ROLLING BALL IN BREATHING PLANE-TREE ALLEY PARADIGM

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Abstract

Sinai's famous geometric explanation of equilibrium thermodynamics is extended towards non-equilibrium thermodynamics and cryodynamics. Specifically, "breathing smooth Sinai trees" are introduced. And so are "breathing Sinai funnels." The example demonstrates that deterministic chaos lies at the root of two fundamental physical disciplines: statistical thermodynamics and statistical cryodynamics. Quantum mechanics, computer simulation, energy technology and cosmology profit from the new consistency.

Keywords: Smooth far-from-equilibrium Sinai paradigm; inverted Sinai paradigm; hypothesis of molecular chaos; geometrico-topological proof; cryodynamics

Introduction

Fourty-three years ago, Yakov G. Sinai (1970) reduced equilibrium statistical thermodynamics to deterministic-chaos theory in modern parlance. The method was unique in its geometric parsimony. First, he reduced the repulsive interactions of Hamiltonian billiard balls in 3D, down to two frictionless hard disks interacting in 2 D. Second, he without loss of generality nailed the one disk onto the middle of the quadratic billiard table. Third, he doubled this disk's diameter while shrinking the moving disk to a point without change of trajectory. Fourth, he identified opposite sides of the table. The result was the "tennis ball in an orchard" game, as Harry Thomas would call it (personal communication 1976): The ball follows a straight path in the orchard – except for a locally symmetric reflection whenever hitting a tree stem. As a consequence, almost all paths diverge from their infinitesimally close neighbors. Exponential divergence of initial conditions would later be named "chaos" following work on dissipative dynamical systems by T.Y. Li and J.A. Yorke (1975).

Equilibrium thermodynamics got based on deterministic-chaos theory by Sinai. The traditional probability-theoretic approaches thus got effectively replaced by causal deduction more than 4 decades ago. In the following, Sinai's result will be generalized further towards covering non-equilibrium thermodynamics and more.

The New Paradigm

We simplify the motion on Sinai's billiard table by focusing on a single unstable trajectory. The latter grazes the quadratic table on one side. To represent it uniquely, we for convenience take two neighboring Sinai cells and make them our new unit cell. The path of interest then is the symmetric middle path between the two repulsive Sinai disks contained, the one lying to the left, the other to the right. The double cell is automatically repeated in a strip-like tessellation space. We call the resulting picture the "Plane-tree Alley Problem" because our unstable path follows the middle-line of an infinitely long alley of trees as it were (reminiscent of the "Plane-Alley" in the town of Tubingen).

In recompense for this strong simplification, we introduce two complicating ingredients into our Sinai world:

(1) Smooth rather than hard potentials: The repulsive trees in our alley are to possess, instead of a step-function on either side (box-shaped potential), a soft hyperbolic exterior governed by a 90-degree hyperbola. For example, take the repulsive part of a 12th-order Lennard-Jones potential, or else use long-range Newtonian repulsion (it makes no qualitative difference).

(2) Time-dependence: The trees are assumed to be "breathing." That is, they expand and shrink slowly in a periodic fashion. (Equivalently, symmetric motions of the trees, equidistributed across all directions can be assumed, but the breathing case is more intuitive.)

This is the "Rolling Ball in Breathing Alley Problem." The rest is implications. What is it that is going to happen to our frictionless point-shaped light-weight ball as it rolls along the middle of the Plane Alley?

A First Result

Consider first the non-breathing subcase – point (1) above: Then the central path in the alley is no longer traversed at an everywhere constant speed as in the original hard-tree case. Rather the ball has to climb up to negotiate a potential maximum in the form of a smooth mountain col whenever it is passing in between the middle of two trees. The ball hence periodically loses speed, while going up to that saddle point, in order to thereafter re-gain its old speed on sliding down on the other side again on the way toward the boundary of the next double Sinai cell, and so on. That is all which occurs – a time-periodic straight motion since we do not consider deviations at first.

Now the full case – points (1) and (2) taken together: At first sight, one is prepared to bet that the ball on average once more re-gains just as much speed as it loses, now that the col in the potential ridge between two trees no longer has a fixed height but rather is "breathing" upwards or downwards while the ball is rolling on. This breathing is the only change introduced. On closer inspection it turns out that the loss and the gain are not equal to each other but rather differ lawfully in their amplitude. The reason lies in the fact that the oscillation-in-height between minimum and maximum, at the location of the col, has a non-sinusoidal shape: the upward swings are larger than the downward swings. This is owed to the two hyperbolic tree flanks as they approach and recede symmetrically. They do come closer together by as much as they recede from the middle position, in the next half-phase: however, this generates an unequal height difference in the two cases: The lift-up exceeds the let-down. In other words, the potential of the col rises up higher when the trees are approaching than it decreases when they are receding.

This observation finishes our differential-topological game. We have found a direction-of-time invariant, on average linear, increase in the kinetic energy of our frictionless ball as it recurrently negotiates two symmetrically breathing trees in an infinite alley of synchronously expanding and shrinking hyperbolic tree stems. Q.e.d.

The result is robust. It can be generalized towards non-periodic recurrent breathing motions of the trees, and towards recurrently approaching and receding symmetric motions of the trees in all directions, and towards asymmetric placements of the breathing or moving trees, whereby the path ceases to be straight (which fact makes no difference as far as the ball's gaining energy on average is concerned).

Implications

We have encountered energy dissipation in a time-reversible deterministic setting since we implicitly invoked conservation of the total kinetic energy. Note that the slow heavy tree stems dissipate while breathing (or equivalently moving) part of their energy of motion into that of the passing-by low-mass fast point particle while the overall energy is conserved. This behavior occurs in both directions of time since we did not specify the direction of time beforehand.

This result can be seen as representing the essence of non-equilibrium thermodynamics – energy dissipation derived from deterministic first principles under far-from-equilibrium conditions (since the trees have much more energy than the ball). However, the observed energy dissipation in both time directions stands not alone. This is because the result is valid only for "uncommitted" initial conditions. Hence there do also exist "committed" initial conditions – namely, all those that have already been running for a short non-zero (or even arbitrarily long) stretch of time either in the one or the other direction of time. In case the direction of motion is continued, the initial condition in question is indistinguishable from an uncommitted initial condition in the direction of time in question. But in the other direction of time the same initial condition reveals a radically different behavior: it shows "anti-dissipation" up to the uncommitted original initial condition in order to from that moment on show dissipation just like any other uncommitted initial conditio**n i**n the new direction of time.

We now turn to an uncommitted initial condition obtained by a random pick. These initial conditions illustrate (and prove) L. Boltzmann's (1895) "hypothesis of molecular chaos" under deterministic conditions. One sees that the hypothesis is not direction-of-time specific (as was generally believe up until now). This "Boltzmann phenomenon" was encountered here (and hence proved in a deterministic setting for the first time). Quantitative and numerical illustrations are easily possible.

Thus the famous "time's arrow" of statistical physics was successfully reproduced in both directions of time in a deterministic Sinai-type setting. The **main** difference to the original case was that two particle classes with differing energies were introduced. This corresponds to a far-from-equilibrium initial condition.

The result may appear trivial despite the fact that it is valid in both directions of time. Note that a constantly shaken box with a frictionless ball in it likewise heats the latter up in its kinetic energy in both directions of time. What then is the conceptual gain obtained here? The mysterious difference between "uncommitted" and "committed" initial conditions (cf. H. Price 1997) was made palpable to the eye under deterministic conditions. If this appears trivial, there exists an at least equally mysterious corollary.

The Dual Result

Inverting the potentials from "repulsive" towards "attractive" was not an option in the original hard-spheres case of 1970. However, it is an option in the present case of smooth potentials. The inversion of all potentials yields a "dual case" to the bidirectional energy dissipation demonstrated above. The special case of a repulsive potential – of the trees being "anti-Newtonian" – considered above for convenience – is especially natural to focus on here. For potential inversion leads directly to the familiar Newtonian attraction. Hence this case ought to be well-known even though it is not as it turns out. The original potential mounds (trees) are now pointing downwards rather than upwards. Downwards-pointing mirror-symmetric Newtonian potential troughs (funnels) do now correspond to the former potential mounds (trees).

This scenario "below the glass ceiling" represents a second explicit smooth Sinai ballpark. It is even more perplexing. It yields the opposite result to the previous one in a nontrivial "duality." Since the potentials are mirror-inverted below the upper-world alley now, the ball that is moving down there now systematically loses rather than gains in kinetic energy when it is again started out from a non-selected initial condition. For the downwards-pointing inverted breathing trees (funnels) now descend more deeply with their potential in the middle of the col during the mutually approaching phase than they are rising up with their potential during the mutually receding phase. Thus while the geometry is mirrored, the speed effects are the opposite of what held true upstairs. The fruit of this "second smooth Sinai paradigm" as it can be called is a causal explanation, not of energy dissipation in thermodynamics but of "energy anti-dissipation in cryodynamics." Cryodynamics was recently proposed as "sister discipline" to thermodynamics (O.E. Rossler 2011).

Discussion

Sinai's geometric paradigm successfully explained equilibrium thermodynamics from first deterministic principles (Y. Sinai 1970). This famous geometric paradigm was extended above towards covering non-equilibrium thermodynamics. The trick consisted in using, (1) soft potentials, and (2) a periodic perturbation – the two oldest ingredients of Poincaréan chaos theory. The obtained new ballpark rehabilitated Boltzmann's "hypothesis of molecular chaos." Even more surprising, it possesses a natural "dual" which lets it explain also non-equilibrium Cryodynamics, a recently discovered dual theory to thermodynamics (O.E. Rossler 1011). Results presented previously – of a qualitative (O.E. Rossler et al. 2003; Rossler and Movassagh 2005), numerical (K. Sonnleitner 2010; R. Movassagh 2013) and analytical (R. Mocassagh 2013) kind – hence got confirmed and extended by a geometrico-topological proof. A numerico-didactic illustration of the above presented proof is herewith solicited.

The new Sinai-derived paradigm of the "breathing alley" confirmed with its downward-reflected dual the existence of cryodynamics as a sister discipline to thermodynamics in fundamental physics. The new joint status of both disciplines – one being more than 150 years old, the other new – now entails some general consequences:

Firstly, the combined deterministic classical discipline needs to be reconciled with quantum mechanics in a new way. This is because cryodynamics with its anti-entropic character is much more sensitive to minor perturbations than thermodynamics. In a system that is governed by both thermodynamics and cryodynamics, like nature at large, the anti-entropic character of cryodynamics, with its strong correlations to the past, severely constrains quantum mechanics. The currently accepted die-tossing Copenhagen interpretation contains too much leeway formally so that Everett's theory may represent the only choice left.

Secondly, cryodynamics generates novel numerical problems. The fact that it escaped detection in thousands of many-particle computer simulations performed over decades reveals a deep shortcoming in the latters' numerical accuracy. This is a highly unexpected state of affairs.

Thirdly, technological applications – like a proposal to improve the Tokamak reactor of hot nuclear fusion by the addition of a cooling mechanism based on even hotter electrons (O.E. Rossler et al. 2013) – can be mentioned.

Fourthly, cosmology is severely constrained by cryodynamics (O.E. Rossler 2011).

To conclude, a venerable paradigm in fundamental physics due to Yakov Sinai was taken up and extended towards covering two different mass classes of particles. A deterministic geometric derivation of non-equilibrium thermodynamics could be offered as an implication of the slightly extended Sinai ballpark. That ballpark turned out to possess a "natural dual" under potential inversion. The dual enabled a deterministic geometrico-topological derivation of cryodynamics. Numerical illustrations promise to become a bonanza. The overarching new chaos-based discipline of "deterministic statistical mechanics" – with its two sub-disciplines of "deterministic thermodynamics" and "deterministic cryodynamics" – has predictable implications for quantum mechanics, numerical simulation, energy technology and for cosmology.

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