time, in the discrete case, nil value does not only mean that the impossible event is happening, but it also means that there is a continuous process. The zero value here provides an explanation of the limit value for $\lim_{n\to\infty} \frac{1}{2^n} = 0$. $Result\ 3'$

If the density of the probability $f(z)$ is subject to two conditions: (a) $f(z) \geq 0$ for all z in (a, b). (b) $\int_a^b f(z)\, dz = 1$. However, in all cases, the largest possible interval may be taken from $-\infty$ to $+\infty$. To this end, it suffices to *define* the density outside the originally given interval as being $= 0$. Thus, the density is defined for all real values of z and satisfies the conditions: (a) $f(z) \geq 0$ for all z. (b) $\int_{-\infty}^{+\infty} f(z)\, dz = 1$. Furthermore, the probability for x to be in any interval (c, d) will be given by $\int_c^d f(z)\, dz$. In particular, taking $c = -\infty$ and writing t instead of d, $F(t) = \int_{-\infty}^{t} f(z)\, dz$ represents the probability that x will not exceed or will be less than t. Considered as a function of $t, F(t)$ is never decreasing (Uspensky, 1937).

From another point of view, the negative part in $result$ 2 is not defined as being $= 0$. However, it is satisfying the axioms if result 1 is solely considered as enough condition that is satisfying the axioms. Thus, the sum's axiom is explained by the inequalities instead of equality. If that is the case, then the importance of probability axioms will lie in that,
$0 \leq \int_{-\infty}^{+\infty} f(z)\, dz \leq 1$. Therefore, it is significant to know how the sum in this interval $(-\infty, +\infty)$ can spread between these two inequalities.

On the other hand, if $\sum_{0\leq\, i\, \leq\pi} \mathcal{P}_H(\wp_i) = \frac{1}{2}$,

then $\sum_{0\leq\, i\, \leq\frac{\pi}{2}} \mathcal{P}_H(\wp_i) = \frac{1}{2} - \sum_{\frac{\pi}{2}<\, i\, \leq\pi} \mathcal{P}_H(\wp_i)$. In addition, $t + 1 \geq n$ implies that both quarters of every half are included in $t + 1$, where $0 \leq \theta \leq t$, $t_0 = 0$ and $t = 1$, Here, if $t = 1$, then $t + 1$ implies that "1" is an interval. In the discrete case, "1" is a point, and n are also points (Discussion-Supposition). Thus, $\sum_i^{t+1} \mathcal{P}_H(\wp_i) = \sum_i^{t} \mathcal{P}_H(\wp_i) + \sum_i^{t} \mathcal{P}_H(\wp_i)$.

The integration is $\mathcal{P}_H(t_0, t) = \frac{1}{t+1} \int_{t_0}^{t} [\frac{t-t+(t-t_0)}{t}]\, d\theta$. Also,

$$\frac{1}{t(t+1)} \int_{t_0}^{t} [t - t + (t - t_0)]\ d\theta = \frac{1}{t(t+1)} \int_{t_0}^{t} [\theta]\ d\theta = \frac{1}{t(t+1)} \left[\frac{\theta^2}{2}\right]_{t_0}^{t} =$$
$$\frac{1}{2t(t+1)} [(t)^2 - (t_0)^2] = \frac{1-0}{2(2)} = \frac{1}{4}.\ Result\ 4.$$

Consequently, the ball is divided into 4 quarters and every face has two attached quarters. Therefore, $\sum_{0\leq\, i\, \leq\frac{\pi}{2}} \mathcal{P}_H(\wp_i) = \sum_{\frac{\pi}{2}<\, i\, \leq\pi} \mathcal{P}_H(\wp_i) = \frac{1}{4}$ means that $\sum_i^{t+1} \mathcal{P}_H(\wp_i)$ is already multiplied by $\frac{1}{2}$, since $\wp_n$ is always equal to 1.

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Also, if the value of t increases infinitely such that $\theta \longrightarrow +\infty$, then the value of probability will also increase to approach the value of $\frac{1}{2}$. However, it does not exceed it. Numerically, this proves that $\frac{1}{2}$ is a limit of $\mathcal{P}_H(\wp_i)$ (Table 2). At the same time, as the value of t decreases infinitely such that $\theta \longrightarrow -\infty$, the value of probability decreases to approach zero. However, it does not exceed it. This further proves another limit of $\mathcal{P}_H(\wp_i)$, which is explained in results 1, 2, and 3.

Table 2. Whatever the acceleration of $\boldsymbol{t}$ is, the function has less acceleration

t	t^2	$\frac{t^2}{2t^2+2t}$
1	1	0.25
2	4	0.333333333
⋮	⋮	⋮
1,000,000,000	1E+18	0.5

Actually, this case is not seen as one continuous process. It is rather seen as two separate continuous processes on $[0,1]$, where every process repeats the other and they both cease at point 1 where the sum is $\frac{1}{4}$. Hence, they both serve separately to give a discrete quantity, where repeating is an aspect of the discrete processes, $Result\ 4'$. Therefore, $\mathcal{P}_H(\wp_i)$ is continuous at 0 to the right, while 1 is a discontinuity point.

Also, in continuous processes, the conditional concept could be considered. So, for every n events (interval) ; $\wp_{i+1\uparrow}$ occurring if $\wp_{i\uparrow}$ occurs is given. Additionally, it is sufficient that every point in $\wp_{i\uparrow}$ is also in $\wp_{i+1\uparrow}$ which is satisfying that the points are in $\wp_{i\uparrow}\wp_{i+1\uparrow}$.

For a probability space $(\mathfrak{F}, \mathfrak{B}, \mathcal{P}(\cdot))$, if $\mathfrak{F}$ "is uncountable", then $\mathfrak{B}$ "cannot in general be the set of all subsets." But "once one has one probability defined over" $(\mathfrak{F}, \mathfrak{B})$, "then one can define other probabilities that are called conditional probabilities" (Tucker, 1967).

In the present case, if $\mathcal{P}_{j=1,..,m}(\wp_0) = 0$ (impossible: nothing happened: no condition: zero), it is excluded, and this is also the case for $\mathfrak{F} \in \mathfrak{B}$ and all $\wp_i \in \mathfrak{B}$. Nevertheless, the nature of $\mathfrak{F}$ allows it to contain probabilistic values ($\wp_0 = P(\emptyset), \wp_n = P(S)$). Thus, even if $\mathfrak{F}$ is considered a fundamental set, $\mathfrak{F}$ could be considered a set of subsets of $\{(P(H), P(\ T)\}$. In a sure event, S means the probability of 1, which takes the connective (or) $(1\ or\ 1)$. It also means the occurrence of all events of S, which also takes the connective (or) for its elements' occurrence. So, in definition VI in the discrete case (Jughaiman, 2023), it is necessary to take into account that it would have been better if "all $\wp_i \in \mathfrak{F}$" was extended to "all $\wp_i \in \mathfrak{B}$". Also, the mutually disjoint sets in $\mathfrak{B}$, as well in the present case should be put into consideration.

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Also, $\wp_i$ should be considered to represent, implicitly, any event such that $[\wp_{j(i),j(i)}, \cdots, \wp_{j(i),j+1(i)}, \cdots, \wp_{j+1(i),j+1(i)}, \cdots, \wp_{j+1(i),j(i)}]$.

Moreover, the difference in the discrete case is that the events are independent in every throwing, where in the present case every event is supposed to be dependent on the previous event. “Among Markov’s own significant contributions to probability theory were his pioneering investigations of limit theorems for sum of *dependent* random variables and the creation of a new branch of probability theory, the theory of dependent random variables, that form what we now call a Markov chain”, (Shiryaev, 2016).

Corollary II (From Multiplication Rule): For every n events $\wp_1, \dots, \wp_n$ for which $\mathcal{P}_{j=1,..,m}(\wp_1 \dots \wp_{n-1}) > 0$. Then, $\mathcal{P}_{j=1,..,m}(\wp_1 \dots \wp_n) = \mathcal{P}_{j=1,..,m}(\wp_1)\mathcal{P}_{j=1,..,m}(\wp_2|\wp_1) \cdots \mathcal{P}_{j=1,..,m}(\wp_n|\wp_1 \dots \wp_{n-1})$.
For the theorem and proof, see Tucker (1967).

Corollary III (From Theorem of Total Probabilities): If $\mathcal{P}\left(\bigcup_{j=1}^{m} \bigcup_{i=1}^{n} \wp_{ji}\right) = 1$, where $\{\wp_{j=1,\dots,m.i=1,\dots,n}\}$ is a finite or denumerable sequence of disjoint events, if $\mathcal{P}_{j=1,..,m}\left(\wp_{i=1,\dots,n}\right) > 0$ for every i, and if $A \in \mathfrak{B}$. Then, $\mathcal{P}_{j=1,..,m}(A) = \sum_{j=1}^{m} \sum_{i=1}^{n} \mathcal{P}\left(A \middle| \wp_{ji}\right) \mathcal{P}\left(\wp_{ji}\right)$. For the theorem and proof, see Tucker (1967). Moreover, for the expectation, if it exists and “as soon as the given σ-fields are not generated by countable partitions, the descriptive approach remains possible, thanks to the Radon-Nikodym theorem” (Loève, 1978).

3. If the ball moves in one direction but it diffuses in all directions at a velocity that is higher than the velocity of its motion, then the points turn away. Also, at every instant, there would be unknown points (uncountable) (Figure 7).

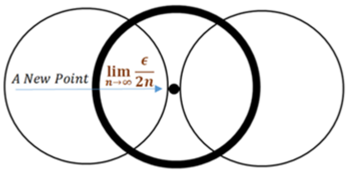

Figure 7. As point is undefined, it is also unknown (uncountable)

For example, the predominant circumstance in the process of tossing a true coin is the one-direction circumstance, where resistance is a bit like a flat surface. Also, if every face of a coin loses weight randomly in a continuous process, then by the integration $\sum_j^m \sum_i^n \mathcal{P}_j(\wp_i) \geq 0$.

Circumstance 2: Different Directions

Every sub-interval of time will be corresponding to fractions’ interval i in a motion of different rotation. So, for an infinite number of sub-intervals

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of time, sample space will be all points on the ball. If one considers that all events are equally likely, then it will also bring Result 3 or 4 (Figure 8).

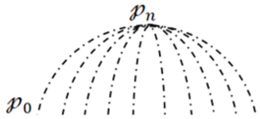

Figure 8. The infinite curves that the ball can move on, where every curve represents a single direction. That means the infinite motion in different directions brings the same result of one direction infinite motion

Definition I: "Let Ω be a measurable subset of n-dimensional Euclidean space with positive, finite Lebesgue measure. Furthermore, let $\mathcal{A}$ be the set of all measurable subsets of Ω and $\mu(A)$ the n-dimensional Lebesgue measure of the measurable set A. Let $P(A)$ be defined by $\frac{\mu(A)}{\mu(\Omega)}$,"and "$[\Omega, \mathcal{A}, P]$ is a Kolmogorov probability space. In this probability space, probabilities may be obtained by geometric determination of measures" (Rényi, 1970).

"The selections of measurable sets and the concepts of limit in range-spaces are rooted in the properties of the Euclidean line: Real line R =$(-\infty, +\infty)$ with euclidean distance $|x - y|$ of points (numbers, reals) x, y. Species of spaces vary according to the preserved amount of these properties, an amount which increases as we pass from separated spaces to metric spaces, then to Banach spaces and to Hilbert spaces" (Loève, 1977).

Theoretically, if one bends the portion of the real line of the closed interval $[0,1]$ as shown in Figure 9, this will represent the infinite uncountable fractions (circles).

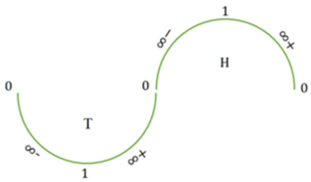

Figure 9. The portion of the real line that bends over every quarter of the ball

"It is well known that we can establish a one-to-one correspondence between all real numbers and all points on the line. Also, a similar correspondence may be established between all pairs of real numbers (x_1, x_2) and all points in a plane, or between all triplets of real numbers (x_1, x_2, x_3) and all points in a three-dimensional space." Moreover, "the length of a finite interval (a, b) in $R_1$is the non-negative quantity $b - a$." Consequently, "for a degenerate interval, the length is zero. The length of an infinite interval can be defined as $+\infty$. Thus with every interval $i = (a, b)$, we associate a definite non-negative length, which may be finite or infinite. Also, we may express

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this by saying that the length $L(i)$ is a *non-negative function of the interval* i, and writing $L(i) = b - a$, or $L(i) = +\infty$, as the interval i is finite or infinite" (Cramér, 1946). For example, in the present case, interval $\wp_{i=[0,0]}$ is a degenerate interval. Intuitively, if area (or length) is used instead of numbering (frequency), then the probability of head will be the area of head / the area of the coin for the process of tossing a true coin. Also, in this present case for each face, if the two intervals $[0, 1]$ and $(0, 1)$ are considered separately, then, for some successive sub-intervals without common points, the sum of sub-intervals will take a length of less than or equal to 1. So, it is possible in principle to represent the function $L(i)$ by $\mathcal{P}_j(\wp_i)$.

"Ensembles de mesure nulle.- Soit un ensemble E constitué par les points de plusieurs intervalles extérieurs les uns aux autres ; on appelle mesure de cet ensemble la somme des longueurs des intervalles qui le constituent. La mesure l ainsi définie est liée à la probabilité ; si l'ensemble E est tout entier situé dans un intervalle de longueur égale à l'unité, et que l'on considère un point choisi au hasard dans cet intervalle suivant une loi de probabilité telle que deux intervalles égaux soient également probables, la mesure de l'ensemble E représente la probabilité que le point choisi soit dans cet intervalle" (Lévy, 2006). Also, if the end points of any interval will be from an infinite uncountable sample space, then the intervals are infinite and uncountable intervals.

Geometrically, "for random directions in the space R^3, the unit sphere serves as sample space; each domain has a probability equal to its area divided by 4π. Choosing a random direction in R^3 is equivalent to choosing at random a point on this unit sphere." "A *random vector* in R^3 refers to a vector drawn in a random direction with a length L, which is a random variable independent of its direction. The probabilistic properties of a random vector are completely determined by those of its projection on the x-axis. Thus, by using the latter, it is frequently possible to avoid analysis in three dimensions" (Feller, 1966). As the ball moves in different directions, with various velocities, it would result to both vectors and scalars. Every vector has initial point v_0, terminal point v_1 and length as a magnitude, which will be the time interval length. This is also dependent on the velocity. Simply, every point a has a probability value equal to $\frac{a}{t}$.

On the other hand, in the case where the ball is still without movement, every point at a random position on the ball could sketch a ball. Thus, as the points are uncountable, there will be uncountable balls. Also, when the ball moves, every point takes the position of the other one. So, if there are uncountable balls, then the probability to pull one of them will be such as the experiment of drawing ball randomly in a continuous process. However, since

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there are random velocities and random directions, value will behave as follow:

1. The points on the peak remain around their position; if the ball move around its axle, the value remains within a bounded range.
2. The points on the peak remain around their orbit; if the ball move around this orbit, the value remains within some great circles.
3. The point moves in random directions, which is changing the value randomly.

From these, if one supposes that every point is moving on random orbits, then through the use of these orbits instead of points, there are infinite and uncountable orbits. Thus, the argument will be: What is the probability value of every orbit the moving ball may take? And, how long is the interval for the ball to remain moving at a specific half (head or tail)? Therefore, there would be some kinds of behaviors which include:

1. The behavior at a specific time interval.
   It will be subjected to the conditions of the existing case. This means that events occurrence will not be equally likely (limited-time interval).
2. The behavior at an infinite time interval.
   It will reflect the value behavior which depend on the ball movement behavior. This means that events occurrence will be equally likely.

Approach

To approach the ball motion problem to the stochastic processes' problems, the following examples are considered: "In the Bohr model of the hydrogen atom, the electron may be found in one of certain admissible orbits." This is "a Markov chain with an infinite number of states (although in principle only)" (Gnedenko, 1963). Also, for example: One "starts at the origin and takes a step in any direction of length Δ." The one "then stops, selects a new arbitrary direction, and proceeds to take another step of length Δ in this new direction." Thus, he "continues his walk for n steps. The angles through which" the one "proceeds on the n steps are chosen independently and at random and thus may be taken as independent random variables. Therefore, let a_k be a random variable whose value determines the angle made with the x axis in the k^{th} step and assume that it is uniformly distributed from 0 to 2π (that is, the frequency function is $\frac{1}{2\pi}$ for $0 \leq a_k \leq 2\pi$ and zero elsewhere)." Another example is that "of a stationary and ergodic random process, we consider the motion of a perfectly elastic billiard ball on a frictionless circular table with perfectly elastic boundaries. Also, we assume the diameter of the ball to be zero and its speed to be a constant v." Thus, "it is clear that upon

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each impact with the boundary of the table, the direction of motion of the ball changes by the fixed amount α. From elementary geometry, it was found that all paths are tangent to an inner circle of radius $r_0 = R\cos\left(\frac{\alpha}{2}\right)$ and that each chord length is $2R\sin\left(\frac{\alpha}{2}\right)$" (Laning et al., 1956).

Discussion

As soon as an alternative design is considered (Figure 10 to the right), then there would be discontinuity points. Figure 10 (second from the right) consider finite sample space for the structure of single-member event, which "is an event that contains exactly one description" (Parzen, 1960). In this case, the exact description will be a positive finite interval or a negative finite interval.

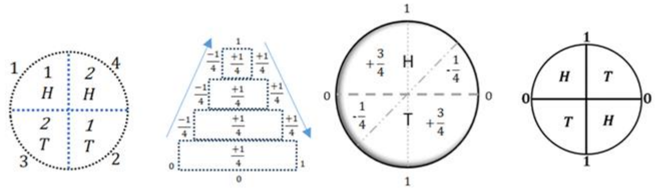

Figure 10. To the left, where every face has two appearances (paths). A possible visualization for the sum in Result 4, each quarter has either height or width sum $(\frac{1}{4} + \frac{1}{4})$ (Second from the left). To the last right, it cannot remain a continuous process to flip the coin from face to face directly, or without zero fraction, $\emptyset$.

Furthermore, it is possible to record negative values (probabilistic values) on any quarter of every half, where the quarters of every half are equally likely, no matter what the directions are. At the same time, the probability of probability will be positive.

Also, besides the importance of $result\ 4'$, and to give more meaning to $result\ 4$, this paper, based on a *mere purpose of giving the frequency*, recorded any velocity less or more than $t_i = 1$. This shows that it is enough in considering both functions as one function in rotation of π and multiplied it by v^{-1} as shown below:

$$\mathcal{P}_H(t_0, t_{i+r}) = \frac{v^{-1}}{t_i + 1}\int_{t_0}^{t_{i+r}} \frac{t_i - t_i + (t_{i+r} - t_0)}{t_i}\, d\theta$$

Hence, this is because the both quarters of every half are symmetric. For instance, if $v = \frac{3}{2}$, $\mathcal{P}_H(0, \frac{3}{2}) = \frac{3}{8}$. But for $v = \frac{1}{2}$, $\mathcal{P}_H(0, \frac{1}{2}) = \frac{1}{8}$. In this case $0 \le \frac{\theta}{\Delta t} \le t + 1$, while $0 \le \frac{\theta}{\Delta t} \le t$ in the results of 1, 2, and 3. Regarding the displacement in general, see (Kane et al., 1988). Also, "suppose that it is in principle possible to continue the trials indefinitely and that the probabilities

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$P_r\{(E_{j_1}, \dots, E_{j_n})\}$ of the outcomes of the first n trials are defined consistently for all n, we shall investigate classes of events defined by certain repetitive patterns" (Feller, 1950).

Regarding the displacement in general, see (Kane et al., 1988) and for some different ideas around the mechanics of a tossed coin (Keller, 1986).

On the other hand, for unequal fractions intervals, there would be unequal areas. "Indeed, one cannot assign an event to every subset of the area since the area, as it is well known, cannot be defined for every subset such that it is completely additive and that the areas of congruent figures are equal. In general, the distribution of probability is said to be *uniform*, if the probability that an object situated at random lies in a subset can be obtained according to the definition (I) from a geometric measure μ invariant under displacement" (Rényi, 1970). Also, in the continuous process, the transition from H to T cannot be the discontinuity point. They are considered as one phenomenon, which is often considered as a homogenous physical body. "Sometimes, the basic set is not even countable. For example, consider the random phenomenon of spinning a pointer on a dial and, when it comes to rest, measuring-in radians, say-the angle θ, it makes with some reference direction." Therefore, it would be natural to take the basic set in our mathematical model to be $E = \{\theta: 0 \leq \theta < 2\pi\} = [0, 2\pi)$. Here, by thinking of the case of a fair pointer, it is natural to begin by assigning to each interval I in this set a probability proportional to its length:

$$P(I) = \frac{length\ of\ I}{2\pi}.$$

Then $P(E) = \frac{2\pi}{2\pi} = 1$ (Botts, 1969).

Also, for more cases in geometric probabilities problems, see Mcginty (2004). On the other hand, "a random probability measure construction is a technique for specifying a probability measure (prior) on the space of probability measures. The most familiar priors of this type are those defined on some parametric family of probability distributions" (Monticino, 2002).

To accommodate and admit the negativity, in $results$ $1, 2$ and 3, we will consider the following: First of all, probability theory has precisely treated the non-decreasing and non-increasing functions to find the density functions. In this case, "the function $g(\cdot)$ is differentiable at every real number x and, furthermore, either $g'(x) > 0$ for all x or $g'(x) < 0$ for all x" (Parzen, 1960). Here, function is characterized by an inverse function (Velleman, 1997). So, "if y is a decreasing function of x, x is a decreasing function of y and hence $\frac{dx}{dy} < 0$. Thus, by using the absolute-value sign around $\frac{dx}{dy}$, we may combine the result of the increasing function and the result of the decreasing function and obtain the final form of the theorem" (Meyer, 1970). For details and

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theorem, see the latter. Also, the negative probabilities have been suggested in many texts, such as the problem of half of a coin (Szekely, 2005). Subsequently, before then, the study was treated by Feynman (1987) and Blass et al. (2015). On the other hand, Corollary I is understood to be compatible with probability axioms. Nonetheless, in principle, it is important to note that all fractions are positive. As a particular trial, when probability values are in non-increasing sequence, this sequence should be a sub-sequence that follows a non-decreasing sub-sequence that in turn expresses a value that is satisfying the probability axioms which is $\wp_n$. From here, if the probability's value is admitted to take a negative sign, it takes it to reflect the continuous process in some directions. Thus, this could be explained by Values from 0 to < 1 being in the success intervals, which is considered for the occurrence that precedes or tends to the sure event. At the same time, values from 1 (0) to > 0 (-1) are in the failure intervals, especially for the occurrence that precedes or tends to the impossible event. Consequently, "there are at least two directions from which the fundamental axioms of probability theory may be approached: probability theory concepts may be built up, mechanically, from the concept of the event and its probability, or they may be derived as special applications of the theory of measurable spaces" (Allen, 1976).

For infinite time sub-intervals, there would be infinite fractions sub-intervals which are necessary with common points. Moreover, for random velocities, these sub-intervals have random lengths, which are considered as random variables. In addition, the endpoints of intervals could be considered as random variables, which are not necessarily independents. For more discussion, see Justicz et al. (1990).

Supposition: In the discrete case, if the arguments were $\sum_{i=1}^{n=1} \frac{1}{n}\left[1 - \frac{(n-i)}{n}\right]$, then $\mathcal{P}_H(\wp_{i=n}) = 1$. However, this is also contrary to the probability axioms. So, $\wp_0 = 0$ is necessary as φ is necessary, but only for H or only for T (Mutually Exclusive Condition) as shown in Figure 11. This means the unoccurrence probability of H. Thus, $\mathcal{P}_H(\wp_0) + \mathcal{P}_H(\wp_n) = \frac{1}{2}$, where $\mathcal{P}_H(\wp_0)$ is the image of $\mathcal{P}_T(\wp_n) = \frac{1}{2}$, and the reverse is true. So, by supposing that $\mathcal{P}_H(\wp_0)$ is a *limit value*, where in general $P(\varphi) = 0$, and $\wp_0$ should be included in t. Hence, $n + 1$ (discrete) and $t, t + 1$ (continuous) are always true. Also, if $\wp_n = 1$ and $\overline{\wp_n} = \varphi = 0 = \wp_0$, then $\wp_0, \wp_n \in \mathfrak{F}$, which implies that $\mathfrak{F}$ is a set of subsets.

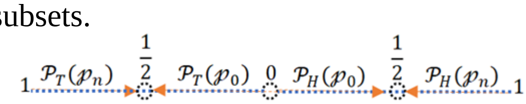

Figure 11. Mutually Exclusive Conditions

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Conclusion

This paper begins with an introduction that distinguishes the concept of randomness within levels of uncertainty. It also briefly presents the concept of continuity and its relationship to the probabilistic nature. This paper uses a method that relies on carefully designed experiment to reflect the physical nature and to ensure consistency with the mathematical logic of probability. This is done so that it contains the possible greatest number of applicable concepts, and to help in proffering answers to many questions related to probability theory. As for the results, they explained the method in more detail. Consequently, most of the paper's results were based on a typical circumstance which is based on a structure composed of elements that are subject to the axioms of probability. Here, this circumstance can be transformed into a probabilistic circumstance by adding more elements gradually. The outcomes flowed through mathematical techniques, starting with geometric probability as a logical conception of the nature of the experiment. Also, each circumstance helps to reflect some aspects of the experiment and to deduce some logical imperatives as results. The paper also contained many carefully explained illustrations that make them an important complement to the content of the paper. Also, this paper added more techniques in the form of discussions, examples, and suppositions as part of the experiment and the results.

In conclusion, the experiment of tossing a true coin can be represented geometrically, especially where the continuous process can explain clearly the theoretical concept for the probability values of 0 and 1. It also concluded that the sum of probability values in a motion of rotation of 180° can represent the smallest picture of the continuous infinite rotation towards $\pm\infty$. Also, this paper concluded that the randomness in continuous process is explained by the sample space and the rarity of recurrence (repeating) of event. In the discrete process, the randomness is explained by the extent of the difference of probability values that the event takes, while the continuity could be considered as the nature of all events. Moreover, the present experiment can interpret the continuity by the small difference of fractions, compared to the large difference of fractions in the discrete case. In addition, this paper proposes the counting principle in the experiments design, which is supported by the experiment and the results.

Subsequently, negative probability values do not appear in the discrete processes but they appear in the continuous processes. Therefore, the random processes could be described in more details in the continuous processes. It also concluded that negative values are considered as part of particular trial, and they cannot be discontinuity points from the continuous case as negative values, but as positive discrete values. Also, it is clear that the probability of non-decreasing sequence is equal to the probability of non-increasing

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sequence, of which they are cancelling each other. On the other hand, for a given purpose, it is possible to assume that non-increasing values take the meaning of failure for the event under study. This, thereafter, is followed by the meaning of success for the complement event. Also, the adoption of negative probability value is dependent on the extent of the explanation of the non-increasing sequence, and on the extent of the need to negative value. By some amendments of the experiment design, the distribution and composite probability functions, negative probability values can be calculated by the composite probability function $\mathcal{P}_j(\wp_i)$ to give positive values with $\mathcal{P}_H(\wp_{i=0,1,..n}) + \mathcal{P}_T(\wp_{i=0,1,..n}) = 1$. Therefore, the probability of probability has a mathematical necessity. In addition, this paper concluded that there are maxima and minima limits for $\mathcal{P}_j(\wp_i)$. These limits are proven numerically to reflect probability value behavior and to demonstrate the nature of $\mathcal{P}_j(\wp_i)$. Also, this paper consider that $\mathcal{P}_j(\wp_i)$ can be used in the conditional probability, if $\mathfrak{F}$ is uncountable. On the other hand, it is possible in principle to represent the non-negative function of the interval i by $\mathcal{P}_j(\wp_i)$. Also, this paper concluded that mutually exclusive condition cannot be achieved, except by the existence of $\wp_0$ (zero limit value) or φ in general.

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