MODELLING ECONOMIC GROWTH IN GHANA

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Abstract

The main aim of this paper was to use the Augmented Cobb-Douglas production function as a basis to model the economic growth of Ghana during the period 1991 to 2011.

There has not been equality of economic growth in all economies around the world for a long time. In all economies around the, some grow faster than others. Economists have predicted after some few years to come that the slower growing economies will eventually converge to the faster growing economies.

Starting from the estimates of the parameters from other studies, the growth model was simulated for the period 1991 to 2011, using Matlab, SPSS and Excel spreadsheet. The estimations from the model were compared with the actual figures from the Ghana Statistical Service, World Bank and Ministry of Finance and Economic Planning (MOFEP). The model in this paper provides a better approximation of the changes in the Ghanaian economy for the period from 1991 to 2011, with respect to the changes of the real aggregate in the Gross Domestic Product (GDP) growth and to the ratios of the main macroeconomic variables, like production per worker, capital-output ratio or capital per worker.

The matlab simulation results showed a very close relationship between the actual and calculated growth rates over the periods. The actual average growth rate over the period was 4.5% as compared to the calculated average value of 4.21%.

In conclusion, the correlation coefficient between the actual growth rates and the calculated was 0.298, which indicates that they are correlated, but the strength of correlation was weak meaning the model is good for prediction of any countries economic growth.

Keywords: Capital, Labour force, Total Factor Productivity, Total Production, Economic growth, Ghana, Matlab simulation, GDP, Cobb-Douglas production, Solow model.

1.0 Introduction:

Economic growth is defined as the growth in individual human welfare: - but on a practical level, it is a sustained increase in per capita or per worker product. We often use real per capita GDP as a proxy, not just total GDP, for measuring the rate of economic growth.

Mathematical Economists have long been interested in the factors which cause different countries to grow at different rates and achieve different levels of wealth. This issue is especially relevant today as it was in the 1940's where developmental economics was born. The historical record shows a broad range of outcomes in achieving sustained economic growth. Some countries, particularly in Eastern Asian have achieved very rapid rates of growth and catching up with already wealthy countries while others, particularly Sub-Saharan Africa, have achieved little or no growth. The reasons for these differences remain an important theoretical and empirical task.

A review of recent theoretical advances in growth theories is potentially relevant for policy development and analysis of the determinants of economic growth. Although neoclassical economic theory has become dominant in economic analysis, development economists have been reluctant to adopt it, as it predicts stable growth independent of policy decisions. Cherery [1986] makes the case for the inadequacy of the neoclassical equilibrium approach for developing countries as it does not take into account disequilibrium factors such as internal demand constraints, external market constraints, economies of scale, learning by doing, and imperfect factor markets. In recent years, economists working within neoclassical theory have provided models which address a number of issues raised by the development economists. In particular, new models of endogenous economic growth have been developed to deal with general issues of growth with regards to policies such as the operation of financial markets, trade policies, and government expenditure and taxation.

Caraiani [2007] argued that a country with a higher saving rate will experience faster growth, e.g. Singapore had a 40% saving rate in the period 1960 to 1996 and annual GDP growth of 5-6%, compared with Kenya in the same time period which had a 15% saving rate and annual GDP growth of just 1%. This relationship between savings growth was considered in the Cobb-Douglas model. This was retained in the Solow model; however, in the very long-run capital accumulation appears to be less significant than technological innovation in the Solow model.

While there is no doubt about the fact that the economic growth record of the last two decades, following reforms, differed from that of the first two decades in terms of consistency, it is also clear that the factors behind the growth experiences of shorter periods in-between show remarkable similarity. Whenever there has been considerable capital injection into the economy, this has been followed by significant growth. It is the difficulty in making those injections consistently in the absence of structural change that has left the economy still fragile after five decades of independence.

A critical look at the growth rate of Ghana indicates that the growth rate has been positive since 1984, though the rate of growth has been fluctuating. The fluctuating nature brings up the curiosity for some empirical analysis. Cobb-Douglas [1928], production function has been used in determining growth rates of the American economy. This production function was later modified by Solow in 1956 and the resulting model predicted the growth rate of the American economy and elsewhere. Though the model predicted growth rate of developed and some developing countries, it has not been used here in Ghana to predict the growth rate. It is against this background that the study sought to model, and tests a model of economic growth of Ghana with Solow's production function as the bases to determine whether the model predicts the growth pattern of Ghana from 1991 to 2011well enough relative to the real dynamics of the economy.

The targeted rate of growth requires increased productivity in all sectors of the economy, especially agriculture, and an expansion of the range of goods and services, produced at internationally competitive prices. This was to be assisted by major improvements in all types of economic infrastructure. Accelerated growth of over 8% of GDP per annum will require a major shift in sectoral composition of production, with the share of agriculture falling to below 20% of GDP and industry's share rising to 37% by 2020 (Ghana Vision 2020).

However, available data indicates that not only has the Ghanaian economy been characterized by the non-attainment of the targeted rate of growth but also non-attainment of the shift in sectoral composition of the economy. For instance the contribution of agriculture 8 to GDP averaged 43.8% (1960-2008), 47.53% (1960-1983) and 40.23% (1984-2008), which are all over 100% higher than the targeted 20%.

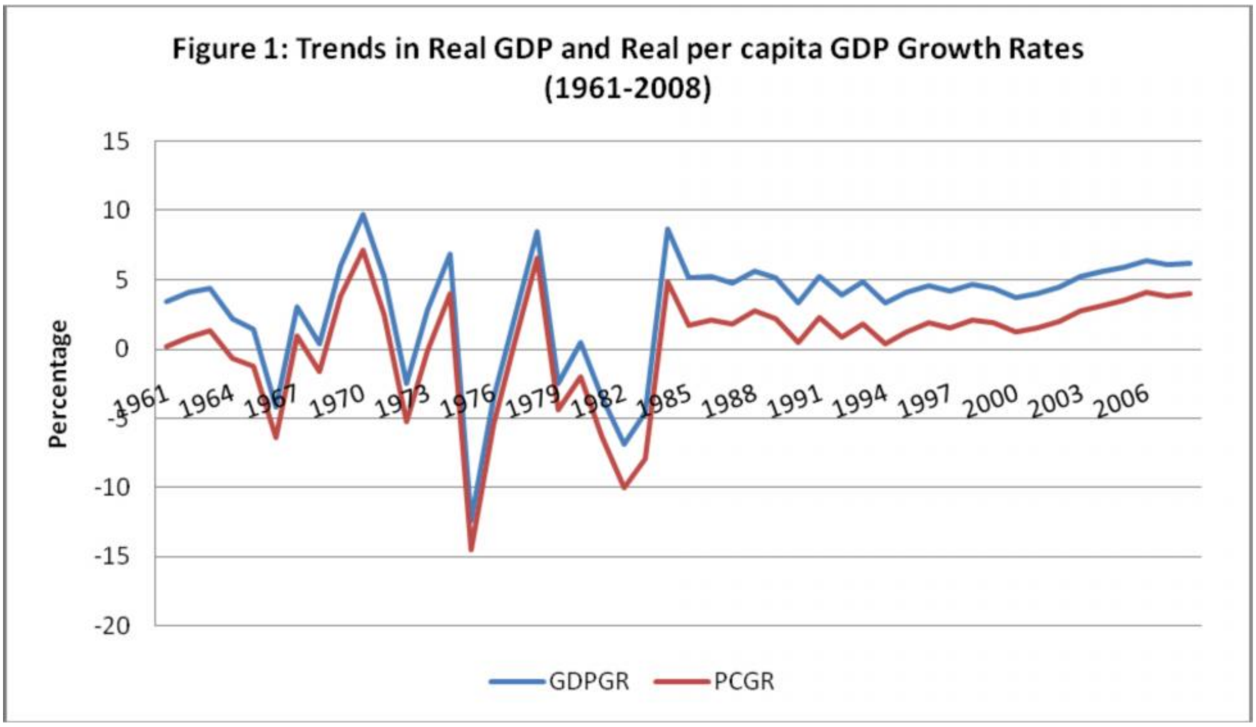

Figure 1 shows the trend in the growth rates of real per capita GDP and real GDP from 1961 to 2008 inclusive. In the figure, real GDP growth is represented by GDPGR whilst the growth rate of per capita income is indicated with PCGR. The stable co-movement of per capita growth and real GDP growth indicates that over the period 1960 to 2008, population growth rate has been constant. The growth rates were highest in 1970 under the second republic constitutional administration headed by Prime Minister K.A. Busia. In the 1970, the recorded real GDP growth was 9.72% whilst per capita income grew at 7.2%. Unfortunately this record level growth was not sustained following the February 1972 Coup headed by General Acheampong. By 1975, the growth rates of per capita income and real GDP hit an all time lowest rates of -14.49% and -12.43 % respectively. Growth remained poorly and negative in most years until the reform period in the mid 1980s.

2.0 Related Works

Since independence in 1957, Ghana has tried a number of approaches to achieving acceptable rates of growth and development. Governments after governments tried various approaches to achieve economic growth that is acceptable, but most approaches couldn't materialize because of political instability, weak commodity demand etc. When Ghana gained her independence she was the world's leading producer of cocoa and this supported a well-developed infrastructure to service trade, and enjoyed a relatively advanced educational system, Aryeetey and Fosu [2005].

The growth rate record of Ghana has been unstable when the post-reformed period is compared to the earlier period. With logically high GDP growth in the 1950s and early 1960s, the economy began to experience a reduction in GDP growth in 1964. According to Aryeetey and Fosu [2005], 'growth was turbulent during much of the period after mid-1960s and only began to stabilize after 1984. In 1966, 1972, 1975-1976, 1979 and 1983, the growth rate of real GDP was negative for Ghana'. The GDP growth has been negative for a number of years. This is mainly due to political instability between these years, even though some years recorded some positive growth in 1974, 1977 and 1978. From 1984 to 2006, the GDP growth has averaged about 3.9 to 4.5 percent, Baafi [2010].

There has always been the view that the economy of Ghana could and should grow faster than it has done. The recent growth record is deemed inadequate for the desired transformation of the economy, Aryeetey and Fosu [2005]. Ghana in 1993 set itself the target of becoming an upper middle income country by 2020 under its *Vision 2020* programme according to Aryeetey and Fosu [2005]. To achieve the targeted *per capita* income by that year, using a simple Harrod-Domar type model, it was reckoned that the economy needed to grow at an average of 8% for the period as indicated in the economic review, 1992 by the Institute of Economic Affairs.

The government sought to use the apparent stability of the Ghanaian economy as a springboard for economic diversification and expansion and began the process of moving Ghana from a primarily agricultural economy to a mixed agricultural industrial one, Aryeetey and Fosu [2005]. But unfortunately, the price of cocoa collapsed in the mid-1960s, destroying the fundamental stability of the economy. Since then, Ghana has been caught in a cycle of debt, weak commodity demand, and currency overvaluation, which has resulted in the decay of unproductive capacities and a crippling growth rate.

Since then the economy has not shown a capacity to move towards the target. The performance of the economy and economic growth have been characterised by the non-attainment of macroeconomic targets. In particular, whereas the GDP was expected to grow between 7.1% and 8.3% in the period 1996-2000, actual growth was between 4.2% and 5.0%.

The significant deviation between set targets and the actual results was translated into low per capita GDP growth and poor sectoral growth, (Cited in Baafi, 2010).

The quite positive growth performance in response to the reform has persisted since 1984 with relatively little variance particularly in the early 1990s.Despite a strong desire to achiever faster growth rates this has not been easy. The inability of growth to move beyond 6.0 percent per annum has been linked to the absence of structural transformation (Aryeetey and Fosu, 2004) and these are driven by the fact that macroeconomic policies have not been anchored in comprehensive and credible longer term development frameworks. Baafi [2010] suggested that, per capita GDP growth closely tracks that of GDP suggesting a seemingly stable growth of population. The slow rate of per capita income growth in the economy hovering below the 4 percent mark after 1984 is largely attributed to low productivity (O'Connell and Ndulu, 2000). Growth since 2001 has been rising slowly as a result of the recovery of agricultural production and general improvement in economic management, particularly in the area of fiscal and monetary policies (AfDB/OECD, 2003). However, this is not regarded as sufficient to drive the economy towards the achievement of the Millennium Development Goals (MDGs) (Aryeetey and McKay 2005).

To achieve sustained economic growth, increased production and productivity must be at the centre of an economic recovery strategy. To formulate strategies for achieving sustained increased production and the rapid growth necessary for poverty reduction, relevant information is absolutely necessary. It is therefore important to decompose the structure of Ghana's economy and its growth rate, to gain a better understanding of those factors that have produced differences in growth rates in the various periods. That is to model the economic growth of Ghana from 1990 to 2010, considering the Solow model on economic growth.

Aryeetey *et al*. [2001] presented a relatively simplistic growth accounting model in the form of a Cobb-Douglas production function for Ghana's economy. The model which was developed using data from 1961-96 is specified as:

$$q = \alpha + \beta l + \delta k + d + \varepsilon \tag{2.1}$$

Where

q : GDP growth

l : labour growth

k : growth in capital (measured as investment to GDP ratio)

d : a dummy variable representing economic liberalization (d = 1 from 1969-72 and 1983-96; d= 0 from 1961-68 and 1973-1982)

ε : the error term

The results of the estimation, admittedly crude, indicated that most of GDP growth seemed to be accounted for by factors outside of the model. The results show that the only significant variable is the economic liberalization dummy variable, which has a positive coefficient. Growth in labour and capital has supposedly not contributed significantly to GDP growth.

The results suggested that total factor productivity (TFP) may have played a more important role in the observed pattern of GDP growth, and that TFP is affected by economic regimes. In particular, liberal regimes apparently positively contribute to TFP and to growth.

The Solow residual is defined as per-capita economic growth above the rate of per-capita capital stock growth, so its detection indicates that there must be some contribution to output other than advances in industrializing the economy (cited in Baafi, 2010). The fact that the measured growth in the standard of living, also known as the ratio of output to labour input, could not be explained entirely by the growth in the capital/labour ratio was a significant finding, and pointed to innovation rather than capital accumulation as a potential path to growth. Solow assumed a very basic model of annual aggregate output over a year (*t*). He said that the output quantity would be governed by the amount of capital (the infrastructure), the amount of labour (the number of people in the workforce), and the productivity of that labour. He concluded that the productivity of labour was the factor driving long-run GDP increases.

Romer (1989) suggested five stylized facts that growth theorists should be able to explain.

- In cross-section, the mean growth rate shows no variation with the level of per capita income.
- The rate of growth of factor inputs is not large enough to explain the rate of growth of output; that is, growth accounting always finds a residual.
- Growth in the volume of trade is positively correlated with growth in output.
- Population growth rates are negatively correlated with the level of income.
- Both skilled and unskilled workers tend to migrate towards high-income countries.

However, the most famous model which has been the corner stone of most economic growth analysis is the Solow Model. Although, many economists felt that a much more sophisticated model was needed to accurately depict the complex process of economic growth. Solow’s simple neoclassical model still dominates the economic growth literatures.

Succession of attempts has been to create models of economy's growth that could explain the variation in growth experienced by different regions of the world.

We have noticed from the previous section that, the neo-classical Solow model explains economic growth as resulting from the combination of two elements, namely Capital and Labour. Now the question arising is how much of the output growth can be attributed to other factors apart from capital and labour. To answer this question, Solow decomposes the growth in output into three components,- with each identifiable as contribution of one factor of production, that is labour, capital and total factor productivity. This type of measurement of total factor productivity is often referred to as the Solow residual. The term residual is appropriate because the estimate present the part of measured GDP growth that is not accounted for by the weighted-average measured growth of the factors of production (capital and labour). To account for this,

Solow used the Cobb-Douglas production function and started from his simple growth equation. For simplicity, we repeat the equation as

$$Y(t) = f(K, L, A) \tag{2.2}$$

Where A : total factor productivity

L : labour

K : capital

Using Cobb-Douglas production function, Solow stated the following equation

$$Y(t) = AK^{\alpha}L^{\beta} \tag{2.3}$$

From this, Solow defined his other factor (total factor productivity) to be technology as noted earlier. Solow acknowledged the convenience of the Cobb-Douglas production function because it exhibits constant returns to scale which is consistent with his model. We should note that the variable *A* is not constant but varies with different production functions based on the factors studied. Different authors like Mansouri [2005] have used different factors to account for the total factor productivity.

In accounting for the determinants of Morocco's economic growth, Mansouri [2005] used the aggregate production function model. He used the aggregate production of the following general form: (2.3)

$$Y = f(A, L, K)$$

Where *Y* is real GDP, *A* is total factor productivity, and *L* and *K* stand for labour and capital inputs respectively. Mansouri [2005]argued that *A* is determined by economic factors and in the case of Morocco, *FDI and FDI interaction with trade openness (TR)* are the vehicle through which technology travels,- hence;

$$A = g(FDI , FDI * TR) \quad (2.4)$$

Substituting (2.3) into (2.4), gives:

$$Y = f(FDI, FDI * TR, L, K) \quad (2.5)$$

To account for the isolated impact of trade openness on economic growth, Mansouri [2005] introduced *TR* as an explanatory variable. Considering the specificities of the Moroccan economy, Mansouri [2005] accounted for the impact of drought cycles on economic growth in the particular case of Morocco. Mansouri finally added a proxy for drought (DR) to equation (2.5), to yield:

$$Y = G(FDIt ,TRt , FDIt * TRt , Lt ,Kt ,DRt) \quad (2.6)$$

Where *DR* is a proxy for drought, is the inverse of the cereal yield per hectare. The operational model that was finally selected by Mansouri [2005] to explain Moroccan growth is:

$$In(Y) = \beta_0 + \beta In(FDI) + \beta_2 In(TR) + \beta_3 In(FDI) * TR + \beta_4 In(L) + \beta_5 In(K) + \beta_6(DR) + \varepsilon \ldots \quad (2.7)$$

Because Robert Solow used the ‘marginalist’ thinking of the 19th century neoclassical economist, his model is usually referred to as Neoclassical Growth Model. The basic structure of the Solow model is quite simple. To differentiate his model from the Harrod-Domar model and its fixed capital-output ratio, Solow defined a production function that permits factors to be continuously substituted for each other such that the marginal product of each factor is variable; depending on how much of the factor is already used in production and how many other factors it is combined with. This continuous substitutability of the factors of production is what makes Solow’s model neoclassical in nature (Hendricks Van den Berg, 2001).

Solow furthermore assumed that each factor of production is subject to diminishing returns. Meaning equal increments of one factor are added to a fixed amount of the other factors of production, output increases, but it increases by ever-smaller amounts.

Thomas Malthus [1798], had assumed that labour was subject to diminishing returns when it was combined with a fixed stock of agricultural land. Solow’s aim was to show that the Harrod-Domar model was wrong in concluding that a constant rate of saving and investment could bring everlasting economic growth. Solow showed that, with diminishing returns, continuous investment could not, by itself, generate permanent economic growth because diminishing returns would eventually cause the gains in output from investment to approach zero. Solow's model thus clashed with what many development economist were advising policy makers to do in order to increase economic growth, which was to increase saving and investment any way possible. (Hendricks Van den Berg, 2001).

But if investment is not the determinants of an economy's long-run rate of growth, what is? Solow's identified that; long-run growth must come from another source: technological progress. Only if an economy keeps increasing the amount of output that it can produce from a given amount of input then can it avoid diminishing returns and keep it per capita output growing forever. (Hendricks Van den Berg, 2001).

Slavin [2005] expressed the view that, model of economy's growth was developed by economists, Robert Solow in 1957. In his model, he assumed that an economy wide production function can be written in the simple form:

$$Y=AK^{0.3}L^{0.7} \tag{2.8}$$

Where Y is aggregate output, A is a number based on the current state of technology, K is a quantitative measure of the size of the stock of manufactured capital, and L the quantity of labour used during the period of time. K and L are the only factors of production explicitly included in the model. Both capital and labour are needed for the production of output, with the exponents in the equation reflecting their relative contributions.

A is called total factor productivity, and includes all contributions to total production not already reflected in the levels of K and L. Often, total factor productivity has been interpreted as reflecting the way in which technological innovation allows capital and labour to be used in more effective and valuable ways. For example, the development of computer word-processing greatly increased efficiency compared to the use of typewriters. Typewriters, which seem antique to us today, were themselves a huge productive advance over clerical work using pen paper. This process of improved technological methods has resulted in an increase in labour productivity; more output can now be produced with fewer labour-hours.

In a more recent study, Brunnschweiler and Bulte (2008) critically evaluated the empirical basis for the resource curse using two-stage least squares (2SLS) estimator for a cross country sample of 60 countries using data from 1970 to 2000. They concluded that despite the popularity of the resource curse thesis, the apparent paradox of plenty may be a red herring. They argued that the most commonly used measure of resource abundance (the ratio of primary commodity exports-to-GDP) can be more usefully be interpreted as a proxy for resource dependence which is endogenous to the underlying structural factors of an economy. In multiple estimations that combine resource abundance and dependence, constitutional and constitutional variables, Brunnschweiler and Bulte (2008) show that 16 resource abundance, constitutions and institutions determine resource dependence. They maintained that resource dependence does not affect economic growth but rather resource abundance positively affects growth and institutional quality. The positive effect on growth of natural resources have been confirmed in studies such as Ding and Field (2005) Brunnschweiler (2008), and Butkiewicz and Yanikkaya (2010).

3.0 Model Development

The following are assumptions made;

- Savings and investment decisions are exogenous (no individual optimization).
- Factor accumulation and technological growth are generated outside the model.
- It is assumed that each factor of production is subject to diminishing returns. That is, as equal increments of one factor are added to a fixed amount of the other factors of production, output increases, but it increases by ever-smaller amounts.
- It is assumed also that labour supply grows by itself.
- The Economy is closed to external forces and government intervention.
- Depreciation of capital is ignored.

3.1 The Production Function

Solow begins with a production function in which, Y, is a function of quantity of capital,

K and labour L:

$$Y = f(K, L) \tag{3.1.1}$$

Solow assumed that this production function exhibits constant returns to scale, which means that if all inputs are increased by a certain multiple, output will increase by exactly that same multiple. Specifically, if equation 2.1 represents a constant-returns-to scale production function, then for any positive constant c the following must also hold:

$$cY = F(cK, cL) \tag{3.1.2}$$

We now take advantage of this characteristics of constant-returns-to scale production functions and let $c = 1/L$, which give us

$$Y/L = F(K/L, 1) \tag{3.1.3}$$

Equation 3.1 can be conveniently rewritten as

$$y=f(k) \tag{3.1.4}$$

If we define Y/L and K/L as y and k, respectively, and let the function $f(k)$ represent $F(k, 1)$. Equation 3.1.3 describes output per worker as a function of capital per worker. This representation of the production function in per-worker terms is quite appropriate given that we define economic growth as the change in per capita output. In judging whether welfare in society increases, output per person must increase. In terms of the variables defined above, economic growth requires an increase in y, not just Y.

In addition to assuming constant returns to scale, Solow further assumed positive but diminishing marginal returns to any single inputs. That is the slope of output continuously decreases because each additional increase in K relative to L causes smaller and smaller output (Hendrick *Van den Berg, 2001*). This is the inherent characteristics of the Solow model that brings convergence to light.

The General production function, with physical capital K, labor L and knowledge or technology A: is given as

$$Y(t) = F(K, A, L) \tag{3.1.5}$$

Time is discrete: t = 1, 2, 3…

The Solow growth model can be described by the interaction of five basic macroeconomic equations:

- Macro-production function
- GDP equation
- Savings function
- Change in capital
- Change in workforce

We have so far specified a neoclassical production function with the general form $Y=f(K, L)$, in which f represent the functional relationship between output and inputs. But such a general form has its limitations. We can reach many useful qualitative conclusions, but specific quantitative solutions are not possible. To reach more specific quantitative conclusion, Solow applied the Cobb-Douglas production to his model. The Solow model also identified total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by some other factors apart from capital and labour that is exogenous to the model.

The Solow residual measures total factor productivity, but is normally attached to the labour variable in the macroeconomy because return on investment doesn't seem to change very much in time or between developing nations, and developed nations—not nearly as much as human productivity seems to change.

The intensive-form production function is assumed to have the following properties:

$$\left.\begin{array}{l} f(0) = 0 \\ f'(k) > 0 \\ f''(k) < 0 \end{array}\right\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3.1.6)$$

And Inada conditions:

$$\left.\begin{array}{l} lim_{k \to 0} f'(k) = \infty \\ lim_{k \to \infty} f'(k) = 0 \end{array}\right\} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.1.7)$$

In macroeconomics, the Inada conditions (named after Japanese economist Ken-Ichi Inada) are assumptions about the shape of a production function that guarantee the stability of an economic growth path in a neoclassical growth model.

The six conditions are:

- the value of the function at 0 is 0,
- the function is continuously differentiable,
- the function is strictly increasing in t,
- the derivative of the function is decreasing (thus the function is concave),
- the limit of the derivative towards 0 is positive infinity,
- the limit of the derivative towards positive infinity is 0.

According to Borelli, P and Pessoa, S [2003], it can be shown that the Inada conditions imply that the production function must be asymptotically Cobb–Douglas.

The production function is not different from the general production function 3.1.5 where

$$Y(t) = F(K, L, A).$$

Unlike the general production function, Solow coupled labour supply, L and Productivity, A as one factor. That is;

$$Y(t) = F(K, A, L). \quad (3.1.8)$$

Productivity in the Cobb-Douglas Production Function was constant, with a value of 1. In Solow's model, productivity was made to grow proportional to it.

We shall begin with the production function of Solow's (1956) type as follows;

$$Y(t) = K^{\alpha}(A.L)^{\beta} \quad (3.1.9)$$

If we couple productivity and labour supply, i.e. A(t)*L(t) = Z(t), then the production function will be

$$Y(t) = K^{\alpha}Z^{\beta} \quad (3.1.10)$$

Where

Y(t):Total output (GDP).

A (t): Total Factor productivity, TFP (i.e. Knowledge and Technological change)

K(t) :Capital accumulation

L(t):labour Supply

α and β: are the output elasticities of capital and labour.

3.2 Definition of Parameters

The models contain certain parameters that are important to mention them here in order to put the results into perspective.

(τ-δ): This parameter represents the rate of labour supply. This constant of proportionality may not exist in the real world. Labour may not grow exogenously. The growth of Labour may vary according to certain natural or economic conditions. '**τ**' shows that labour will grow upwards forever. '**δ**' indicates that labour will grow downward forever, until a point of equilibrium is achieved. That is, the point at which '**τ= δ**'. '**δ**' is the parameter for exit rate of labour in the economy.

β: Solow assumed that the marginal productivity of labour is proportional to the amount of production per unit of labour. The constant of proportionality represented as β. The constant of proportionality may vary from time to time. The value of β may not remain the same over a period of time.

φ: This is the constant of proportionality, depicting the rate of growth of Total factor Productivity. Solow assumed Total Factor productivity to be exogenous. That is the growth rate is not affected by the national income or amount of capital.

TFP and labour supply are assumed to be exogenously determined.

We derived expressions (models) for the determinants; Labour supply, TFP, Capital Accumulation, and Production Function.

3.3 The Labour supply model

From the assumption bullet four, labour supply can be written mathematically as;

$$\frac{dL}{dt} = \tau L - \delta L = (\tau - \delta)L \qquad (3.3.1)$$

Where

τ : the rate of labour supply and

δ: rate at which people leave the labour sector (exit rate)

Equation 3.3.1 is a separable differential equation. Therefore it can solve by separating variables and integrating.

$$\frac{dL}{dt} = (\tau - \delta)L$$

Dividing both sides by L and integrating both sides, we get

$$\int \frac{dL}{dt} = \int (\tau - \delta)dt$$

$$In(L) = (\tau - \delta)t + C$$

Taking natural logarithm on both sides

$$L(t) = e^{(\tau-\delta)t+c}$$

$$L(t) = e^{c}e^{(\tau-\delta)t}$$

Let, $L_0 = e^c$, (Initial capital stock of the economy)

Therefore; $L(t) = L_0 e^{(\tau-\delta)t}$ (3.3.2)

This represents the supply of labour model.

3.4 Total Factor Productivity Model (TFP)

Diewert (1992) define productivity as the ratio of output index to an input index. The model again assumed that TFP is endogenous. That is it grows proportional to itself. Hence;

$$\frac{dA}{dt} = \varphi A(t) \qquad (3.4.1)$$

$$\frac{dA}{dt} = \varphi A(t)$$

Solving by separating variables

$$\int \frac{dA}{dt} = \int \varphi A(t) dt$$

$$\int \frac{dA}{A} = \int \varphi dt$$

$$In(A) = \varphi t + c$$

It follows that

$$e^{In(A)} = e^{\varphi t + c}$$

Let $A_o = e^c$ (Initial level of productivity)

And consequently, $A(t) = A_0 e^{\varphi t}$ (3.4.2)

Where

φ: parameter, indicating the growth of productivity.

A_0: *initial level of productivity*

3.5 Capital Accumulation Model

Let $\frac{dK}{dt}$ denote change of stock of Capital with respect to the time (t). In a closed economy total output, denoted by Y is equal to Total Income in a closed economy. In this case, the fraction of income saved which is defined as investment constitutes change in capital. This change in capital is denoted by **'s'** and therefore the amount saved will be equivalent to $sY(t)$, where $Y(t) = F(K, A, L)$ by equation 3.1.8.

In a closed economy without government involvement, we assume investment is equal to savings, it follows that;

$\frac{dK}{dt} = sF(K, L, A)$ (3.5.1)

From equation 3.1.8, we know that

$$Y(t) = F(K, A, L)$$

This implies that

$\frac{dK}{dt} = sY(t) = I$ (3.5.2)

This equation can be written as

$\frac{dK}{dt} = sK^{\alpha}(A.L.)^{\beta}$

(3.5.3)

Furthermore substituting $L(t)$ for $L_0e^{(\tau-\delta)t}$ and $A(t)$ for $A_0e^{\varphi t}$ in this equation yields;

$$\frac{dK}{dt} = sK^{\alpha}(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}$$

Dividing both sides by K^{α} and integrating

$$\frac{dk}{K^{\alpha}} = s(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}dt$$

$$\int K^{-\alpha}dk = \int s(A_0e^{\varphi t}L_0e^{(\tau-\delta)t})^{\beta}dt$$

$$\frac{K^{1-\alpha}}{1-\alpha} = s(A_0L_0)^{\beta}\int (e^{\varphi t}e^{(\tau-\delta)t})^{\beta}dt$$

$$\frac{K^{1-\alpha}}{1-\alpha} = \frac{s(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)t}}{(\varphi+\tau-\delta)\beta} + C$$

But $1-\alpha = \beta$

$$\frac{K^{\beta}}{\beta} = \frac{s(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)t}}{(\varphi+\tau-\delta)\beta} + C$$

Simplifying;

$$K^{\beta} = \frac{S(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)\beta t}}{(\varphi+\tau-\delta)} + C \quad \dots\dots\dots. \qquad (3.5.3)$$

If $K(0) = K_0$ (initial capital stock)

$$K_0^{\beta} = \frac{S(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)\beta(0)}}{(\varphi+\tau-\delta)} + C$$

$C = K_0^{\beta} - \frac{s(A_0L_0)^{\beta}}{\varphi+\tau-\delta}$ (3.5.4)

Replacing C in the equation 3.5.3, we have

$$K^{\beta} = \frac{s(A_0L_0)^{\beta}e^{(\varphi+\tau-\delta)\beta t}}{(\varphi+\tau-\delta)} + K_0^{\beta} - \frac{s(A_0L_0)^{\beta}}{(\varphi+\tau-\delta)}$$

$$K^{\beta} = \frac{s(A_0L_0)^{\beta}(e^{(\varphi+\tau-\delta)\beta t}-1)}{(\varphi+\tau-\delta)} + K_0^{\beta}$$

This then follows that the model for capital accumulation is given by

$$K(t) = \left[\frac{s(A_0L_0)^{\beta}\left(e^{(\varphi+\tau-\delta)\beta t} - 1\right) + (\varphi + \tau - \delta)K_0^{\beta}}{(\varphi + \tau - \delta)}\right]^{\frac{1}{\beta}}$$

$$K(t) = \left[\frac{s\mu^{\beta}(e^{\omega\beta t}-1)+\omega K_0^{\beta}}{\omega}\right]^{\frac{1}{\beta}} \qquad (3.5.5)$$

Where

μ= A_0L_0

ω= $\varphi + \tau - \delta$

In the next model, we replace the determinants with their appropriate expressions (models) in equation 3.1.9.

3.6 **The Production Model**

Using Robert Solow's production function, thus equation 3.1.8

$$Y(t) = K^{\alpha}(A.L.)^{\beta}$$

And substituting equation 3.3.2, 3.4.2 and 3.5.5, we have

$$Y(t) = \left\{\left[\frac{s(A_0L_0)^{\beta}(e^{(\varphi+\tau-\delta)\beta t}-1)+(\varphi+\tau-\delta)K_0^{\beta}}{(\varphi+\tau-\delta)}\right]^{1/\beta}\right\}^{1-\beta} \left[A_0L_0e^{(\varphi+\tau-\delta)t}\right]^{\beta}$$

$$Y(t) = \left[\frac{s(A_0L_0)^{\beta}\left(e^{(\varphi+\tau-\delta)\beta t} - 1\right) + (\varphi + \tau - \delta)K_0^{\beta}}{(\varphi + \tau - \delta)}\right]^{\frac{1-\beta}{\beta}} \left[A_0L_0e^{(\varphi+\tau-\delta)t}\right]^{\beta}$$

$$Y(t) = \left[\mu^{\beta}e^{\omega\beta t}\right]\left[\frac{s\mu^{\beta}(e^{\omega\beta t}-1)+\omega K_0^{\beta}}{\omega}\right]^{\rho} \qquad (3.6)$$

Where

μ= A_0L_0

ω= $(\varphi + \tau - \delta)$

ρ= $\frac{1-\beta}{\beta}$

This is the production model of the economy.

3.7 The Growth Model

The growth model is represented by

$\Delta Y = Y(t) - Y(t-1)$ (3.7.1)

$\Delta t = 1$ (3.7.2)

Using the equation 3.7.1 and substitution in time, t_2= t and t_1= t-1, we have

$$\Delta Y = \mu^{\beta} e^{\omega\beta t}] \left[\frac{s\mu^{\beta}(e^{\omega\beta t}-1)+\omega K_0^{\beta}}{\omega}\right]^{\rho} - [\mu^{\beta} e^{\omega\beta(t-1)}] \left[\frac{s\mu^{\beta}(e^{\omega\beta(t-1)}-1)+\omega K_0^{\beta}}{\omega}\right]^{\rho} \quad \ldots\ldots (3.7.3)$$

Therefore the percentage change (Q) of Total output over time, thus the rate of growth becomes;

$$Q_t = \left[\frac{Y(t)-Y(t-1)}{Y(t-1)}\right] \times 100\% \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7.4)$$

This is the equation of the economy's growth rate at the time (t).

4.0 Model application

The real data received from the Ghana statistical service and MOFEP on Ghana from 1991 to 2011, the results was computed using the capital Model, the Labour Model, the TFP Model, and the Production Model and finally computed the Growth rate using the Growth Model. These Models are applied to 1991 year's data as a base year, and the models are simulated for the periods 1991 to 2011.

4.1 Computations for the year 1991

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 1, into the Matlab code L (t) = 5962958*exp (0.032*t) yields 6,156,900.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 1; into the Matlab code; A(t)= 100*exp(0.032*t); yields 103.25.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=1;into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 893,680,000.00.

Production

Using equation 3.5.2 which is and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064 , β=0.3, ρ=(1-β)/β and t=1;into the Matlab code; Y(t)=((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,172,500,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y(t-1) =3,266,886,838.00 and Y (t)=3,172,500,000.00, into the Matlab code; Q (t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 2.89.

4.2 Computations for the year 1992

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 2, into the Matlab code L (t) = 5962958*exp(0.032*t) yields 6,357,100.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 2; into the Matlab code; A(t)= 100*exp(0.032*t); yields 106.61.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=2;into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 943,000,000.00.

Production

Using equation 3.5.and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064, β=0.3, ρ= (1-β)/β and t=2; into the Matlab code; Y(t)=((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,303,200,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y(t-1) =3,172,500,000.00 and Y(t)=3,303,200,000.00, into the Matlab code; Q(t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 4,12.

Computations for the year 1993

Labour

Using equation 3.2.2 and substituting the values for L_0 =5,962,958, n-δ=0.032, and t= 3, into the Matlab code L(t)=5962958*exp(0.032*t) yields 6,563,800.

Total factor Productivity

Using equation 3.3.2 and substituting the values for A_0=100, g=0.032 and t= 3; into the Matlab code; A(t)= 100*exp(0.032*t); yields 110.08.

Capital

Using equation 3.4.5 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, β=0.3, ω=0.064 and t=3; into the Matlab code K(t)=((s*(μ^β)*(exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^(1/β) yields 994,860,000.00.

Production

Using equation 3.5.2 and substituting the values for K_0=846773730; s= 0.06; μ=596295800, ω=0.064, β=0.3, ρ= (1-β)/β and t=3; into the Matlab code;
Y(t) = ((μ^β)*exp(ω*β*t))*(((s*(μ^β)*exp(ω*β*t)-1)+ω*(846773730.1^β))/ω)^ρ; yields 3,440,100,000.00.

Growth Rate

Using equation 3.6.3, and substituting the values for Y (t-1) =3,303,200,000.00 and Y(t)= 3,440,100,000.00, into the Matlab code; Q(t)= ((Y(t)-Y(t-1))/Y(t-1))*100; yields 4.14%.

	Total Labour Force -		**Total Factor Productivity**		**Total Capital ($ US)**	
Year	**Actual**	**Calculated**	**Assumed**	**Calculated**	**Actual**	**Calculated**
1991	5,962,958	5,962,958	100	100.00	846,773,730.10	846,773,730.10
1992	6,170,246	6,156,900	100	103.25	1,044,279,211.00	893,680,000.00
1993	6,378,379	6,357,100	100	106.61	816,711,991.40	943,000,000.00
1994	6,630,762	6,563,800	100	110.08	1,418,975,139.00	994,860,000.00
1995	6,890,063	6,777,200	100	113.66	1,228,167,337.00	1,049,400,000.00
1996	7,154,239	6,997,600	100	117.35	1,364,512,238.00	1,106,700,000.00
1997	7,422,616	7,225,100	100	121.17	1,405,789,969.00	1,166,900,000.00
1998	7,695,831	7,460,100	100	125.11	1,640,841,947.00	1,230,200,000.00
1999	7,974,689	7,702,700	100	129.18	1,671,456,113.00	1,296,700,000.00

2000	8,260,518	7,953,100	100	133.38	1,578,001,319.00	1,366,600,000.00
2001	8,554,240	8,211,800	100	137.71	1,149,706,878.00	1,440,000,000.00
2002	8,808,936	8,478,800	100	142.19	1,439,998,829.00	1,517,100,000.00
2003	9,068,519	8,754,500	100	146.81	1,156,455,431.00	1,598,100,000.00
2004	9,331,979	9,039,200	100	151.59	1,748,749,261.00	1,683,100,000.00
2005	9,585,053	9,333,100	100	156.52	2,517,616,120.00	1,772,400,000.00
2006	9,852,131	9,636,600	100	161.61	3,109,129,779.00	1,866,200,000.00
2007	10,120,320	9,949,900	100	166.86	4,411,164,569.00	1,964,600,000.00
2008	10,376,027	10,273,000	100	172.29	4,953,021,277.00	2,067,900,000.00
2009	10,647,454	10,608,000	100	177.89	6,119,680,499.00	2,176,400,000.00
2010	10,925,982	10,952,000	100	183.68	5,122,231,687.00	2,290,200,000.00
2011	11,211,796	11,309,000	100	189.65	5,424,443,356.53	2,409,700,000.00

Table 1: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

	(Production)		(Production)		
	GDP (constant 2000 US$)		GDP Growth Rate		**Periods**
Year	Actual	**Calculated**	Actual	**Calculated**	**(t)**
1991	**3,266,886,838.00**	3,266,886,838.00	**3.33**	3.33	0
1992	**3,439,438,121.00**	3,172,500,000.00	**5.28**	2.89	1
1993	**3,572,868,343.00**	3,303,200,000.00	**3.88**	4.12	2
1994	**3,746,152,457.00**	3,440,100,000.00	**4.85**	4.14	3
1995	**3,869,775,489.00**	3,583,500,000.00	**3.27**	4.17	4
1996	**4,028,916,869.00**	3,733,600,000.00	**4.02**	4.19	5
1997	**4,214,346,194.00**	3,890,900,000.00	**4.6**	4.21	6
1998	**4,391,195,243.00**	4,055,600,000.00	**4.2**	4.23	7
1999	**4,597,598,580.00**	4,228,200,000.00	**4.57**	4.26	8
2000	**4,799,892,758.00**	4,409,200,000.00	**4.55**	4.28	9
2001	**4,977,488,790.00**	4,598,800,000.00	**3.74**	4.3	10
2002	**5,176,588,342.00**	4,797,700,000.00	**4.18**	4.33	11
2003	**5,409,534,817.00**	5,006,200,000.00	**4.46**	4.35	12
2004	**5,690,830,628.00**	5,224,900,000.00	**5.34**	4.37	13
2005	**6,009,517,143.00**	5,454,300,000.00	**5.58**	4.39	14

2006	**6,364,078,886.00**	5,695,000,000.00	**5.86**	4.41	15
2007	**6,771,379,934.00**	5,947,600,000.00	**6.43**	4.44	16
2008	**7,208,793,173.00**	6,212,800,000.00	**5.7**	4.46	17
2009	**7,816,530,776.00**	6,491,100,000.00	**7.23**	4.48	18
2010	**8,180,601,366.00**	6,783,400,000.00	**4.14**	4.5	19
2011	**8,664,892,967.00**	7,090,300,000.00	**5.92**	4.52	20

Table 2: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

	GDP/Production per labour		**Capital per Labour**		**Rate of Change of Capital**		**Rate of Change of Labour**	
Year	**($ US)**		**($ US)**		**(%)**		**(%)**	
-	**Actual**	**Calculated**	**Actual**	**Calculated**	**Actual**	**Calculated**	**Actual**	**Calculated**
1991	547.86	547.86	142.01	142.01	-	-	-	-
1992	557.42	515.28	169.24	145.15	23.32	5.54	3.48	3.25
1993	560.15	519.61	128.04	148.34	-21.79	5.52	3.37	3.25
1994	564.97	524.10	214.00	151.57	73.74	5.50	3.96	3.25
1995	561.65	528.76	178.25	154.84	-13.45	5.48	3.91	3.25
1996	563.15	533.55	190.73	158.15	11.10	5.46	3.83	3.25
1997	567.77	538.53	189.39	161.51	3.03	5.44	3.75	3.25
1998	570.59	543.64	213.21	164.90	16.72	5.42	3.68	3.25
1999	576.52	548.92	209.60	168.34	1.87	5.41	3.62	3.25
2000	581.06	554.40	191.03	171.83	-5.59	5.39	3.58	3.25
2001	581.87	560.02	134.40	175.36	-27.14	5.37	3.56	3.25
2002	587.65	565.85	163.47	178.93	25.25	5.35	2.98	3.25
2003	596.52	571.84	127.52	182.55	-19.69	5.34	2.95	3.25
2004	609.82	578.03	187.39	186.20	51.22	5.32	2.91	3.25

2005	626.97	584.40	262.66	189.90	43.97	5.31	2.71	3.25
2006	645.96	590.98	315.58	193.66	23.49	5.29	2.79	3.25
2007	669.09	597.75	435.87	197.45	41.88	5.27	2.72	3.25
2008	694.75	604.77	477.35	201.29	12.28	5.26	2.53	3.25
2009	734.12	611.91	574.76	205.17	23.55	5.25	2.62	3.26
2010	748.73	619.38	468.81	209.11	-16.30	5.23	2.62	3.24
2011	772.84	626.96	483.82	213.08	5.90	5.22	2.62	3.26

Table 3: Actual and Calculated Estimations

Sources of actual data: http://data.worldbank.org/country/ghana, MOFEP.

From Table 1 it was observed that the actual data for labour over the periods 1990 to 2010 seem to be building up over the period. But, the calculated figures of labour seemed to be growing faster than that of the actual labour supply.

The calculated figures for TFP seem to be growing over the period, indicating that in the long run when growth rate of capital is constant, growth in GDP may be as a result of change in the level education, change of technology, etc. This was not the case in Cobb-Douglas model, where TFP was assumed to be constant.

The actual Production (GDP) figures were evenly spread over the period. They were higher in certain periods and lower in other periods. The calculated figures for production consistently build up over the periods. It increases year after year; highest in the last year under consideration, 2010 with a figure of 7,090,300,000.00.

Considering the production/GDP growth rate, the following findings were apparent. The growth rates were evenly spread for most of the years under consideration, especially, 1997, 2001, and 2002.

The actual average growth rate from 1984 to 2010 according to http://data.worldbank.org/country/ghana and African Development indicator 2007 CD Rom for Ghana was 4.5% which is relatively close to the calculated average growth rate of 4.21%. The graphs below show these trends.

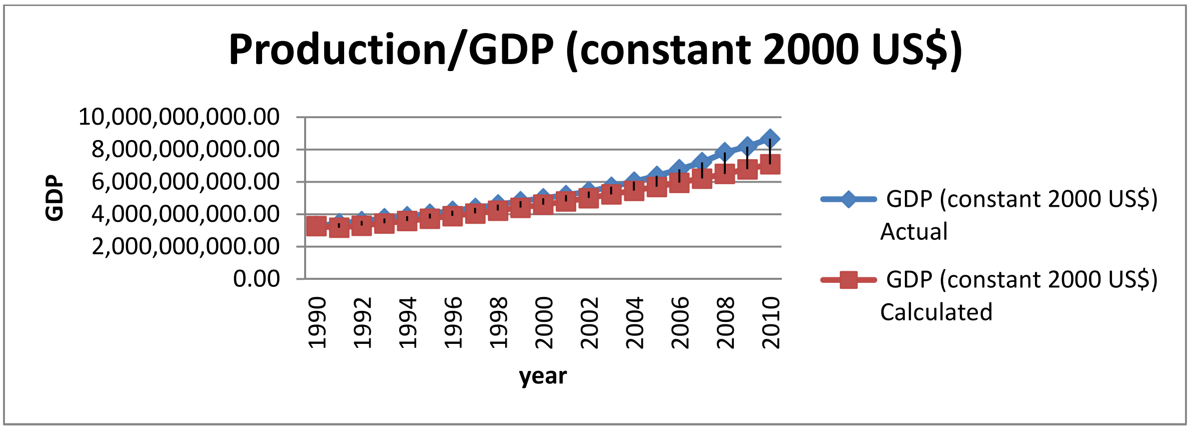

Figure 1: Production/GDP (constant 2000 US$)

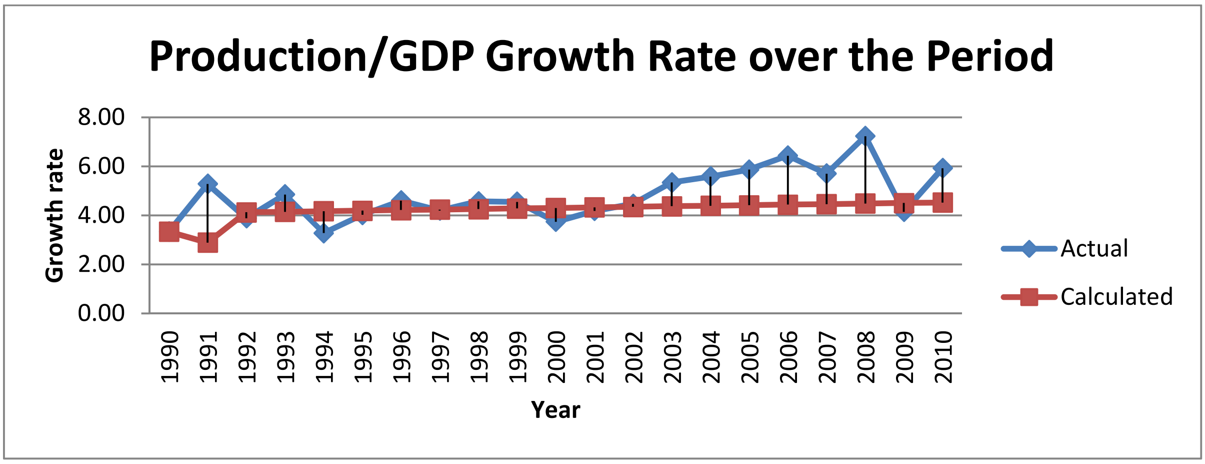

Figure 2: Production/GDP Growth Rate

Figure 1 depicts the production/GDP for the economy, which builds up from year to year. The actual production values were always above that of the calculated throughout the periods under consideration.

Figure 2 depicts the rate at which production builds up from year to year. From the graph it is apparent to see that growth rates were positive throughout the period under consideration.

It was also interesting to know that the difference between the growth rates were marginal. The actual growth rates for some periods were always above that of the calculated throughout the periods, except 1992, 1994, 2000 and 2009.

The actual average growth rate over the period was 4.5% as compared to the calculated value of 4.21%. From the graph the computed figures appear in red line. This is so because the variances between the rates were very small. This showed that the growth rate of Ghana has been slow as was noted by Aryeetey and Fosu (2004).

Production per Labour

The actual production per labour started from a high figure of $ 547.86 per labour in 1990 and kept increasing throughout the period.

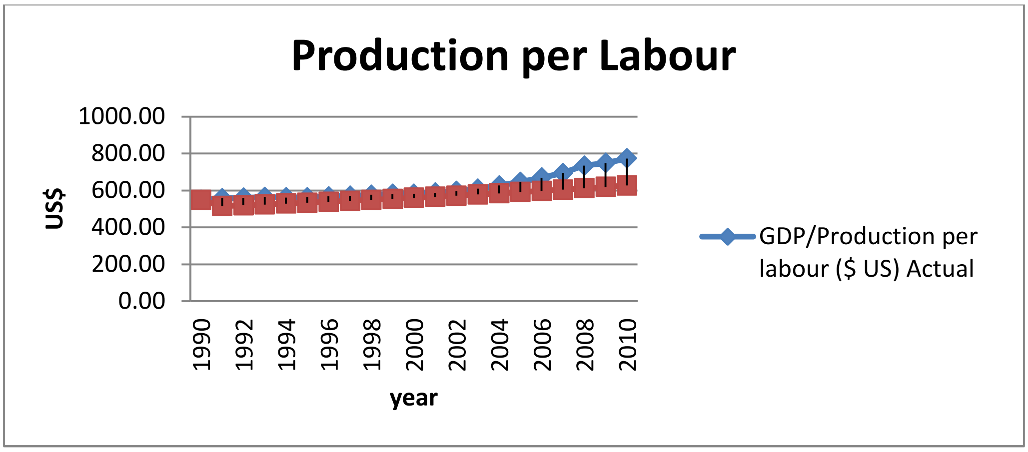

Figure 3: Production per Labour

The calculated production per labour started at $547.86 and reduced to $515.28 in 1992 and started increasing from1994. The variances between the figures were very small from 1991 to 2005 and afterwards the actual figures started growing faster than the calculated.

Capital per Labour

Actual capital per labour was uneven over the periods under consideration as is depicted on the graph below. It increases and decreases throughout the period. It is neither increasing nor decreasing throughout the periods. The figure ranges between $142.01 per labour in 1990 and $483.82 in 2011.

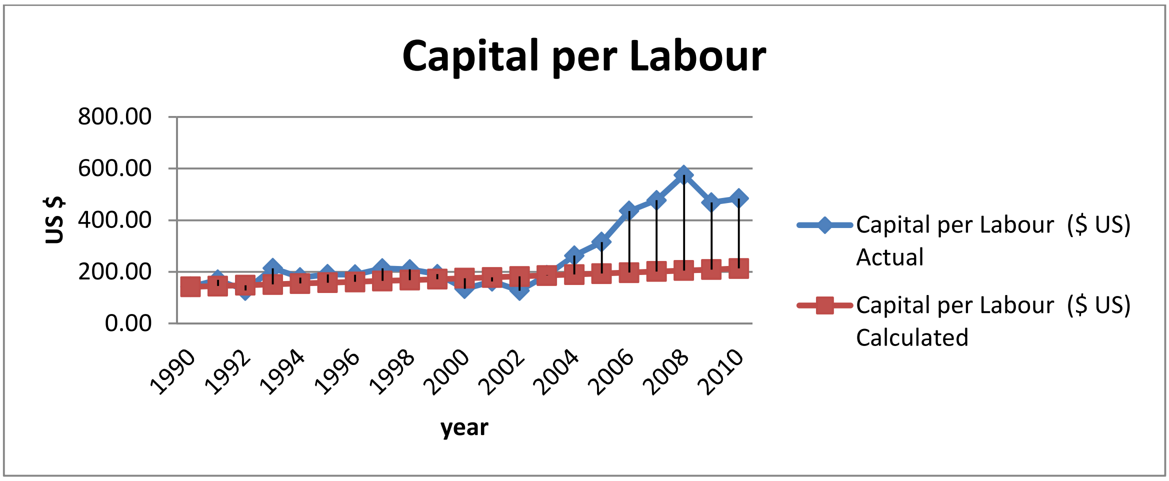

Figure 4: Capital per Labour

The calculated capital per labour started with a figure of $145.15 in 1991 grew steadily over the period to record $213.08 per labour in 2011.

Percentage change in Capital

As is depicted in figure 5, the actual percentage change was uneven over the periods. It ranges between a low figure -27.14 percent in 2001 to as high a figure of 73.74 percent in 1994. There were fluctuations in the rate of change of capital over the periods.

The calculated change in capital ranges between 5.22 percent in 2011 and 5.54 percent in 1991. But generally the percentage change lies between 0 and 100% making the graph smooth out around the horizontal axis.

Rate of change of Labour Supply

It is amazing figure 6 showed similarities between actual and calculated rate of change in Labour Supply. The only difference was that the calculated rates of change were always higher for the periods 2002 to 2011 and vice versa for the periods before 2002.

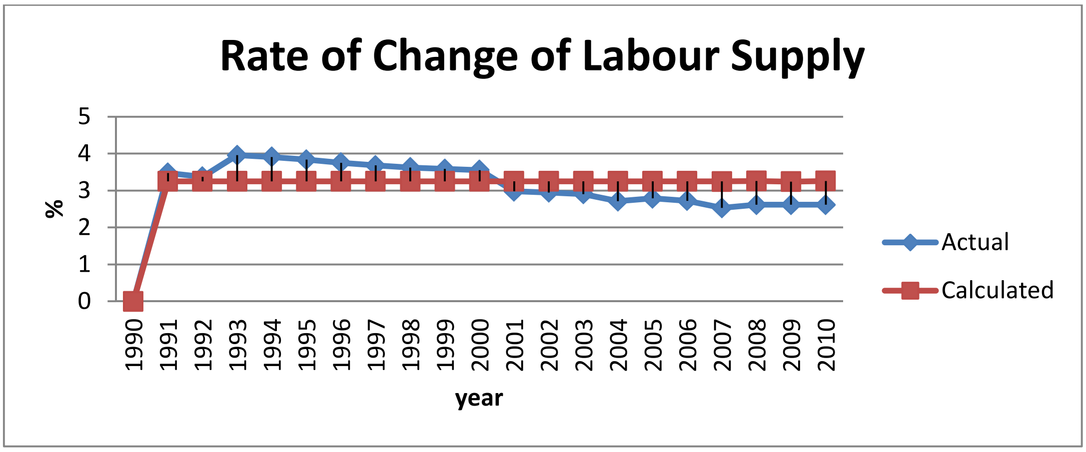

Figure 6: Rate of change of Labour Supply

Though the changes were uneven, the two lines showed similar patterns. The actual figure ranges between 1% and 4.2% whilst the calculated ranges between 2% and 5%.

Also we observed that the correlation coefficient between the actual growth rates and calculated was 0.298, indicating that the two variables are correlated, but the strength of correlation is weak.

5.0 Conclusions:

Economic growth is defined as the increase in the amount of goods and services produced by an economy over time. Economy's performance is generally measured using GDP (Gross Domestic Product) by Economists. Being able to discern and measure progress more comprehensively that with GDP, is a key prerequisite for improved decision making.

The principal determinants of the economy's growth as used in this study were capital, labour and Total Factor productivity (T.F.P).

The determinants have the following effects on the growth of the economy; α and β are the output elasticities of labour and capital respectively are constants determined by available technology. Output elasticity measures the responsiveness of output to a change in levels of either labour or capital used in the production, ceteris paribus. Cobb-Douglas model assumed that α= 0.7 and β= 0.3, which was one of the assumptions used in the models. For example if: α= 0.2, a 1% increase in labour would lead to approximately a 0.2% increase in output. Further, if: α+ β= 1, the production function has a constant returns to scale. That is, if L and K are each increased by 20%, Y increases by 20%. If: α+ β<1, returns to scale are decreasing, and if: α+ β> 1, returns to scale is increasing. Assuming perfect competition, α and β can be shown to be labour capital's share of output.

Solow simplify that an economy-wide production function as $Y = K^{\alpha}(A.L.)^{\beta}$ Solow assumed that TFP is proportional to itself, unlike Cobb-Douglas, which estimated TFP to be equal to 1 in their function. Solow again argued that, since technology changes over time and increases in the level of education also changes over time, it was prudent to allow TFP to also grow proportional to it.

A little work needs to be done on the parameters α, β from time to time in the production function.

The appropriate model is the model that answers all the questions of the system being described by the models. For example our production model of the economy's growth at equation 3.7.3, could be said to describe the system of economy's growth if it computes the intended output of the economy appropriately.

Finally, comparing the outcome of our model to the actual data we gathered from the Ghana Statistical Service, World Bank Group and MOFEP. The results showed a very close relationship between the actual (4.5%) and calculated (4.21%) average growth rate over the periods 1990 to 2010. This model predicted the growth pattern very well and it is our thinking that a little work needs to be done on the parameters to put the results into perspective. Therefore, Solow’s model predicts the growth pattern of Ghana’s economy.

Finally, as the economy strives to achieve middle income status, savings and investment rates should be encouraged to increase the parameter, s. This will lead to an increase in the capital stock and thus shift the rate of growth of real GDP from its current average of 4.5% to about 9% or higher. The study finds a significant positive relationship between real GDP and the level of the capital stock in both the short-run dynamic and the long-run static models.

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