MODELING, SIMULATION, CONTROLLER SELECTION AND DESIGN OF ELECTRIC MOTOR FOR MECHATRONICS MOTION APPLICATIONS, USING DIFFERENT CONTROL STRATEGIES AND VERIFICATION USING MATLAB/SIMULINK

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Abstract

For comprehensive study and analysis purposes of most used motion control models and strategies for comparison and analysis purposes, this paper addresses design, modeling, simulation, dynamics analysis and controller selection and design issues of Mechatronics single joint robot arm, where a electric DC motor is used, modeled, simulated and a control system is selected and designed to move a robot arm to a desired output position, θ corresponding to applied input voltage, V_{in} and satisfying all required design specifications, controller was selected and designed to make the system robust, adaptive and improving the system on both dynamic and steady state performances. This paper is intended for research purposes as well as for the application in educational process.

Keywords: Modeling, simulation, control strategy selection and design, MATLAB m.file

Introduction

The term control system design refers to the process of selecting feedback gains that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant (Ahmad A. Mahfouz et al, 2003)(D'Azzo et al, 1988)(Hedaya Alasooly, 2011). There are many motor control system design strategies that may be more or less appropriate to a specific type of application each has its advantages and

disadvantages. The designer must select the best one for specific application. In this paper, we are to design, apply, verify and compare different control strategies in order to suggest the best control strategy that can be applied to control the output angular position, θ of a given DC motor, corresponding to applied input voltage V_{in} to meet the following specifications; applied voltage of 0 to 12 volts corresponds linearly to an output arm angle of 0 to 180, The designed system should respond to the applied input with an overshoot less than 5%, since speed faster than the reference may damage the equipment , and a settling time less 0.2 second, gain margin greater than 20 dB, phase margin greater than 40 degrees and a zero steady state error.

The mathematical model of the basic open loop system, to be derived and analyzed, this will be followed by designing the closed loop system, with sensor and controller models added, different control strategies will be applied with the closed loop, tested, analyzed compared and finally, based on system controller selection and design, overall controller effect, system performance analysis and comparison, the most suitable control strategy will be selected, design and dynamic analysis will be verified by MATLAB/Simulink.

Mechatronics robot systems and low-to-medium power machine-tools often use DC motors to drive their work loads. These motors are commonly used to provide rotary (or linear) motion to a variety of electromechanical devices and servo systems. There are several well known methods to control DC motors such as: Proportional-Integral PI, Proportional-Integral-Derivative PID or bipositional (M. S. Rusu et al , 2008). (D'Azzo et al, 1988) covered how it is possible to improve the system performance, along with various examples of the technique for applying cascade and feedback compensators, using the methods root locus and frequency response. It also covered some methods of optimal linear system design and presentation of eigenvalues assignments for MIMO system by state feedback (Hedaya Alasooly, 2011). In (M. S. Rusu et al , 2008) different closed loop control strategies and compensator designs were compared to eliminate the steady state error and enhance the electric motor system transient response in terms of output speed, similar approach will be applied in this paper in terms of output position. In (Hedaya Alasooly, 2011) and (Frank. L. Lewis, 1986), good description of the optimal control design, including linear state regulator control, the output regulator control and linear quadratic tracker. The MATLAB SISO Design Tool (MathWorks,2009) will be used for design and verification of compensators by root locus, Bode diagram, and Nichols plot, as well as a set of MATLAB commands that can be used for a broader range of control applications. An important advantage of the root-locus method is that the roots of the characteristic equation of the system can be obtained directly, which results

in a complete and accurate solution of the transient and steady-state response of the controlled variable. The frequency-response approach yields enough information to indicate whether the system needs to be adjusted or compensated and how the system should be compensated (Jamal A. Mohammed, 2011).

II. Robot arm system characteristic; Modeling of the Permanent Magnet DC Motor

Single joint robot arm system shown in Fig.1, consists of three parts; arm, connected to actuator through gear train with gear ratio, n. The actuator most used in Mechatronics applications is a PMDC motor. The DC motor is an example of electromechanical systems with electrical and mechanical components, a simplified equivalent representation of DC motor's two components are shown in Fig.2 DC motor turns electrical energy into mechanical energy and produce the torque required to move to the desired position, θ_L, or rotate with the desired angular speed, ω_L.

In (Ahmad A. Mahfouz et al, 2003) detailed derivation of PMDC dynamic models are introduced, including refined, linear, nonlinear and simplified models. The PMDC motor open loop transfer function without any load attached relating the input voltage, $V_{in}(s)$, to the motor shaft output angle, $\theta(s)$, given by:

$$G_{angle}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{K_t}{L_a J_m s^3 + (R_a J_m + b_m L_a)s^2 + (R_a b_m + K_t K_b)s} \tag{1}$$

The PMDC motor open loop transfer function relating the input voltage, $V_{in}(s)$, to the angular velocity, $\omega(s)$, is given by:

$$G_{speed}(s) = \frac{\omega(s)}{V_{in}(s)} = \frac{K_t}{\left\{\left[L_a J_m s^2 + (R_a J_m + b_m L_a)s + (R_a b_m + K_t K_b)\right]\right\}} \tag{2}$$

Using these equations, the block diagram and Simulink model of the open loop PMDC motor system shown in Fig.3(a)(b) are built,
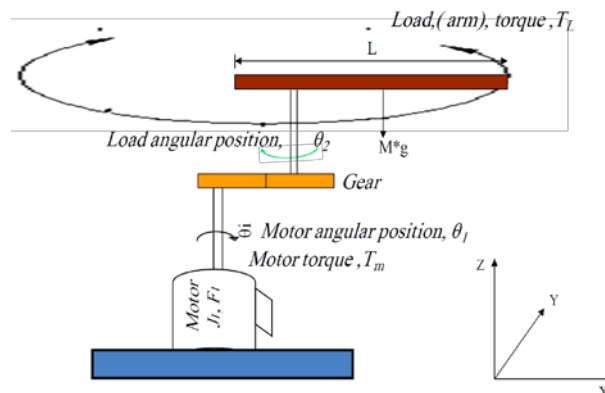

Fig.1: Schematic model of a single joint (one DOF) robot arm driven by an armature-controlled DC motor (Ahmad A. Mahfouz et al, 2003).
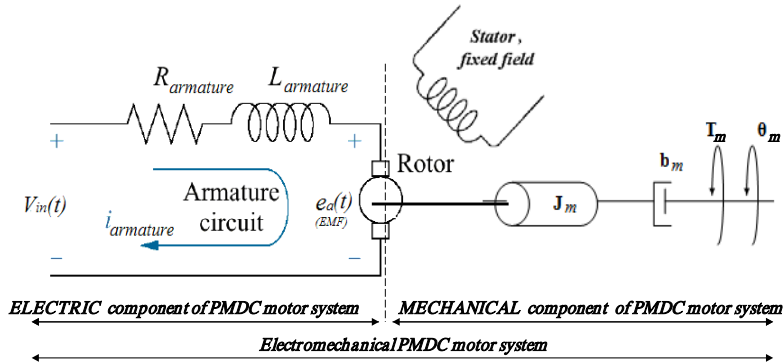
Fig.2: a simplified equivalent representation of the PMDC motor's electromechanical components (Ahmad A. Mahfouz et al, 2003)
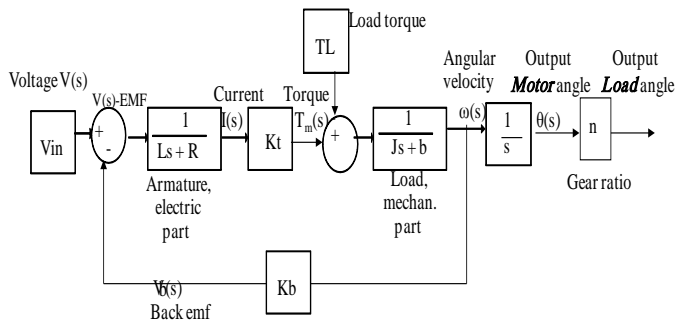

Fig.3(a) The block diagram representation of PMDC motor system(Ahmad A. Mahfouz et al, 2003)
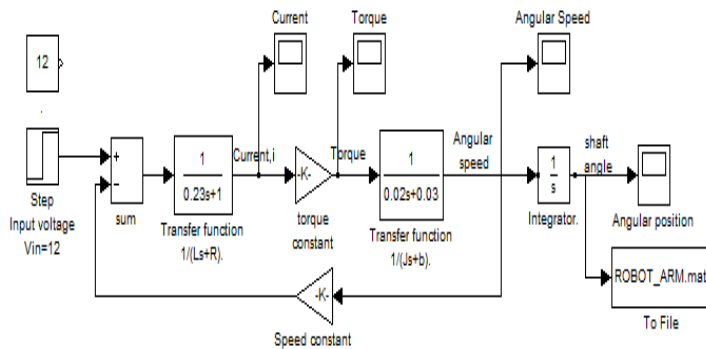

Fig.3(b) Simulink model of PMDC motor open loop system

III. Controlling of PMDC motor output angular position using different control strategies and verification by MATLAB/Simulink

We are to design, apply and compare different control strategies in order to suggest the best control strategy that can be used to control the output angular position, θ_m of PMDC motor, corresponding to applied input voltage V_{in} to meet the desired specifications. The following nominal values

for the various parameters of a PMDC motor used : $V_{in}=12$ Volts; $J_m=0.02$ kg·m²,; $b_m =0.03$; $K_t =0.023$ N-m/A ; $K_b =0.023$ V-s/rad, $R_a =1$ Ohm ; and $L_a=0.23$ Henry ; $T_L=$ no load attached, gear ratio, n=1:2, and design criteria : applied voltage of Vin = 0 to 12 volts corresponds linearly to an output arm angle θ =0 to 180, PO% < 5%, Ts < less 2 second, gain margin, $G_m > 20$ dB, phase margin, $Φ_m > 40$ degrees , Ess =0.

The following set of MATLAB commands can be used to derive and obtain open loop transfer function, as well as,   for analysis and comparison purposes:

```
clc, clear all, close all
Vin=   12;Jm=0.02;bm   =0.03;Kt   =0.023;    Kb=0.023    ;Ra
=1;La=0.23;angle =180; Kpot=Vin/angle;
% Jm = input('  Enter  moment of inertia of the rotor, (Jm)
=');
% bm = input('  Enter  damping constant of the mechanical
system ,(bm)=');
% Kt = input('  Enter  torque constant,  Kt=');
% Kb = input('  Enter  electromotive force constant,  Kb=');
% Ra = input('Enter electric resistance of the motor armature
(ohms),  Ra =');
% La =input('  Enter   electric inductance  of the motor
armature (Henry), La=');
% Vin = input('  Enter  Max. applied input voltage, Vin = ');
% angle=input(Enter  Max. required output angle, Theta out
=')
% Kpot=Vin/angle;
G_open_motor_num=[Kt];
G_open_motor_speed_den=[La*Jm, (Ra*Jm+bm*La), (Ra*bm+Kt*Kb)];
G_open_motor_angle_den=[La*Jm,  (Ra*Jm+bm*La),  (Ra*bm+Kt*Kb),
0];t=0:0.001:50;G_open_motor_speed=tf(Kt,[La*Jm,
(Ra*Jm+bm*La), (Ra*bm+Kt*Kb)]);
G_open_motor_angle=tf(Kt,[La*Jm,              (Ra*Jm+bm*La),
(Ra*bm+Kt*Kb),
0]);G_close_motor_angle=feedback(G_open_motor_angle,Kpot)
% controller_Tf= P. PI.PD,PID, ...
controller_TF=10;
G_open_controller_angle=
series(controller_TF,G_close_motor_angle)
G_close_controller_speed=
feedback(controller_TF*G_open_motor_speed,1);
G_close_controller_angle                          =
feedback(controller_TF*G_open_motor_angle,Kpot;
subplot(2,2,1) ,step(Vin*G_open_motor_angle,t);
subplot(2,2,2) ,step(Vin*G_open_motor_speed,t);
subplot(2,2,3),rlocus(G_open_controller_angle);
subplot(2,2,4),bode(G_close_controller_angle);
margin(G_close_controller_angle);
[Gm,Pm,Wg,Wp]=margin(G_close_controller_angle);
```

```
[mag,phase,w]=bode(G_close_controller_angle);
y1=step(Vin*G_close_controller_speed,t);
y2=step(Vin*G_close_controller_angle,t);
yA=step(Vin*G_close_controller_angle,t.*1000);
angle_endA= yA(end);
angle_max= max(yA);
Ess_step= 1/(1+ dcgain(G_close_controller_angle));
Ess = angle - angle_endA;
Mp= angle_max- angle;
fprintf('    \n');
fprintf('===========================\n');
fprintf('For chosen Kp, the following results are obtained:
\n');
fprintf('===========================\n');
fprintf('  Error    (Mp)    max value    Final       gain
Phase  \n');
fprintf('  (Ess)            (Peak)       output    margin
margin   \n');fprintf('===========================\n');
answers= [ Ess'    Mp'    angle_max'   angle_endA'    Gm'
Pm'];disp(answers ),fprintf('===========================\n');
```

IV. Analyzing basic used PMDC motor open loop system performance

Substituting values and applying unit step input voltage signal, $R(s) =$ 1/s, to Eq. (1), will result in PMDC motor open loop transfer function given by Eq. (3) as well response curve shown in Fig.4(a). As shown in Fig.4(b), this is type one system when subjected to step input result in $e_{ss}=0$, when to ramp input $e_{ss} =finite$, when to parabolic $e_{ss} =infinity$. Also, running m.file given by (Farhan A. Salem, 2013), or the given set of MATLAB commands, as will running Simulink model given in Fig.4(b).

$$G(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.023}{\left[0.0046s^3 + 0.0269s^2 + 0.03053s\right]} = \frac{0.023}{s(s+4.3069)(s+1.5410)} \quad (3)$$

Different control strategies will be applied and compared in order to suggest the best control strategy that can be applied to control PMDC output angular position to meet the desired performance specifications.
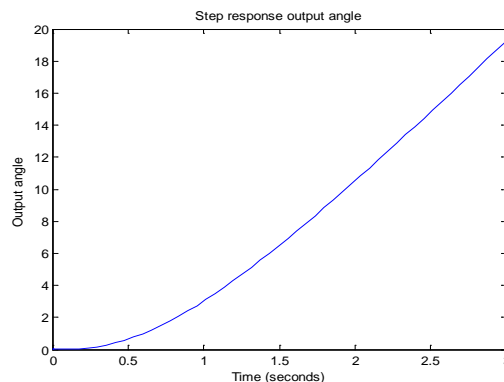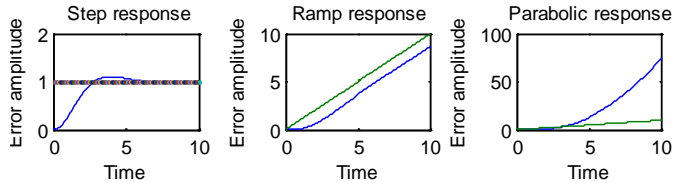

Fig.4(a) Step response, output angle.

Fig.4(b) step, ramp and parabolic responses and corresponding steady

V. Analyzing closed loop PMDC motor system applying different control strategies.

A negative feedback control system with forward controller shown in Fig.5(a)(b) is to be used.
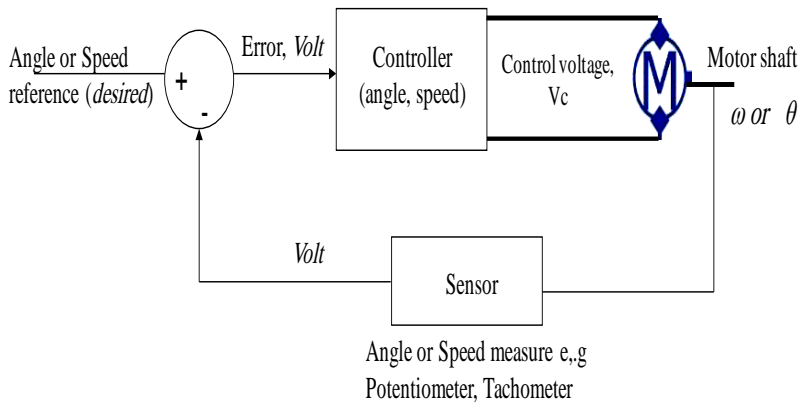

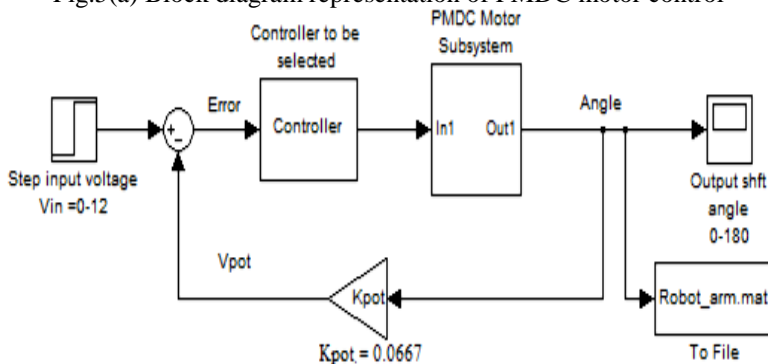Fig.5(a) Block diagram representation of PMDC motor control

Fig.5(b) Preliminary Simulink model for negative feedback with forward compensation

V.I Sensors modeling

The sensing device used to measure and feedback the actual output angular position, θ_m of the motor shaft, is potentiometer. To measure the actual output angular speed, ω_m tachometer is used. The potentiometer output is proportional to the motor shaft angular position, θ_m. The output voltage of potentiometer, V_{pot} is given by: $V_{pot} = \theta_a * K_{pot}$. The potentiometer constant, K_{pot} equals the ratio of the voltage change to the corresponding angle change,

in our case, applied voltage is of 0 to 12 volts corresponds linearly to an output arm angle of 0 to 180, this gives:

$$K_{pot} = \frac{(\text{Voltage change})}{(\text{Degree change})} = \frac{(12 - 0)}{(180 - 0)} = 0.0667 \text{ V / degree}$$

Dynamics of tachometer can be represented using the following equation:

$$V_{out}(t) = K_{tac} * \frac{d\theta_m(t)}{dt} \Rightarrow V_{out}(t) = K_{tac} * \omega_m$$

The transfer function of the tachometer is given by: $K_{tac} = V_{out}(s)/\omega_m(s)$. In case the PMDC , is to be used to drive output shaft with linear velocity of , say, 0.5 m/s, the angular speed is obtained as: ω=V/r = 0.5/ 0.075 = 6.6667 rad/s. Substituting values, we have Tachometer constant, given by: $K_{tac} = 12/6.6667 = 1.8$.

V.I Applying only Proportional controller.

The P-controller amplifies the error and applies a control action proportional to the error to the system. This is useful for improving the response of a stable system and will reduce but never eliminate the steady-state error e_{ss}. Applying Proportional controller with transfer function $G_p(s)$ =K_p =1,will result in overall all closed loop transfer function given by Eq. (4), running m.file ,given by (Farhan A. Salem, 2013) , for values of K_p =[1,5,8] will return angle step response shown in Fig.6(b). In Fig.6(a) shown root locus and bode plots. Analyzing response for K_p =1, shows that applying only proportional controller will not result in meeting all desired response requirements, where the DC motor will reach the desired steady-state angle with finite offset-error e_{ss} = 0.0625, settling time, T_S = 92-120 sec that is very large, reducing the rise time T_R also increasing the gain K_p will result in reducing settling time, increasing overshoot and leads to instability problems for large values. Modifying and running Simulink model shown in Fig.5 to include only P-controller with K_p =1, will result in similar response shown in Fig.6(b),

(4)

$$T_{Overall\_P}(s) = \frac{0.023}{\left[0.0046s^3 + 0.0269s^2 + 0.03053s + 0.001534 \right]} = \frac{0.023}{(s + 4.3344)(s + 1.4608)(s + 0.0527)}$$
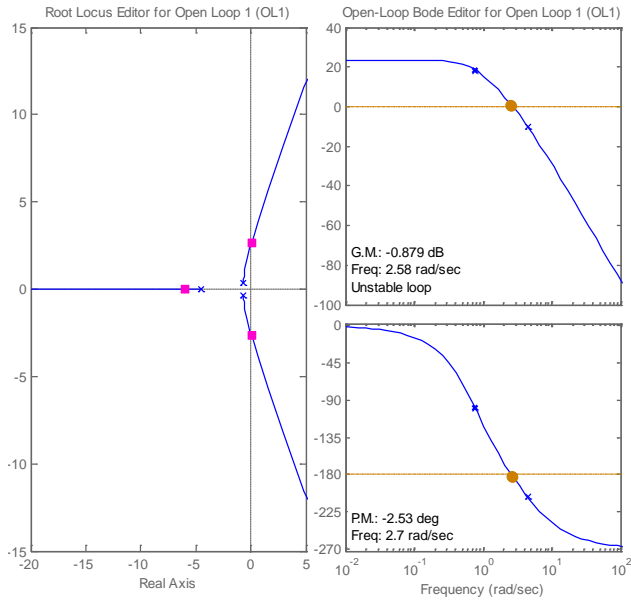
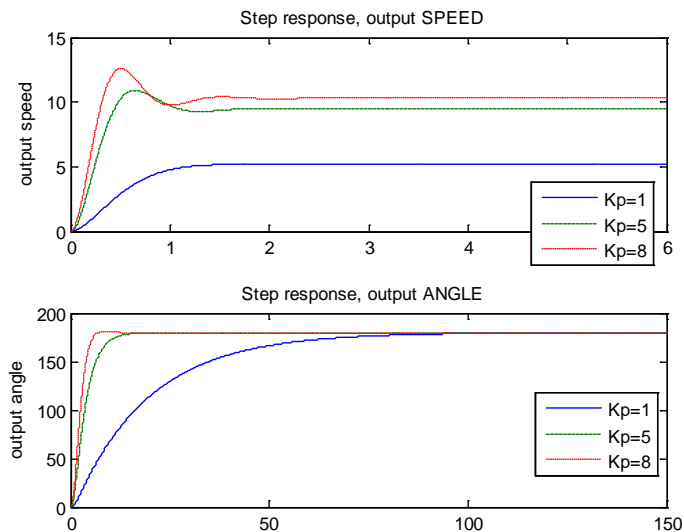Fig.6(a) Root locus and bode plot of the PMDC system with $K_p = 1$.

Fig.6(b) The closed loop step response of the PMDC motor output angular position θ and ω

V.II Applying only Derivative controller:

The D-controller output is proportional to the rate of change of the error (differentiating the error); D- control action only works in transient mode and provides an improved fast response, it has the effect of increasing the stability of the system and reducing the overshoot. The D-controller transfer function is given by $G_D(s) = K_D s$,   meanwhile D-controller should never be used alone, lets analyze its effect; running m.file given by (Farhan A. Salem, 2013) for values of derivative gain $K_D = [20,25,30]$ will return

angle response shown in Fig.7(b), in Fig.7(a) shown root locus and bode plots, analyzing response for K_D =30, shows that by applying only D-controller will not result in meeting all desired response requirements where when a step input voltage is applied to the system, the system stability is improved, the settling time is reduced (T_s=2.3 second), but the DC motor will reach steady-state angle with error, e_{ss} = 0.0632, that is not acceptable, Modifying and running Simulink model shown in Fig.5 to include only D-controller with K_D =30, will result in similar response shown in Fig.7(b), and overall all closed loop transfer function given by Eq. (5)

$$T_{Overall\_D}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.69s}{\left[0.0046s^3 + 0.02s^2 + 0.04655s\right]}$$
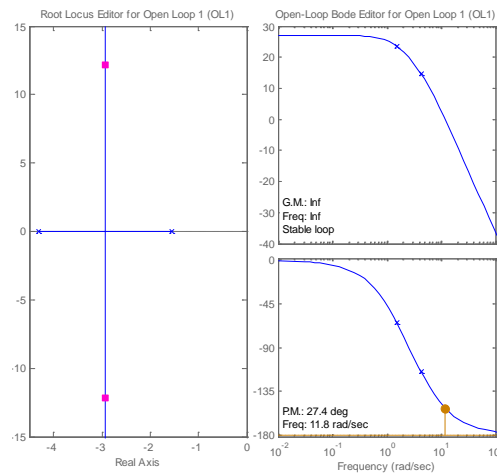
(5)

Fig.7(a) Root locus and bode plot of PMDC system with K_D =30.
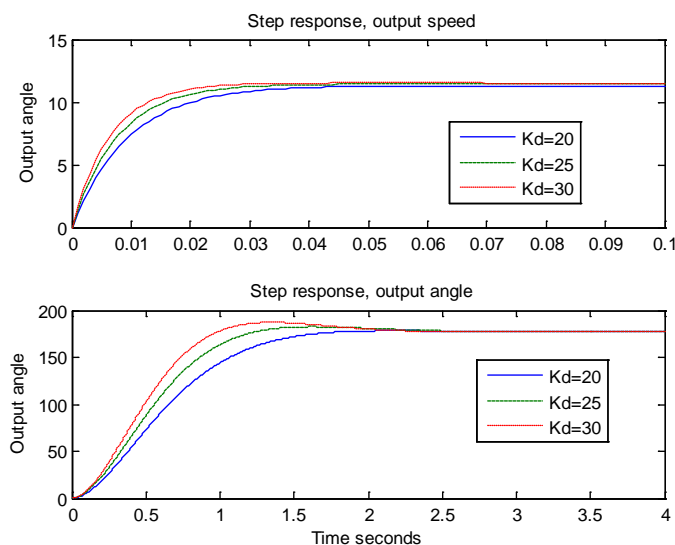

Fig.7(b) The closed loop step response of the DC motor output angular position θ and ω

VIII. Applying only Integral controller:

The I-controller integrates the error and eliminate it, has the unique ability to return the process back to the exact set-point, but it has a disadvantage based on fact that integration is a continual summing, integration of error over time means summing up the complete controller error history up to the present time, this means I-controller can initially allow a large deviation at the instant the error is produced allowing the oscillatory and slow transient behavior that can lead to system instability and cyclic operation. Applying I-controller controller with transfer function $G_I(s)=K_I/s$, will increase system type by one, and   overall all closed loop transfer function given by Eq. (6) . Running m.file ,given by (Farhan A. Salem, 2013), for values of $K_I =[0.01,0.1,3]$ will return angle response shown in Fig.8(b), in Fig.8(a) shown root locus and bode plots, analyzing response for $K_I =30$, shows that by applying only I-controller will not result in meeting all of design requirements, also increasing integral gain, K_I cause instabilities and unsterilized the response. Modifying and running Simulink model shown in Fig.5 to include only I-controller with $K_I =30$, will result in similar response shown in Fig.8(b).

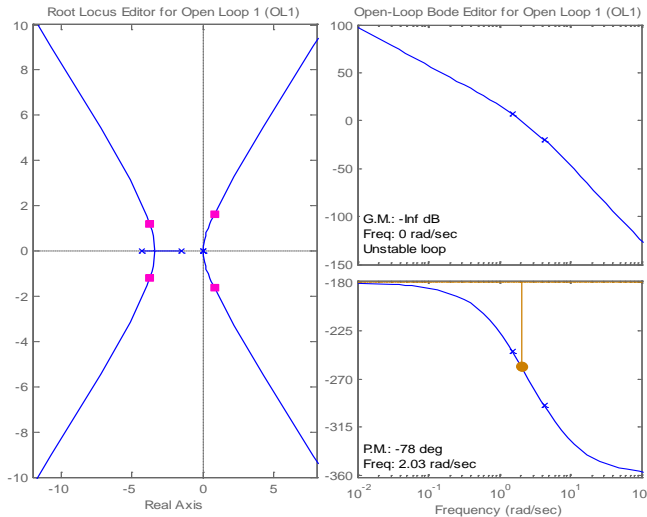$$T_{Overall\_I}(s) = \frac{0.69}{\left[0.0046s^4 + 0.02s^3 + 0.04655s^2 + 0.004602\right]}$$

(6)

Fig.8(a) Root locus and bode plot of PMDC system with $K_I =3$.
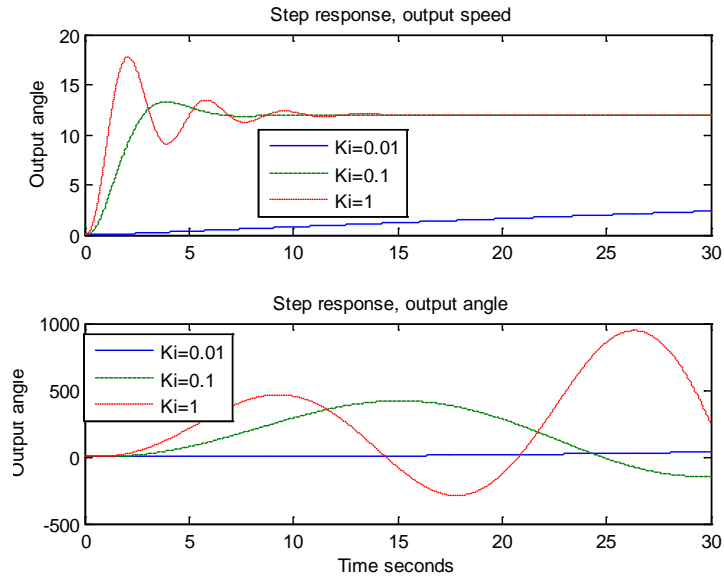
Fig.8(b) The closed loop step response of the DC motor output angular position θ and ω

IX. Applying Proportional Derivative controller:

The output signal of PD-Controller controller is equal to the sum of two signals, resulting from: multiplying the error by a proportional gain K_p plus, differentiating and multiplying the error by derivative gain K_d:

$$m_{PD}(t) = K_P e(t) + K_D \frac{de(t)}{dt}$$

Taking Laplace transform and rearranging, PD-controller transfer function is given by:

$$G_{PD} = K_p + K_D s = K_D \left(s + \frac{K_P}{K_D} \right) = K_D \left(s + Z_o \right)$$

The PD-controller is equivalent to the addition of a simple zero at $Z_o = K_P/K_D$ to the controlled system, resulting in more stable system and improving the system transient response; (T↓, Ts↓, Tp↓, ζ↓, SO%↓) by shifting root locus to the left. Applying PD-controller will result in overall all closed loop transfer function given by Eq. (7). Running m.file, given by (Farhan A. Salem, 2013), for values of K_p=[1,1.5,1.8703] and K_D=[1,2,3.336] will return angle response shown in Fig.9(b). in Fig.9(a) shown root locus and bode plots ,analyzing response for K_p =1.8703, K_D=3.336, shows that applying only PD-controller will not result in satisfactory meeting all of desired response requirements, where when a step input voltage is applied to the system, the system stability is improved, the settling time T_s is reduced but still is large, DC motor will reach steady-state angle with zero error, (e_{ss} =0), Modifying and running simulink model shown in Fig.5 to include only

PD-controller with K_p =1.8703, K_D=3.336 ,will result in similar response shown in Fig.9(b).

$$T_{Overall\_PD}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.07673s + 0.4302}{\left[0.0046s^3 + 0.0269s^2 + 0.3565s + 0.002869\right]}$$
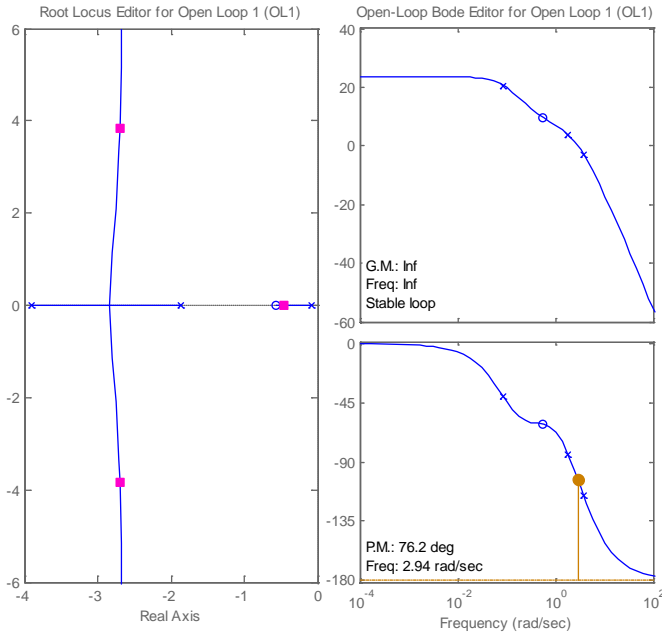
(7)

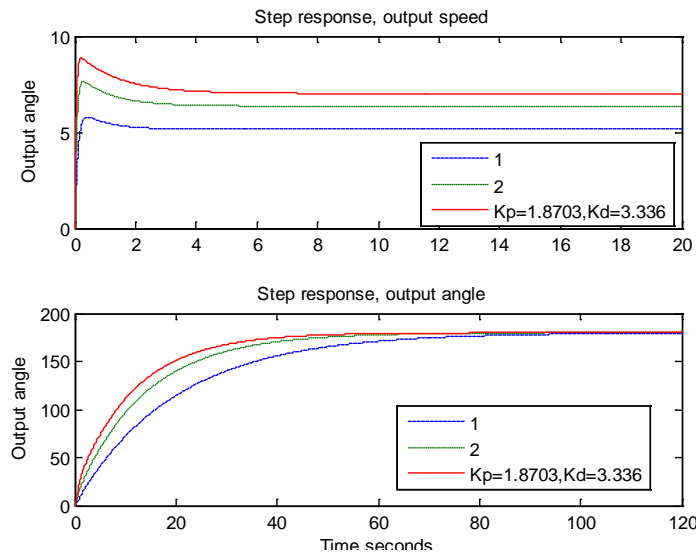Fig.9(a) Root locus and bode plot of the PMDC with $K_P = K_I$ =10.

Fig.9(b) The closed loop step response of the DC motor output angular position θ and ω

X. Applying Lead compensator.

Lead compensator is a soft approximation of PD-controller, PD controller transfer function is given by $G_{PD}(s) = K_P + K_D s$, The PD controller is not physically implementable, since it is not proper, and it would differentiate high frequency noise, thereby producing large swings in output. To avoid this, PD-controller is approximated to lead controller of the following form (Shreyas Sundaram, 2012):

$$G_{PD}(s) \approx G_{Lead}(s) = K_P + K_D \frac{Ps}{s+P}$$

Manipulating and rearranging gives:

$$G_{Lead}(s) = K_P + K_D \frac{Ps}{s+P} = \frac{K_P(s+P) + K_D Ps}{s+P}$$

$$G_{Lead}(s) = \frac{(K_P + K_D P)s + K_P P}{s+P} = (K_P + K_D P)\frac{s + \left[\dfrac{K_P P}{K_P + K_D P} \right]}{s+P}$$

Now, let $\quad K_C = K_P + K_D P \quad$,and $\quad Z = \left[\dfrac{K_P P}{K_P + K_D P} \right]$, we obtain the following

approximated Controller transfer function of PD controller , also called lead compensator is given by:

$$G_{Lead}(s) = K_C \frac{s+Z}{s+P}$$

Where : $Z_o < P_o$, The larger the value of P, the better the lead controller approximates PD control,

Applying Lead compensator will result in overall all closed loop transfer function given by Eq. (8), shifting the locus to the left , improving the transient response, raising bandwidth, speeding up the response, increasing the stability of the system, and reducing steady state error. running m.file ,given by (Farhan A. Salem, 2013) , for values of lead compensator gain K_c=[34, 100, 300], and $Z_o = 1.5410$, P_o= 15.410 will return angle response shown in Fig.10(b), in Fig.10(a) shown root locus and bode plots. Analyzing response, shows that a controller consisting of Lead-controller can not be used to meet all desired response requirements, where when a step input voltage is applied to the system, the system stability is increased, the settling time T_s is reduced (T_s=3.5 sec), and a reduction in steady state error, but for large gain values an overshoot, oscillation and steady state error are introduced. Modifying and running Simulink model shown in Fig.5 to include only Lead-compensator with chosen values will result in similar response shown in Fig.10(b),

$$T_{Overall\_Lead}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{10.35s + 15.95}{\left[0.0046s^3 + 0.09779s^2 + 0.4451s + 1.161 + 1.064 \right]}$$
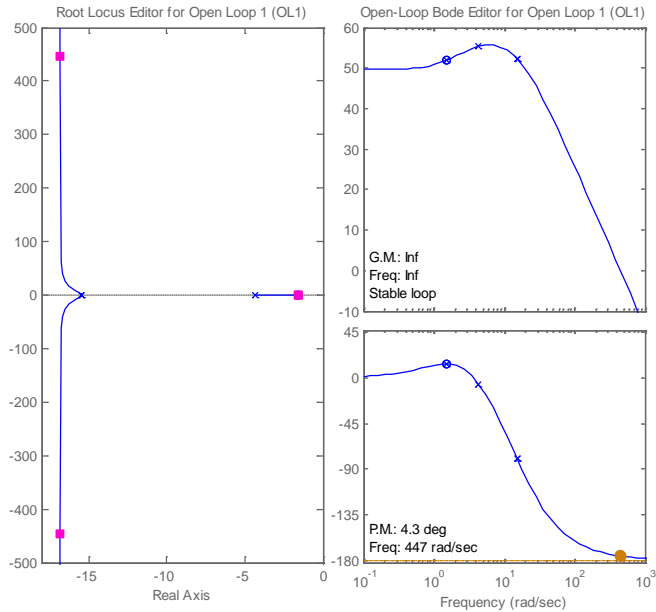
(8)

Fig.10(a) The root locus and bode plot of the controlled system with K_C= 450,1.8703, Z_o = 1.5410, P_o= 15.410
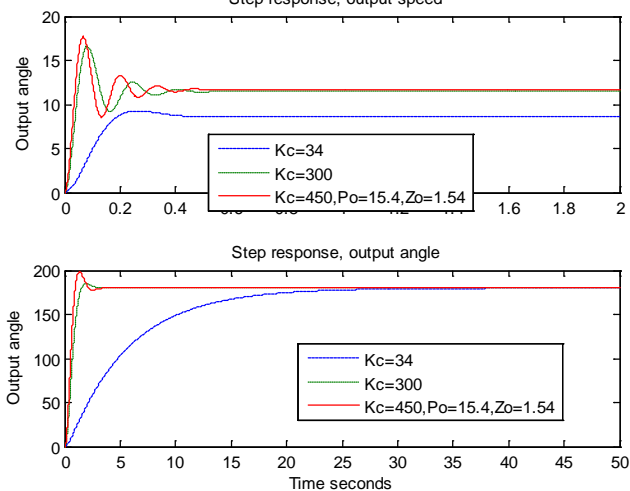

Fig.10(b) The closed loop step response of the DC motor output angular position θ

XI. Applying Lead integral compensator.

Applying Lead integral compensator with transfer function given by Eq. (9) will result in overall all closed loop transfer function given by Eq. (10). running m.file ,given by (Farhan A. Salem, 2013) , for values of compensator gain K_C =[4 7 30], Z_o = 0.15410, P_o= 1.5410 will return angle response shown in Fig.11(b), in Fig.11(a) shown root locus and bode plots, analyzing response for K_C= 4,1.8703, Z_o = 0.15410, P_o= 1.5410, shows that a controller consisting of Lead-Integral controller can not be used to meet all

desired response requirements, where it can be noted that the lead Integral compensator will eliminate the steady state error (Ess = 0), but the transient response settling time (Ts=70 sec) ,and overshoot are large, a also the system is subject to instability problems as the controller gain increased. Modifying and running Simulink model shown in Fig.5 to include only Lead-Integral -controller with chosen values, will result in similar response shown in Fig.11(b),

$$G_{Lead\_Integral}(s) = K_C \frac{1}{s} \frac{(s+Z_o)}{(s+P_o)} = K_C \frac{(s+Z_o)}{s(s+P_o)}$$

(9)

$$T_{Overall\_Lead\_Integral}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.69s + 0.1063}{\left[0.0046s^5 + 0.03399s^4 + 0.07198s^3 + 0.04705s^2 + 0.04602s + 0.007092\right]}$$
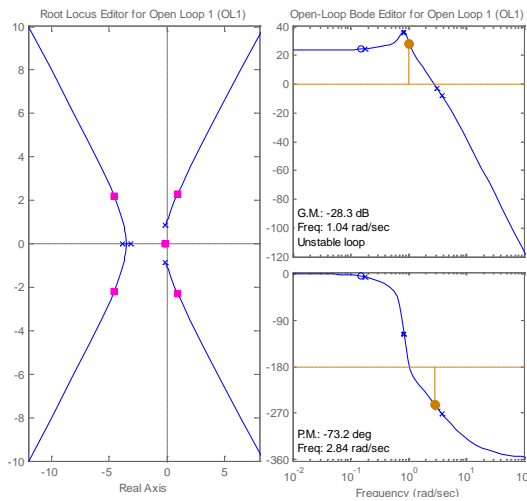
(10)

Fig.11(a) Root locus and bode plot of the controlled system with K_C= 4,1.8703, Z_o = 0.15410, P_o= 1.5410
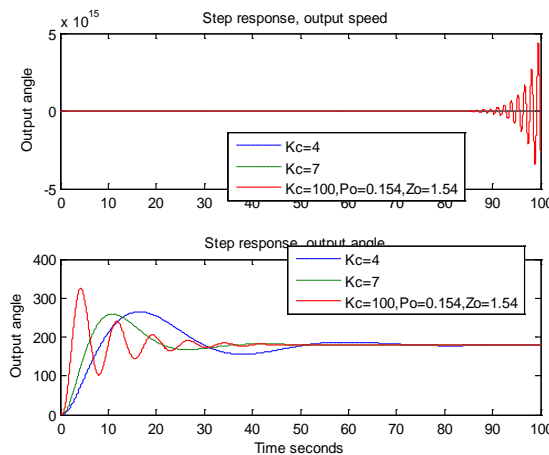

Fig.11(b) The closed loop step response of the DC motor output angular position θ ,

XII. Applying Proportional Integral controller.

Applying PI-controller with transfer function given by Eq. (11) will result in overall all closed loop transfer function given by Eq. (12). PI controller represents a pole located at the origin and a stable zero placed near the pole, at $Z_o = - K_I/ K_P$, resulting in drastically improving steady state error due to the fact that the feedback control system type is increased by one . running m.file ,given by   (Farhan A. Salem, 2013)   for values of compensator $K_I = 0.01$, $K_P = 9$, $Z_o = 0.0011$ will return angle response shown in Fig.12(b), in Fig.12(a) shown root locus and bode plots, analyzing response shows that a controller consisting of PI-controller can not be used to meet all desired response requirements, where it can be noted that the PI controller will eliminate the steady state error ($e_{ss} = 0$), but the transient response settling time ($T_s = 11$ seconds) is still large. Modifying and running simulink model shown in Fig.5 to include only PI -controller with chosen compensators values, will return similar response shown in Fig.12(b):

$$G_{PI}(s) = K_P + \frac{K_I}{s} = \frac{(K_P s + K_I)}{s} = \left[\frac{K_P \left(s + \frac{K_I}{K_P} \right)}{s} \right] = \frac{K_P (s + Z_o)}{s} \tag{11}$$

$$T_{Overall\_PI}(s) == \frac{0.207s + 0.0023}{\left[0.0046s^4 + 0.0269s^3 + 0.03053s^2 + 0.01381s + 1.534\text{e-}5 \right]} \tag{12}$$
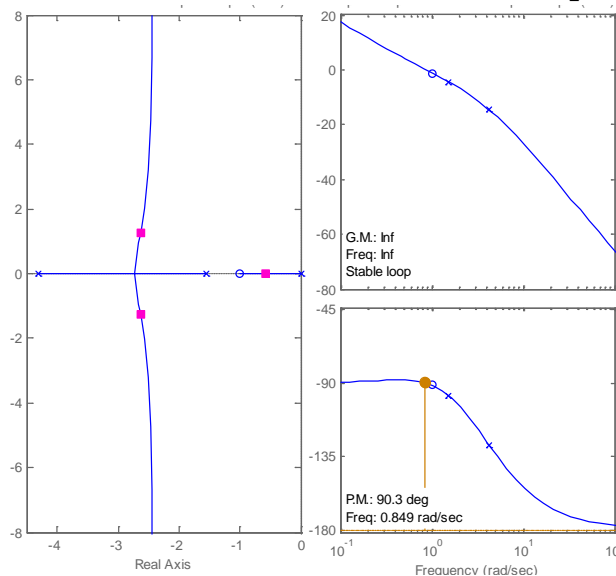

Fig.12(a)   the root locus and bode plot of the controlled system with $K_I = 0.01$ $K_P = 9$, $Z_o = 0.0011$
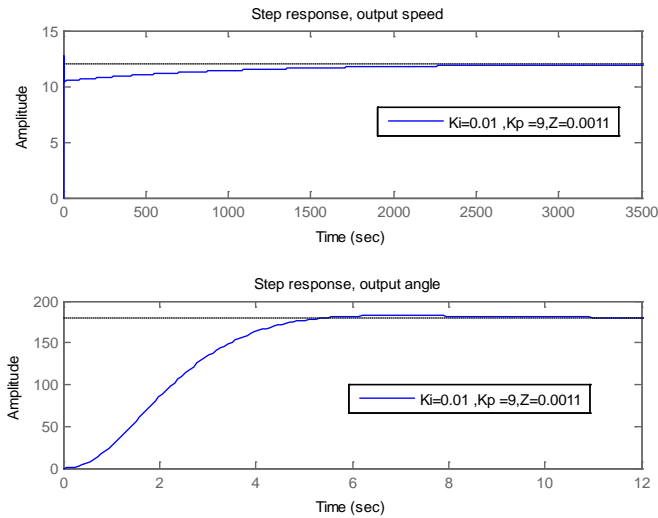
Fig.12(b) The closed loop step response of the DC motor output angular position θ,

XIII. Applying lag compensator:

Lag compensator is a soft approximation of PI-controller and given by Eq. (13), where PI-controller by it self is unstable, pure integrators not easy to physically implement, the lag compensator can be built with passive components only (resistors and capacitors), and thus is easily implemented in analog control systems.

$$G_{PI}(s) \approx \approx G_{lag}(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s} = K_P \frac{\left(s + \frac{K_I}{K_P}\right)}{s} \qquad (13)$$

Where: $Z_o > P_o$ is a small number, Since PI controller by it self is unstable, we approximate the PI controller by introducing value of P_o that is not zero but near zero; the smaller we make P_o, the better this controller approximates the PI controller, the approximation of PI controller will have the form given by Eq. (14):

$$G_{lag}(s) = K_c \frac{(s + Z_o)}{(s + P_O)} \qquad (14)$$

Applying lag-controller, will result in overall all closed loop transfer function given by Eq. (15), in lag compensator design the Z_o and P_o are placed very closed together, and the compenation is closed relatevly to the origin to result in increased gain, and correspondingly reduced steady state error by factor Zo/Po , and a decrease in undamped natural frequency ω_n, and correspondingly an increase in settling time, general system response improvement and the locus slightly shifted to the right. running m.file, given by (Farhan A. Salem, 2013), for values of lag compensator gain K_c = [4 7 10] , Z_o = 0.13410, P_o= 0.013410 , will return angle response shown in Fig.13(b), in Fig.13(a) shown root locus and bode plots, analyzing response,

shows that a controller consisting of lag-controller can not be used to meet all desired response requirements, where when a step input voltage is applied to the system, the system stability is increased, the settling time is reduced ($T_s = 14$ sec) , and reduce in steady state error, but for large gain values an overshoot and transient oscillation are introduced. Modifying and running Simulink model shown in Fig.5 to include only lag-controller with chosen values will result in similar response shown in Fig.13(b),

$$T_{lag}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.69s + 0.09253}{\left[0.0046s^4 + 0.0269s^3 + 0.03089s^2 + 0.04643s + 10.006172 \right]} \qquad (15)$$
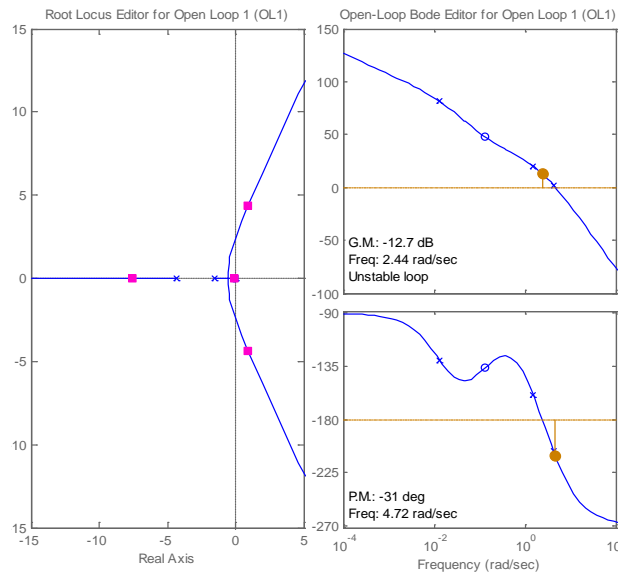

Fig.13(a)   the root locus and bode plot of the controlled system with K_c 10 , Z_o = 0.13410, P_o= 0.013410
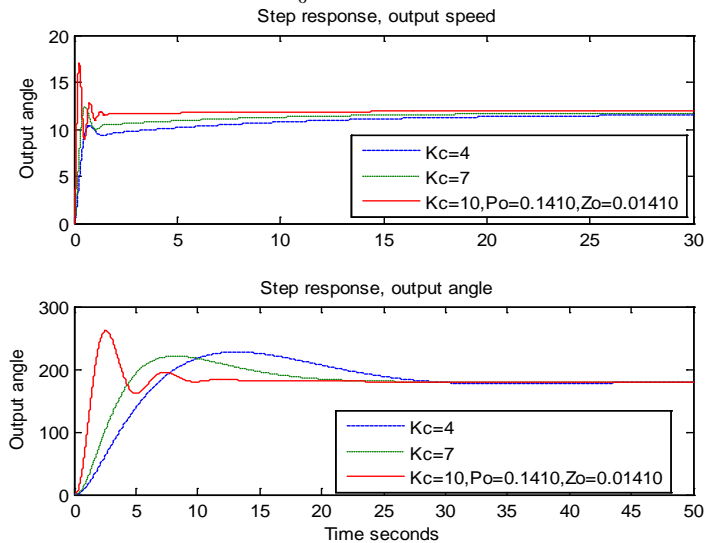

Fig.13(b) The closed loop step response of the DC motor output angular position θ.

XIV. Applying Proportional Integral Derivative PID controller.

PID controllers are commonly used to regulate the time-domain behavior of many different types of dynamic plants (Jamal A. Mohammed, 2011). PID controller is combining all three controllers, P, PI , and PD results in the PID controller, when three controllers combined we get a system that responds quickly to change (derivative), generally track required positions (proportional), and will eventually reduce errors (integral) and is used to improve the dynamic response as well as to reduce or eliminate the steady-state error. Different characteristics of the motor response (steady-state error, peak overshoot, rise time, etc.) are controlled by selection of the three gains that modify the PID controller dynamics (B. Shah, 2004). the PID transfer function is given by Eq. (16)   ,it can be rewritten in terms of derivative time, T_D and integral time T_I to have the form given by Eq. (17). The PID controller has two zeros and one pole at the origin, therefore it transfer function can be rewritten in the form given by Eq. (18), where: One zero ($-Z_{PD}$) is designed as the PD controller zero=K_P/ K_D. The other zero ($-Z_{PI}$) is designed as the PI controller zero =K_I/ K_P , typically taking a value of (-0.1) and the pole at the origin are designed as the PI controller.

$$G_{PID} = K_P + \frac{K_I}{s} + K_D = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D \left[s^2 + \frac{K_P}{K_D} s + \frac{K_I}{K_D} \right]}{S} \qquad (16)$$

$$G_{PID} = K_P \left(1 + \frac{1}{T_I s} + T_D s \right) \qquad (17)$$

$$G_{PID} = \frac{K_D \left(s + Z_{PI} \right) \left(s + Z_{PD} \right)}{s} \qquad (18)$$

The given PMDC motor open loop system transfer function given by (1), and correspondingly the PID controller transfer function is given by Eq. (19), all will result in (20)

$$G_{PID} = \frac{K_D \left(s + 4.3069 \right) \left(s + 1.5410 \right)}{s} \qquad (19)$$

Running m.file ,given by   (Farhan A. Salem, 2013),   for values of compensator gain K_D = [2 2.5 3]; Z_{PI} = 0.15410, Z_{PD} = 4.3069 , will return angle response shown in Fig.14(b), in Fig.14(a) shown root locus and bode plots, analyzing response, shows that when a step input voltage is applied to the system, the system stability is increased, the settling time Ts is decreased and steady state error is eliminated, by proper selection the of the three PID gains, different characteristics of the motor response are controlled. Also running m.file by (Farhan A. Salem, 2013)with K_D = [20,30, 37.783]; K_P =56.5860, K_I =1.2310   will return angle response that meets all required design specifications shown in Fig.14(c),(d).

$$T_{Overall\_PID}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{0.869\,s^2 + 1.301s + 0.02831}{\left[0.0046s^4 + 0.0269s^3 + 0.08849s^2 + 0.08681s + 0.001888\right]} \quad (20)$$
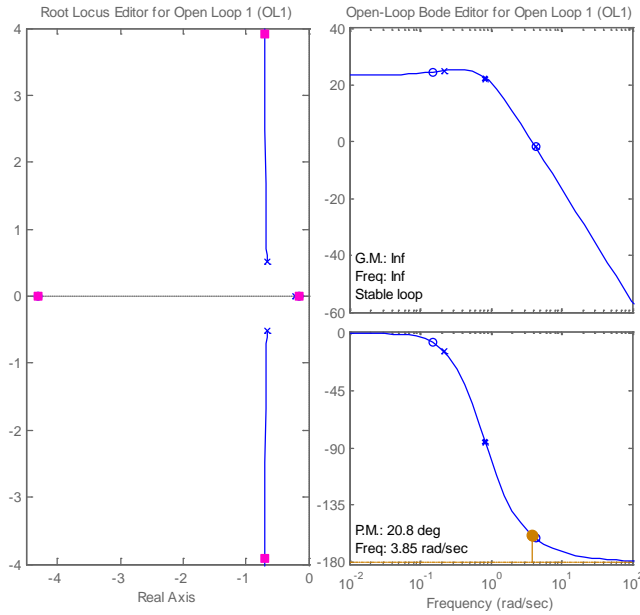

Fig.14(a) Root locus and bode plot with K_D = [2 2.5 3]; Z_{PI} = 0.15410, Z_{PD} = 4.3069
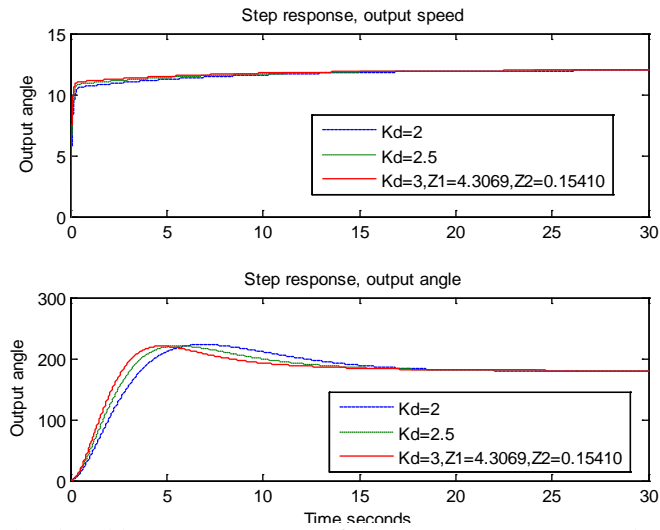

Fig.14(b) The closed loop step response of the DC motor output angular position θ.
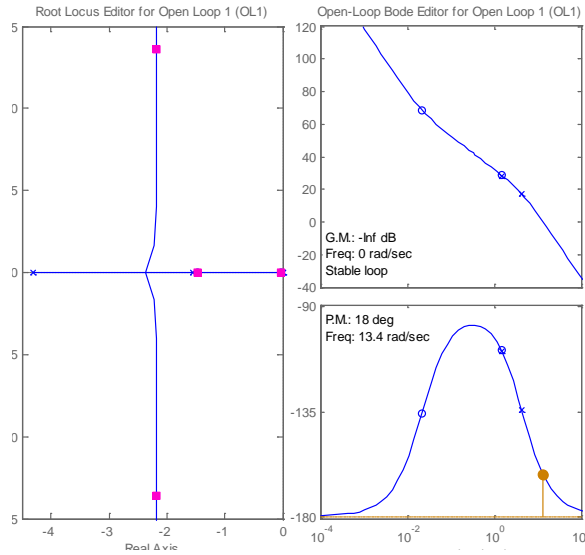
Fig.14(c) Root locus and bode plot with K_D = [20,30, 37.783]; K_P =56.5860, K_I =1.2310
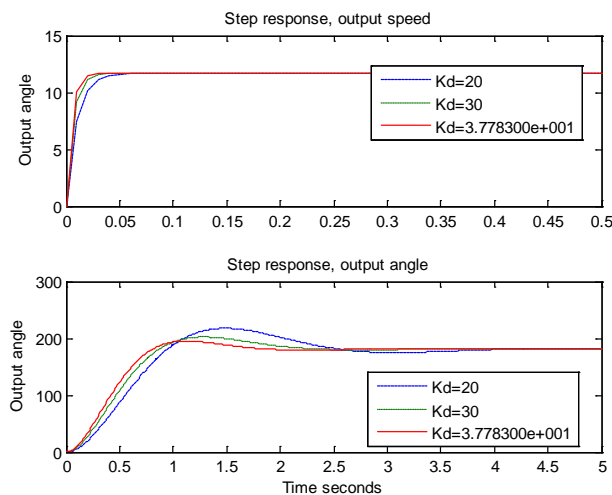

Fig.14(d) The closed loop step response of the DC motor output angular position θ

XV. Applying internal model controller; IMC tuned PID Controller.

The internal model principle states that the control can be achieved only if the control system encapsulates, either implicitly or explicitly, some representation of the process to be controlled, the internal model control, IMC, relies on this principle. Applying IMC will result in improving system response, mainly reducing overshoot, rise time and settling time and disturbance rejection. The general block diagram structure of internal model control is shown in Fig.15(a), where controller, G_c(s) is used to control the actual process G_p(s), and the process model G_{pm}(s), and D(s) is unknown disturbance affecting the process. Designing of IMC controller is simple and

232

easy; first by factoring process model $G_{pm}(s)$ into two terms invertible and noninvertible, where the noninvertible terms are those terms of $G_{pm}(s)$ that if inverted will lead to instability and reliability, gives:

$$G_{pm}(s) = G_{pm}^{inv}(s) * G_{pm}^{non-inv}(s) \qquad (21)$$

By setting controller transfer function, $G_c(s)$ to be the inverse of the invertible term of process model, we have:

$$G_c(s) = \frac{1}{G_{pm}^{inv}(s)}$$

To obtain a practical IMC controller, $G_c(s)$ is set to be in series with a transfer function of the low pass filter, $G_{filter}(s)$, and the practical IMC controller is given by multiplication to give:

$$G_{IMC}(s) = G_c(s) * G_{filter}(s) = \frac{G_{filter}(s)}{G_{pm}^{inv}(s)}$$

The simplest form of the filter is given by (28), where n is the order of the filter and is chosen to result in proper and stable $G_{IMC}(s)$, and τ is the filter parameter that has an inverse relationship with the speed of the close loop response, also low pass filter will help to minimize the discrepancies between the process and model at high frequency:

$$(22)$$

$$G_{filter}(s) = \frac{1}{(\tau s + 1)^n}$$

The structure shown in Fig. 15(a) can be simplified and reduced to conventional closed loop structure shown in Fig. 15(b)(c), therefore used to generate settings for PID controller, given by:

$$G_{PID}(s) = \frac{G_{IMC}(s)}{1 - G_{IMC}(s)G_{pm}(s)} = \frac{G_{filter}(s)}{G_{pm}^{inv}(s)\left(1 - G_{pm}^{non-inv}(s)G_{filter}(s)\right)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$(23)$$

Substitute PMDC motor model equation given by (11) in (29), the IMC- PID tuning parameters K_P, K_I, and K_D are obtained. The responses and comparison are shown in Fig.15(d)(e)
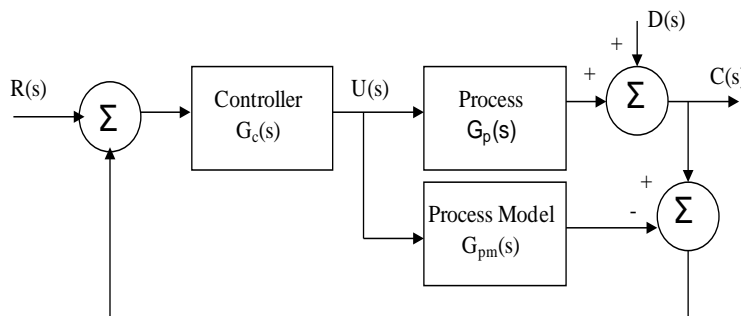

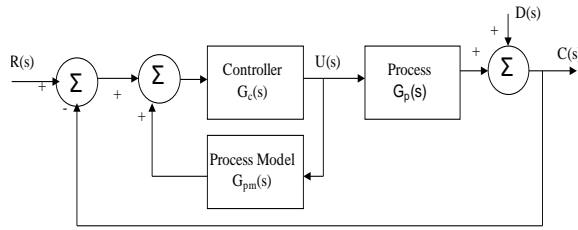Fig. 15(a) Basic structure of internal model controller.
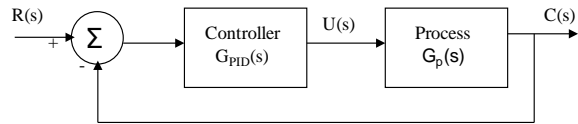
Fig. 15(b) Equivalent simplification.

Fig. 15(c) IMC tuned PID design.
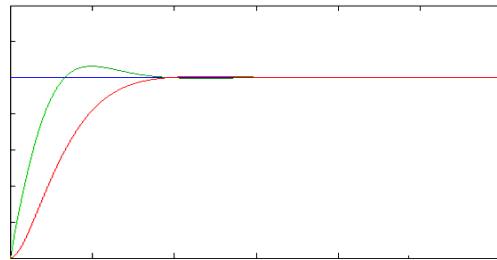
Fig. 15(d) Response for IMC compared with PID
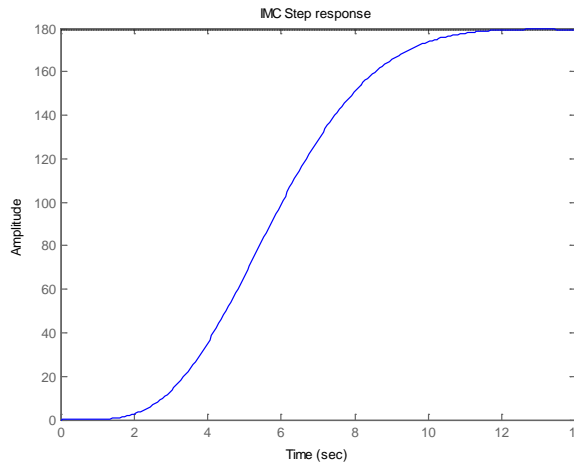
Fig. 15(e) Step response for IMC

XVI. Applying Proportional Derivative controller with deadbeat response

Deadbeat response means the response that proceeds rapidly to the desired level and holds at that level with minimal overshoot (Richard C. Dorf, 2001). The characteristics of deadbeat response include; Zero steady state error, Fast response, (short rise time and settling time) , percent overshoot greater or equal to 0.1%   and less or equal 2 %   and minimal

undershoot, less than $\pm 2\%$ error band. PD-controller transfer function is given by Eq.(24):

$$G_{PD} = K_p + K_D s = K_D \left(s + \frac{K_P}{K_D} \right) = K_D \left(s + Z_o \right) \tag{24}$$

Using simplified first order form of PMDC motor transfer function in terms of output angle given by Eq. (25):

$$G_{angle}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{\dfrac{K_t}{R_a J_m}}{s \left[s + \dfrac{1}{J_m} \left(b_m + \dfrac{K_t K_b}{R_a} \right) \right]} \tag{25}$$

The system forward transfer function is given by:

$$G_{forward}(s) = \frac{\dfrac{(K_p + K_D s) K_t}{R_a J_m}}{s^2 + \left(\dfrac{b_m}{J_m} + \dfrac{K_t K_b}{J_m R_a} \right) s}$$

The system overall closed loop transfer function transfer function is given

by:
$$T(s) = \frac{\dfrac{(K_p + K_D s) K_t}{R_a J_m}}{s^2 + \left(\dfrac{b_m}{J_m} + \dfrac{K_t K_b}{J_m R_a} + \dfrac{K_D K_t K_{pot}}{R_a J_m} \right) s + \dfrac{K_p K_t K_{pot}}{R_a J_m}}$$

Referring to (Richard C. Dorf, 2001), the controller gains K_P and K_D depend on the physical parameters of the actuator drives, to determine gains that yield optimal deadbeat response, the overall closed loop transfer function $T(s)$ is compared with standard second order transfer function given by Eq. (26), and knowing that parameters α and ω_n are known coefficients of system with deadbeat response given by (Richard C. Dorf, 2001), $\alpha = 1.82$ and $\omega_n T_n = 4.82$, $T_n = 2$ and gives the following:

$\omega_n T_n = 4.82$, $\omega_n = 4.82/2 = 2.41$

$$G(s) = \frac{\omega_n^2}{s^3 + \alpha \omega_n s^2 + \omega_n^2} \tag{26}$$

Equating and comparing the actual and desired characteristic equations, gives:

$$s^3 + \alpha \omega_n s^2 + \omega_n^2 = s^2 + \left(\frac{b_m}{J_m} + \frac{K_t K_b}{J_m R_a} + \frac{K_D K_t K_{pot}}{R_a J_m} \right) s + \frac{K_p K_t K_{pot}}{R_a J_m} \tag{27}$$

$s^3 + 10.5707 s^2 + 33.7340 = s^2 + \left(1.5265 + 0.000030667 K_D \right) s + 0.000030667 K_p$

$K_p = 850430$ and $K_D = 294920$. The PD controller with deadbeat response is shown in Fig.16.
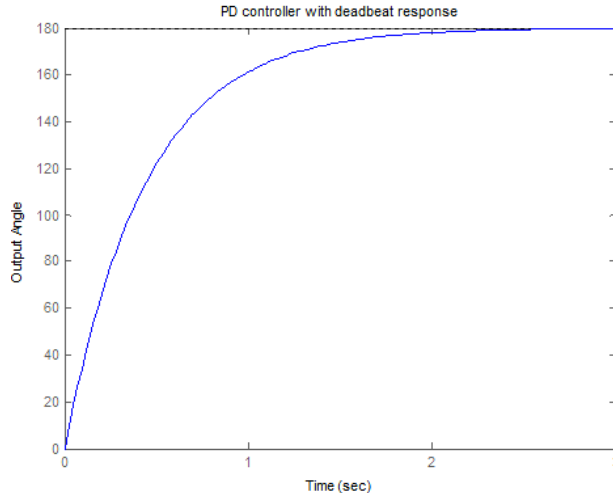
Fig.16 PD with deadbeat response.

XVII. Conclusion

There are many motor control strategies that may be more or less appropriate to a specific type of application each has its advantages and disadvantages. The designer must select the best one for specific application, the current work introduce, for comparison and analysis purposes, comprehensive study of the modeling, analysis and position control design strategies of a PMDC motor, that can be used in Mechatronics output position applications, to suggest the best control strategy to control the output angular position, θ of PMDC motor, corresponding to applied input voltage V_{in} and verification by MATLAB/Simulink.

Analysis and comparison of applied different control strategies show that each control strategy has its advantages and disadvantages, it was found that PID controller enables designer to satisfy, almostly, all required design specifications, providing almostly all the desired response. It found that using a PID controller with, K_P =56.5860, K_I =1.2310 ,K_D = 37.783, all of the design requirements, almostly, was satisfied, the PMDC motor used, reached the desired output angle smoothly and within a desired period of time. It has observed that both PMDC motor and PID transfer functions, control have a large influence upon the response of the system. To achieve a fast response to a step command with minimal overshoot and zero steady state error, so called PD controller with deadbeat response, enables designer to satisfy most required design specifications smoothly and within a desired period of time and can be applied to meet the desired specification., applying internal model controller result in improving system's both dynamic and steady state performances, reducing overshoot, rise time and settling time, as well as, disturbance rejection.

References:

Ahmad A. Mahfouz ,Mohammed M. K., Farhan A. Salem, Modeling, Simulation and Dynamics Analysis Issues of Electric Motor, for Mechatronics Applications, Using Different Approaches and Verification by MATLAB/Simulink (I). IJISA Vol. 5, No. 5, 39-57 April 2013.

D'Azzo, John Joachim, Houpis, Constantine H, "Linear control system analysis and design: conventional and modern", 1988.

Hedaya Alasooly, "Control of DC motor using different control strategies" global journal of technology and optimization, Vol. 2,pp 21-28, 2011.

M. S. RUSU, and L. Grama, The Design of a DC Motor Speed Controller, Fascicle of Management and Tech. Eng., Vol. VII (XVII), 2008, pp. 1055-1060.

Hedaya alasooly, "PhD thesis: Control and Modeling of Facts devices and Active Filters on Power System", Electrical Energy Department, Czech Technical University, 2003, published in www.lulu.com.

Frank. L. Lewis, "Optimal Control", 1. ed. New York : Wiley, 1986.

The MathWorks (www.mathworks.com), Version V5.2 (R2009b).

Jamal A. Mohammed ,Modeling, Analysis and Speed Control Design Methods of a DC Motor Eng. & Tech. Journal, Vol. 29, No. 1, 2011.

 B. Shah, Field Oriented Control of Step Motors, MSc. Thesis, SVMITB haruch, India, Dec. 2004.

Farhan A. Salem, Electric machine performance analysis, controller selection, design and verification for Mechatronics motion control applications, using new MATLAB built-in function and Simulink model. Submitted to International Journal of Intelligent Systems and Applications (IJISA), 2013.

Shreyas Sundaram, Control Systems, Department of Electrical and Computer Engineering University of Waterloo, 2012.

Richard C. Dorf and Robert H. Bishop. Modern Control Systems. Ninth Edition, Prentice-Hall Inc., New Jersey, 2001.

Farid Golnaraghi Benjamin C.Kuo, Automatic Control Systems, John Wiley and sons INC .2010.

Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini, Feedback Control of Dynamic Systems, 4th ed., Prentice Hall, 2002.

M.Saranya , D.Pamela ' A Real Time IMC Tuned PID Controller for DC Motor 'International Journal of Recent Technology and Engineering (IJRTE) Vol.1, Issue-1, April 2012

K. Anil Naik and P. Srikanth, Stability Enhancement of DC Motor using IMC Tuned PID Controller, international journal of advanced engineering sciences and technologies,2011

Chun Htoo Aung, Khin Thandar Lwin, and Yin Mon Myint, Modeling Motion Control System for Motorized Robot arm using MATLAB, World Academy of Science, Engineering and Technology 42 2008.

APPENDIX I, TABLE- NOMENCLATURE.

Symbol	Quantity	UNIT
V, or Vin	The applied input voltage ,(Motor terminal voltage)	Volte, V
R_a	Armature resistance,(terminal resistance)	Ohm ,Ω
L_a	Armature inductance	H
i_a	Armature current	Ampere, A
K_t	Motor torque constant	N.m/A
K_e	Motor back-electromotive force const.	V/(rad/s)
ω_m	Motor *shaft* angular velocity	rad/s
T_m	Torque produced by the motor	N.m
J_m	Motor armature moment of inertia	kg.m^2
J_{total}	Total inertia=Jm+Jload	kg.m^2
L_a	Armature inductance	Henry , H
b	Viscous damping, *friction coefficient*	N.m/rad.s
e_a ,EMF:	The back electromotive force, EMF =$K_b d\theta/dt$	e_a ,EMF:
θ_m	Motor shaft output angular position	radians
ω_m	Motor shaft output angular speed	rad/sec
$V_R = R*i$	The voltage across the resistor	Voltage
$V_L=Ldi/dt$	The voltage across the inductor	Voltage
T_{load}	*Torque* of the mechanical load	T_{load}
Tα	*Torque* du to rotational acceleration	Tα
Tω	*Torque* du to rotational *velocity*	Tω
T_{EMF}	The electromagnetic torque.	TEMF