

# Risk-Based Asset Allocation in Factor Investing: Exploring the Inverse Factor Volatility Strategy

*Claudia Perez, MA* Shanghai University, China

Doi:10.19044/esj.2024.v20n19p1

Submitted: 24 June 2024 Accepted: 22 July 2024 Published: 31 July 2024 Copyright 2024 Author(s) Under Creative Commons CC-BY 4.0 OPEN ACCESS

Cite As:

Perez C. (2024). Caractérisation phytosociologique des anciens sites de traitements sylvicoles et de la réserve naturelle dans le parc national du Banco (Abidjan-Côte d'Ivoire. European Scientific Journal, ESJ, 20 (19), 1. <u>https://doi.org/10.19044/esj.2024.v20n19p1</u>

### Abstract

This study evaluates the effectiveness of the Inverse Factor Volatility strategy within the context of factor investing, comparing its performance to the conventional Risk Parity strategy. Using quantitative techniques, including portfolio construction and performance metrics analysis, this research employs data from five individual equities spanning the years 2000 to 2022. The methodology involves constructing portfolios based on Inverse Factor Volatility and Risk Parity principles and analyzing performance metrics, including mean returns, risk-adjusted returns, and drawdowns. The findings indicate that, compared to Risk Parity, Inverse Factor Volatility offers superior drawdowns, risk-adjusted returns, and mean returns. These results suggest that Inverse Factor Volatility may be a more effective strategy for portfolio management and could represent an advancement over traditional factor investing methods. The conclusions of this study hold significant implications for portfolio managers seeking to optimize their investment strategies.

**Keywords:** Quantitative Finance, Asset Allocation, Investment Performance, Risk Management, Portfolio Optimization

## Introduction

Factor investing is central to contemporary portfolio management, presenting a methodical approach aimed at enhancing returns and diversification. This strategy involves fundamental characteristics known as factors—such as market size, value, and momentum—that significantly affect

asset returns. By concentrating on these factors, which have been extensively studied and documented in seminal research by Fama and French, portfolio managers strive to capture superior returns without proportionately increasing investment risk. For instance, Bessler et al. (2021) demonstrated that factor portfolios outperform sector portfolios over long-term horizons, providing higher returns with lower risk.

As the investment landscape evolves, factor investing strategies have also changed, giving rise to the Risk Parity (RP) strategy. Unlike traditional capital allocation approaches, RP strategies aim to generate a balanced risk contribution from each asset in a portfolio. The rationale is straightforward: by reducing the capital invested in higher-risk assets, the portfolio's overall vulnerability to market downturns can be mitigated. However, the Inverse Factor Volatility (IFV) strategy offers a novel contrast to RP. This strategy suggests that assets with lower volatility are predisposed to higher riskadjusted returns—a principle known as the volatility anomaly. According to Shimizu and Shiohama (2020), IFV portfolios perform better than marketcapitalization-weighted portfolios due to their ability to produce greater riskadjusted returns, streamline risk management, and demonstrate global applicability. Thus, this study aims to evaluate the IFV method against the established RP strategy within the realm of factor investing. Therefore, the research seeks to determine whether the IFV strategy offers superior total returns and risk-adjusted returns compared to the RP strategy, and how their risk profiles, including volatility and drawdowns, differ.

Moreover, this study provides portfolio managers with tangible insights into the benefits of incorporating the IFV strategy into their investment decisions. This investigation employs a combination of historical data analysis from 2000 to 2022—a timeframe of significant market fluctuations and advancements in investment strategies—and performance metrics to provide a rigorous examination of these strategies.

In both academic research and industry practice, factor investing has become a central component of modern portfolio management, leveraging specific drivers of asset returns. The pioneering work of Fama and French (1992, 1993) laid the groundwork by isolating market risk, size, and value as key drivers in predicting stock returns. Their framework has since expanded, integrating elements such as profitability and momentum (Jegadeesh & Titman, 1993; Fama & French, 2015), each augmenting the model's predictive robustness. Although Ang (2014) makes a strong case for factor investing due to its risk-adjusted returns and diversification, there is a lack of clarity in this narrative on the limitations of these factors in different market conditions. Furthermore, there remains a gap in knowledge on the generalizability of these determinants across various asset classes, which this research attempts to fill. Recent studies have emphasized the importance of multi-period portfolio optimization in enhancing investment strategies. Li et al. (2022) found that multi-period models achieve higher Sharpe Ratios and outperform their single-period counterparts in risk-adjusted performance, highlighting the benefits of considering multiple periods in portfolio optimization. This strategy uses sophisticated optimization techniques such as the successive convex program algorithm to improve portfolio management's efficiency and robustness (Li et al., 2022).

In addition to factor investing, the emergence of Risk Parity (RP) strategies has revolutionized traditional capital allocation based on market capitalization weights. Instead of distributing capital, RP aims to distribute risk equally among the different parts of a portfolio. This strategy has been the focus of much discussion and examination, notably by Ray Dalio of Bridgewater Associates and Qian (2016). While Qian (2016) argues that RP, as opposed to conventional market-capitalization-weighted portfolios, is a more equitable way to allocate risk, Bhansali et al. (2012) challenge this theory by pointing out that RP might not adapt well to changing asset correlations and volatility. This highlights an important research gap: a more sophisticated understanding of how adaptable RP is to systemic changes in the market.

Building on traditional RP strategies, Wu et al. (2020) introduced the General Sparse Risk Parity (GSRP) portfolio, which selectively allocates assets to achieve stable performance with lower transaction costs. According to the authors, the GSRP method guarantees a superior balance among performance criteria and is a cost-efficient approach to portfolio management. Although the GSRP portfolio's initial transaction costs were high, its profitability and cost-efficiency make it a valuable strategy for investors looking to minimize costs while maintaining robust performance (Wu et al., 2020).

Furthermore, with the introduction of the Hierarchical Risk Parity (HRP) technique, controlling tail risk-adjusted returns has shown tremendous potential, especially in the unpredictable cryptocurrency market. Burggraf (2021) showed that HRP works better than conventional risk-based asset allocation techniques, offering a superior trade-off between risk and return by skillfully allocating risk among portfolio components. This demonstrates how HRP may be used for more than only cryptocurrencies, highlighting its versatility and resilience in a range of market situations (Burggraf, 2021).

Additionally, Lee and Sohn (2023) discovered that integrated risk parity strategies offer consistent risk-return profiles, particularly in times of extreme volatility. Alpha factors and risk parity together can improve the performance and robustness of a portfolio, though the potential impact of fees and rebalancing costs should be considered.

On the other hand, the effectiveness of Inverse Factor Volatility (IFV) strategies has garnered attention for prioritizing lower-volatility assets to enhance portfolio performance. Strategies centered on less volatile assets have gained traction due to the volatility anomaly, with the theory that they can produce higher risk-adjusted returns (Blitz & Van Vliet, 2007). Clarke et al. (2006) further support this by showing how inverse volatility improves drawdowns and Sharpe Ratios. However, there is a dearth of research on directly comparing inverse volatility strategies to risk parity, especially regarding their performance during sharp market declines. This oversight offers an opportunity for this study to provide empirical support for the relative resilience of different approaches.

Risk management and volatility prediction have significantly improved as a result of recent developments in forecasting techniques. To improve volatility forecasting, Di Persio et al. (2023) developed hybrid models that fuse cutting-edge neural networks–specifically, GRU and LSTM–with traditional statistical techniques, such as GARCH. These hybrid models provide more accurate and reliable risk-controlled investing methods by better capturing volatility clustering. When adopting risk parity methods in turbulent market conditions, this incorporation of machine learning approaches delivers a significant improvement in forecasting accuracy (Di Persio et al., 2023).

Furthermore, Bellini et al. (2021) proposed the use of expectiles as a novel risk measure for risk parity portfolios. Compared to typical volatility metrics, expectile-based risk parity portfolios offer more stability and comprehensive evaluations. This strategy is a major development in the field of factor investing since it increases the precision of risk management and portfolio optimization (Bellini et al., 2021).

Costa and Kwon (2022) explored distributionally robust risk parity portfolios, finding that they yield superior risk-adjusted returns and are resilient in various market conditions. Distributional robustness can improve portfolio performance and optimization, although higher turnover rates might increase transaction costs.

Choi et al. (2021) demonstrated that diversified reward-risk parity strategies produce higher returns and reduced downside risks. Implementing diversified reward-risk measures can optimize portfolio performance, though these strategies are sensitive to model inputs and can be complex in highdimensional spaces.

While factor investing and risk parity are well-documented in the literature, research on the application of inverse volatility methods within factor investing remains comparatively scarce. The present literature calls for an empirical investigation to discern the comparative effectiveness of risk parity and inverse volatility strategies across extensive time horizons and market conditions. By comparing these strategies over a two-decade period, this research attempts to close these gaps by providing insights into their performance, risk profiles, and suitability for investors seeking to improve their factor investing techniques.

In a section on emerging markets, Stankov et al. (2024) highlighted that cost mitigation strategies improve factor investing performance. Implementing factor-based methods in less liquid markets requires effective cost management, however, recent reductions in risk premia should be considered.

Furthermore, Dong et al. (2020) introduced the willow tree method, which offers effective risk management and valuation for variable annuities. Improved methods for evaluating complex financial products can improve pricing precision and risk management, although their wider applicability may be limited by their emphasis on stochastic models.

Neisy and Bidarvand (2019) found that effective techniques for estimating volatility enhance the precision of American option pricing. Better pricing models contribute to more accurate hedging and risk management, despite high computational demands and specific model assumptions.

### Methods

### Data Collection

This study incorporated five stocks: Johnson & Johnson (JNJ), Apple Inc. (AAPL), Chevron Corporation (CVX), UnitedHealth Group Inc. (UNH), and JPMorgan Chase & Co. (JPM). Historical stock prices and trading volumes from 2000 to 2022 were sourced from Yahoo Finance, renowned for its comprehensive financial data. In addition to these primary data, the market risk premium (MktRF), size premium (SMB), and value premium (HML) statistics from the Fama-French three-factor model were obtained from the Tuck School of Business at Dartmouth College's official website. This integration of data provided a robust foundation for subsequent analysis.

## Data Processing

Following collection, the data underwent a systematic normalization process to ensure consistency and comparability across different time frames. Dates were standardized to the appropriate R data type, and any superfluous columns were excluded to streamline the dataset. To facilitate long-term investment analysis, monthly returns were converted to an annualized format, and annualized standard deviations were calculated to accurately measure performance volatility.

## Portfolio Construction

A dualistic portfolio construction methodology was adopted. Initially, the Risk Parity (RP) portfolio was formulated by calculating the real risk

contribution of each asset, subsequently adjusting the capital allocation to equalize the risk contribution following the risk parity principle. Conversely, the Inverse Factor Volatility (IFV) portfolio was constructed based on the inverse volatilities of the identified Fama-French factors. By normalizing these inverse volatilities to sum to unity, a portfolio was established where each asset's weight was inversely proportional to its factor volatility.

## Performance Measurement Techniques

Performance evaluation was multi-faceted, incorporating various metrics to provide a comprehensive assessment. The Welch Two Sample T-test was employed to statistically analyze the mean returns differences between the RP and IFV portfolios. Annualized returns provided insight into Long-term performance, while the Sharpe Ratio offered a measure of risk-adjusted returns. The combined returns yielded information about the overall growth of the portfolios, while the annualized standard deviations offered a risk assessment. Additionally, a drawdown analysis was conducted to observe potential losses and portfolio resilience during market downturns.

### Analytical tools

The analysis leveraged the statistical capabilities of R programming, utilizing specialized packages such as xts for time-series management, quantmod for financial data manipulation, Performance Analytics for calculating performance and risk metrics, and openxlsx for exporting results to Excel. This toolkit enabled a thorough examination of the data, ensuring the validity and precision of the study's conclusions.

#### Hypothesis testing

To rigorously determine the comparative efficacy of the Inverse Factor Volatility (IFV) and Risk Parity (RP) strategies, we posited and tested a series of hypotheses. These hypotheses were grounded in three pivotal areas: overall returns, risk-adjusted returns as gauged by the Sharpe Ratio, and the portfolio risk profile measured by volatility.

#### Hypothesis on Overall Returns:

H0 (Null Hypothesis for Returns): The mean return of the IFV Portfolio is less than or equal to the mean return of the RP Portfolio.

H1 (Alternative Hypothesis for Returns): The IFV Portfolio achieves significantly higher mean returns compared to the RP Portfolio.

#### Hypothesis on Risk-Adjusted Returns:

H0 (Null Hypothesis for Risk-Adjusted Returns): The Sharpe Ratio of the IFV Portfolio is less than or equal to the Sharpe Ratio of the RP Portfolio.

H2 (Alternative Hypothesis for Risk-Adjusted Returns): The Sharpe Ratio of the IFV Portfolio is greater than the Sharpe Ratio of the RP Portfolio.

#### Hypothesis on Portfolio Risk:

H0 (Null Hypothesis for Portfolio Risk): The volatility (standard deviation) of the IFV Portfolio is greater than or equal to the volatility of the RP Portfolio.

H3 (Alternative Hypothesis for Portfolio Risk): The volatility (standard deviation) of the IFV Portfolio is less than the volatility of the RP Portfolio.

### Results

Welch Two Sample t-test

Metric	Value
t-value	-2.6051
Degress of Freedom (df)	444.99
p-value	0.009492
Mean of RP Portfolio (x)	-0.003827228
Mean of IFV Portfolio (y)	0.003262424
95% CI Lower Bound	-0.012438149
95% CI Upper Bound	-0.001741155

Table 1: Risk Parity Portfolio and IFV Portfolio Welch Two Sample t-test

The Welch Two Sample t-test was used to statistically analyze the difference in mean returns between the RP and IFV portfolios. The Welch Two Sample t-test yielded a significant p-value of 0.009492 and a t-value of - 2.6051, with degrees of freedom estimated at 444.99, indicating a statistically significant difference between the portfolios' mean returns. The p-value, lower than the traditional alpha threshold of 0.05, suggests a statistically significant difference between the mean returns of the IFV and RP portfolios, The negative t-value indicates that the RP portfolio had a worse mean return (-0.003827228) than the IFV portfolio (0.003262424).

In addition, the 95% confidence range for the mean difference, which spans from -0.012438149 to -0.001741155, does not include zero, supporting the conclusion that the IFV approach yields higher returns than the RP strategy and that the mean returns are considerably different. This result is consistent with the alternative hypothesis (H1) that was put forth regarding returns. It implies that the IFV technique may provide a better return profile than the conventional RP approach, rather than just a different one. These findings suggest that, in terms of mean returns, the IFV strategy should be preferred over the RP method.

RP		IFV			
Annualized Returns	Annualized SD	Sharpe Ratio	Annualized Returns	Annualized SD	Sharpe Ratio
-0.053414351	0.134991537	-0.140595119	0.036579845	0.079416444	0.070264083

**Performance Metrics** 

**Table 2:** Risk Parity (RP) and Inverse Factor Volatility (IFV) portfolio performance metrics

Through the lens of the outlined research hypothesis, we scrutinize the portfolios across multiple dimensions: annualized returns, risk-adjusted returns via the Sharpe Ratio, and overall risk through annualized standard deviation.

At the forefront, the annualized return provides a stark contrast between the two strategies. The IFV portfolio's annualized return stands at a robust +3.66%, a significant departure from the RP portfolio's -5.34%. This disparity not only suggests that the IFV strategy yields higher returns than the RP strategy but also captures the conversion of losses into gains. This is consistent with our first alternative hypothesis (H1), which proposed that the IFV portfolio would produce a higher mean return than the RP portfolio.

Diving deeper into the risk-adjusted performance, the Sharpe Ratio reveals a telling narrative. The RP portfolio's Sharpe Ratio performs poorly, at -0.1406, signaling underperformance relative to a risk-free investment. On the other hand, the IFV portfolio, with its Sharpe Ratio of 0.0703, exemplifies a positive excess return over the risk-free rate. This result validates our second alternative hypothesis (H2), which states that superior risk-adjusted returns would be indicated by a larger Sharpe Ratio for the IFV portfolio than for the RP portfolio.

Turning to volatility, we measure the portfolio's risk via annualized standard deviation (SD). The IFV portfolio has a lower annualized standard deviation (SD) of 7.94%, juxtaposed with the RP portfolio's higher volatility of 13.50%. This reduction in volatility not only indicates a diminution of risk but also supports our third alternative hypothesis (H3), indicating that the IFV strategy is characterized by lower volatility than its RP counterpart.

In conclusion, the performance metrics analysis evidence that the Inverse Factor Volatility strategy appears to outperform the Risk Parity approach across all examined metrics. The IFV strategy delivered higher returns, exhibited superior risk-adjusted performance, and maintained lower volatility. This strong performance across various timeframes provides evidence to support the effectiveness of the IFV strategy, aligning with our alternative hypotheses.



#### **Cumulative Returns**



The RP portfolio, marked by the blue line, exhibits an overall decreasing trend, culminating in a negative cumulative return by the end of the analyzed period. This performance not only suggests a persistent underperformance in comparison relative to the zero-return baseline but also hints at the strategy's vulnerability, especially during market downtrends. Such patterns suggest that the RP strategy might not have consistently mitigated risk as anticipated.

Contrastingly, the trajectory of the IFV portfolio, illustrated by the red line, demonstrates a resilience that aligns with the theoretical expectations. After a period of initial fluctuation—common in investment portfolios—the IFV strategy started to rise steadily from 2003 onward. Despite encountering occasional setbacks, the IFV portfolio displays a remarkable recovery capability, ultimately achieving a positive cumulative return. This pattern affirms our statistical findings of a higher mean return for the IFV strategy.

Particularly telling is the behavior of both portfolios during episodes of market stress. The RP portfolio experiences sharp declines, while the IFV portfolio exhibits relative stability with quicker recoveries. This divergence is most telling of the strategic resilience each methodology offers, echoing our T-test results that favored the IFV strategy's mean performance.

Furthermore, the IFV portfolio's superior performance is reinforced by its positive Sharpe Ratio, which stands in testament to its commendable riskadjusted returns. The less volatile path of the IFV portfolio, suggested by the smoother incline of the red line, is congruent with the strategy's lower annualized standard deviation, a forecast that the IFV strategy inherently bears less risk than the RP strategy.

Ultimately, the graphical examination of cumulative returns offers a strong visual endorsement of the theories put forward concerning the superiority of the IFV method over the RP technique. It illustrates not only an impressive return profile but also a strategic robustness in risk management. The empirical data, coupled with this graphical analysis, underscores the potential of the IFV strategy to enhance portfolio construction and management through tumultuous and tranquil market periods alike.



#### Drawdowns

Figure 2: Drawdowns IFV Portfolio



**Drawdowns-Risk Parity Portfolio** 

Figure 3: Drawdowns Risk Parity portfolio

Figure 2 and Figure 3 provide a detailed comparison of the drawdown profiles of the RP and IFV portfolios, respectively. These profiles are critical for understanding the extent and duration of losses that each portfolio could potentially experience from their peak values to their lowest during the investment period.

The IFV portfolio's drawdown graph (Figure 2) reveals a pattern of relatively modest declines, seldom exceeding a 15% drop. Both the length of these drawdowns and the recovery period seem to be quite short, indicating that the IFV method is effective in preventing losses and promoting a speedier recovery. The aggregation of data points toward lesser drawdowns underscores the IFV portfolio's robust performance, which is consistent with its notable cumulative returns and reduced volatility levels.

In contrast, the drawdown graph of the RP portfolio displays more profound drawdowns, with depths up to 80%. The graph's points are more widely distributed, which suggests that there is greater variation in the drawdowns' duration and depth as well as in the recovery times. This unpredictability is consistent with the higher volatility and negative cumulative returns previously mentioned, indicating that the RP portfolio is more prone to larger changes and might take longer to recover from losses. The contrasting drawdown profiles yield insights that extend beyond mere performance metrics. The drawdown profiles support the theories that the IFV strategy not only generates larger returns but also does so with a more favorable risk profile when these findings are combined with the previous performance measures and t-test analysis. The IFV strategy's ability to deliver higher risk-adjusted returns is further substantiated by its shorter and shallower drawdowns, echoing the positive findings from our Sharpe Ratio analysis. Conversely, the RP portfolio's deeper and more prolonged drawdowns are reflective of its negative Sharpe Ratio and underscore a performance that has not met risk-adjusted return benchmarks.

The culmination of our drawdown analysis affirms the IFV strategy's superior performance relative to the RP approach. This is evidenced not only by higher returns but also by a robust drawdown profile characterized by resilience and swift recovery from market downturns. The findings from Figures 2 and 3 bolster the assertion that the IFV strategy may be a more effective tool for risk-based asset allocation, providing investors with both enhanced returns and a fortified defense against market volatility.

### Discussion

This study critically examines the Inverse Factor Volatility (IFV) strategy in comparison to the traditional Risk Parity (RP) approach, with a focus on enhancing factor investing. We conducted a thorough investigation of the drawdown characteristics, risk-adjusted performance, and returns of both strategies over an extended period, and we have gained insights into the strengths and weaknesses of each strategy.

Statistical analysis using the T-test demonstrated that the IFV strategy significantly outperformed the RP strategy in terms of average returns, a finding corroborated by a p-value that significantly fell below the accepted alpha threshold. This superior performance of the IFV strategy was further evidenced by its positive Sharpe Ratio, indicating better risk-adjusted returns in contrast to the RP strategy's negative ratio. These findings are consistent with the research on factor investing, which suggests that strategies that leverage particular risk factors can outperform conventional market-capweighted portfolios in terms of excess returns (Fama & French, 1993; Carhart, 1997).

In examining drawdown behaviors, the IFV strategy exhibited a distinct advantage, characterized by its less severe losses and quicker recoveries. Such a performance profile, marked by shorter and shallower drawdowns, is particularly advantageous for risk-averse investors or those with shorter investment horizons, underscoring the strategy's capacity to maintain stability during market volatility. Although the RP method has long

been praised for its benefits in diversification, a larger annualized standard deviation suggested heavier drawdowns and a higher overall risk.

The IFV strategy may be attributed to its strategic allocation, which inversely corresponds to factor volatilities and potentially capitalizes on the mean-reversion of factor returns. The equal weighting of asset risk in the RP strategy, on the other hand, might not be as sensitive to movements in the market, increasing exposure during times of high volatility in particular asset classes. The findings highlight the importance of strategy selection in managing portfolio risk, especially during volatile periods, suggesting that the IFV approach may offer a compelling alternative for investors focused on optimizing risk-adjusted returns.

These discoveries have important ramifications for investors. Particularly in volatile market situations, the IFV approach may offer a more enticing risk-return profile because of its lower volatility and drawdown characteristics. If investors aim to optimize their returns while managing risk, the IFV strategy might be a strong substitute for conventional RP portfolios. Nevertheless, as past performance is not necessarily a reliable predictor of future outcomes, investors should also take overfitting into account and emphasize the value of out-of-sample research.

In conclusion, the research offers empirical backing for the IFV approach as a possible way to improve factor investing. It emphasizes how crucial it is to consider both returns and the risk associated with different investing techniques. These observations provide a useful foundation for investors seeking higher risk-adjusted returns and add to the expanding body of research on sophisticated asset allocation strategies.

## Conclusions

This investigation demonstrates that the Inverse Factor Volatility (IFV) strategy significantly enhances factor investing, outperforming the conventional Risk Parity (RP) strategy in terms of higher mean returns, optimized risk-adjusted performance, and resilient drawdown profiles. These findings underscore the substantial impact of advanced asset allocation strategies in achieving the dual objectives of maximizing returns and managing risk effectively.

Moreover, this study combines data analysis from 2000 to 2022—a period marked by significant market fluctuations and advancements in investment strategies—with thorough performance indicators. The study's important ramifications lie in demonstrating how sophisticated factor-based techniques, such as IFV, can significantly enhance risk management and portfolio performance.

For investors and portfolio managers, adopting the IFV strategy may offer a forward-thinking approach to portfolio management, proving particularly effective in markets characterized by uncertainty and volatility. The dynamic nature of the IFV strategy—prioritizing inverse volatility weighting—presents a versatile tool adaptable to varying market conditions and capable of mitigating potential losses more effectively than the traditional RP strategy.

However, while the study's findings are compelling, they are not exhaustive. Future research should broaden the analysis scope to encompass diverse asset classes and market environments. Examining the IFV strategy's performance, accounting for liquidity constraints and transaction costs will provide a more comprehensive view of its practical applicability. Integrating the IFV strategy with other factor-based investment frameworks, such as those combining alpha factors or utilizing hybrid forecasting methods, could reveal synergistic effects worth exploring.

Additionally, conducting stress tests and out-of-sample testing to examine the IFV strategy is recommended, especially to assess its endurance against extreme market scenarios. Moreover, incorporating advanced optimization techniques, as suggested by recent studies, can further refine the strategy. These future investigations will refine our understanding and potentially cement the role of the IFV strategy in the investment landscape, providing investors with a robust tool for achieving superior risk-adjusted returns.

**Funding:** The author did not obtain any funding for this research.

**Conflicts of Interests:** The author reported no conflict of interest.

Data availability: All of the data are included in the content of the paper.

#### **References:**

- 1. Ang, A. (2014, August 1). Asset Management: A Systematic Approach To Factor Investing. *Oxford Academic*. https://academic.oup.com/book/3342
- 2. Qian, E. E. (2016). Risk parity fundamentals. Chapman & Hall/CRC.
- 3. Bellini, F., Cesarone, F., Colombo, C., & Tardella, F. (2021). Risk Parity with Expectiles. *European Journal of Operational Research*, 291(3), 1149–1163. https://doi.org/10.1016/j.ejor.2020.10.009
- Bhansali, V., Davis, J., Dorsten, M. P., & Rennison, G. (2012). The Risk in Risk Parity: A Factor-Based Analysis of Asset-Based Risk Parity. *Journal of Investing*, 21(1), 102–110. https://doi.org/10.3905/joi.2012.21.3.102
- 5. Bessler, W., Taushanov, G., & Wolff, D. (2021). Factor Investing and Asset Allocation Strategies: A Comparison of Factor Versus Sector

Optimization. *Journal of Asset Management*, 22(6), 488–506. https://doi.org/10.1057/s41260-021-00225-1

- 6. Blitz, D., & Van Vliet, P. (2007, April 17). The volatility effect: Lower risk without lower return. SSRN. https://papers.ssrn.com/sol3/papers.cfm?abstract\_id=980865
- 7. Burggraf, T. (2021). Beyond Risk Parity A Machine Learning-Based Hierarchical Risk Parity Approach on Cryptocurrencies. *Finance Research* Letters, 38, 101523. https://doi.org/10.1016/j.frl.2020.101523
- 8. Choi, J., Kim, H., & Kim, Y. S. (2021). Diversified Reward-Risk Parity in Portfolio Construction. *Social Science Research Network*. https://doi.org/10.2139/ssrn.3871944
- Clarke, R. G., De Silva, H., & Thorley, S. (2006). *Minimum-Variance Portfolios in The U.S. Equity Market. Journal of Portfolio Management*, 33(1), 10–24. https://doi.org/10.3905/jpm.2006.661366
- 10.Costa, G., & Kwon, R. (2020). Data-Driven Distributionally Robust Risk Parity Portfolio Optimization. *Social Science Research Network*. https://doi.org/10.2139/ssrn.3709680
- 11.Di Persio, L., Garbelli, M., Mottaghi, F., & Wallbaum, K. (2023). Volatility Forecasting with Hybrid Neural Networks Methods for Risk Parity Investment Strategies. *Expert Systems With Applications*, 229, 120418. https://doi.org/10.1016/j.eswa.2023.120418
- 12. Dong, B., Xu, W., Sevic, A., & Sevic, Z. (2020). Efficient Willow Tree Method for Variable Annuities Valuation and Risk Management. International Review of Financial Analysis. *International Review of Financial Analysis*, 68, 101429. https://doi.org/10.1016/j.irfa.2019.101429
- 13.Fama, E. F., & French, K. R. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance*, 47(2), 427–465. https://doi.org/10.2307/2329112
- 14. Fama, E. F., & French, K. R. (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics*, 33(1), 3–56. https://doi.org/10.1016/0304-405X(93)90023-5
- 15.Fama, E. F., & French, K. R. (2015). A Five-Factor Asset Pricing Model. Journal of Financial Economics, 116(1), 1–22. https://doi.org/10.1016/j.jfineco.2014.10.010
- 16. Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal* of *Finance*, 48(1), 65–91. https://doi.org/10.1111/j.1540-6261.1993.tb04702.x
- 17.Lee, T. K., & Sohn, S. Y. (2023). Alpha-Factor Integrated Risk Parity Portfolio Strategy in Global Equity Fund of Funds. *International*

*Review of Financial Analysis,* 88, 102654. https://doi.org/10.1016/j.irfa.2023.102654

- 18.Li, X., Uysal, A. S., & Mulvey, J. M. (2021). Multi-Period Portfolio Optimization Using Model Predictive Control with Mean-Variance and Risk Parity Frameworks. *European Journal of Operational Research*, 299(3), 1158–1176. https://doi.org/10.1016/i.eior.2021.10.002
- Neisy, A., & Bidarvand, M. (2019). An Inverse Finance Problem for Estimating Volatility in American Option Pricing Under Jump-Diffusion Dynamics. *Journal of Mathematical Modeling*, 7(3), 287– 304. https://doi.org/10.22124/jmm.2019.13082.1258
- 20. Stankov, K., Schiereck, D., & Flögel, V. (2024). Cost Mitigation of Factor Investing in Emerging Equity Markets. *Journal of Asset Management*. https://doi.org/10.1057/s41260-024-00353-4
- 21.Shimizu, H., & Shiohama, T. (2020). Constructing Inverse Factor Volatility Portfolios: A Risk-Based Asset Allocation for Factor Investing. *International Review of Financial Analysis*, 68, 101438. https://doi.org/10.1016/j.irfa.2019.101438
- 22. Wu, L., Feng, Y., & Palomar, D. P. (2020). General Sparse Risk Parity Portfolio Design Via Successive Convex Optimization. *Signal Processing*, 170, 107433. https://doi.org/10.1016/j.sigpro.2019.107433