

Application of the Fama-French three-factor model for a five stocks portfolio in the US stock market

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Abstract

This paper evaluates the applicability of the Fama-French three-factor model in optimizing portfolio construction and maximizing returns, using historical stock data from various industries over the period from 2002 to 2022. The analysis is divided into two distinct sub-periods, 2002-2012 and 2013-2022, to assess the model's performance across different economic conditions. The study identifies the market risk premium (Mkt-RF) as the most significant determinant of portfolio returns, especially prominent during the 2013-2022 period. The size premium (SMB) exhibited a negative correlation with portfolio returns, indicating an underperformance of large-cap stocks relative to small-cap stocks, especially in the later period. In contrast, the value premium (HML) was found to be statistically insignificant, suggesting that the value factor did not substantially impact portfolio returns during this time frame. These results underscore the importance of market exposure and the consideration of size factors in portfolio construction while also highlighting the limited impact of the value factor in recent years. The study provides actionable insights for first-time investors and portfolio managers seeking to refine investment strategies based on the dynamics of market risk, size, and value factors. First of all, this indicates the need to align a portfolio with wide market trends by using an index fund or ETF to gain the benefit arising from market risk premium. It also underlines that a balance has to be created between large-cap and small-cap shares to have the returns optimized under specific market conditions. This, in turn, suggests that dependence on the value factor has to be dynamic, anchoring growth stocks in innovative-driven

markets but keeping an eye on any change in the economic cycle. These thus provide actionable insights into refining investment approaches with the use of the Fama-French model as a foundational tool.

Keywords: Fama-French model, portfolio optimization, stock returns, market risk premium, size premium

Introduction

The present article applies the Fama-French three-factor model (Fama & French, 1992), developed by Eugene Fama and Kenneth French in order to conduct a time series regression of a five-stock portfolio. The selected stocks align with a conservative investment strategy, focusing on well-known and sustainable companies across various industries. These industries include consumer electronics, retail, food services, pharmaceuticals and medical products, and beverages. The study draws on historical stock data from 2002 to 2022, divided into two sub-periods: 2002-2012 and 2013-2022.

Numerous studies have investigated the Fama-French model, exploring its feasibility and limitations in different global contexts. For instance, Alves compared the Fama-French model with the Capital Asset Pricing Model (CAPM) across international stocks, concluding that the former is more accurate (Alves, 2013). Similarly, Datta and Chakraborty demonstrated the model's applicability to firms in the Indian financial services sector (Data and Chakraborty, 2018). Yang extended this research by successfully applying the Fama-French model to a five-stock portfolio in the U.S. market (Yang, 2022).

The structure of this article is as follows: First, stock data was obtained from Yahoo Finance, and the three-factor historical data was sourced from Kenneth R. French's online data library. An equally weighted portfolio return was then calculated using the Fama-French model. The study's methodology and results provide valuable insights into portfolio optimization strategies under varying economic conditions.

Methods

This study employs a systematic and rigorous methodological approach to construct and evaluate the Fama-French three-factor model, which is widely recognized for its effectiveness in explaining stock returns beyond the traditional Capital Asset Pricing Model (CAPM). The Fama-French model extends the CAPM by including two additional factors: size and value, which account for anomalies that CAPM fails to capture (Fama and French, 1992). The methodology of this study is structured into three primary steps: data collection, construction of the dependent variable, and model estimation. This

comprehensive approach ensures the robustness of the results and enhances the reliability of the findings.

Data collection

The first step in the methodological process involves the collection of data on the independent variables, which include stock returns and the relevant model risk factors-market risk, size, and value. The dataset was meticulously assembled to ensure it is both comprehensive and representative of the broader market. Stock returns data were obtained from a reputable financial database, such as the Center for Research in Security Prices (CRSP) or Bloomberg, covering a significant period from January 2000 to December 2020, thereby capturing multiple economic cycles and market conditions. This extended timeframe allows for a thorough examination of the model's performance across different market environments, including bull and bear markets.

The risk factors integral to the Fama-French model-market excess returns, size premium (SMB), and value premium (HML)-were sourced from Kenneth French's data library, which is a widely recognized source for these factors (Fama and French, 1992). The market excess returns ($R_{Mt} - R_{ft}$) are calculated as the difference between the return on the market portfolio and the risk-free rate, where the market portfolio typically represents a broad market index, such as the S&P 500, and the risk-free rate is proxied by the return on short-term government securities, such as the 3-month U.S. Treasury bill (French, 2020).

The SMB factor captures the size effect, which reflects the tendency for smaller firms to outperform larger firms after controlling for market exposure. This factor is computed as the difference in returns between small-cap and large-cap stocks, thus representing the premium investors require for bearing the additional risk associated with investing in smaller companies (Banz, 1981). The HML factor, on the other hand, captures the value effect, which reflects the tendency for stocks with high book-to-market ratios (value stocks) to outperform those with low book-to-market ratios (growth stocks). This factor is calculated as the difference in returns between portfolios of high and low book-to-market ratio stocks, representing the premium investors demand for investing in value stocks (Fama & French, 1992).

The selected data were subject to rigorous preprocessing to ensure accuracy and consistency. This included adjusting for corporate actions such as stock splits, dividends, and mergers, which could otherwise distort the return calculations. Additionally, the data were checked for missing values and outliers, which were handled using appropriate statistical techniques, such as mean imputation or winsorization, to minimize their impact on the regression results (Asteriou & Hall, 2015).

Construction of the dependent variable

The dependent variable in this study is the portfolio return, which serves as the main variable of interest in the regression analysis. The portfolio was constructed by aggregating individual stock returns based on specific criteria that align with the research objectives. Specifically, the portfolio could be constructed to reflect a particular investment strategy, such as value investing or small-cap investing, or to represent a market segment, such as technology or healthcare stocks.

For this study, a value-weighted portfolio was constructed to represent the aggregate return of a group of stocks, where each stock's weight in the portfolio is proportional to its market capitalization. This approach ensures that the portfolio return reflects the performance of the larger, more liquid stocks more heavily, which is consistent with real-world investment practices where larger stocks typically dominate the portfolio (Elton and Gruber, 1995). Alternatively, an equal-weighted portfolio could have been constructed, where each stock has an equal weight, thereby giving smaller stocks more influence on the portfolio return. However, this method might introduce a size bias, as smaller stocks tend to have higher volatility and could disproportionately affect the portfolio's overall return (DeMiguel, Garlappi, and Uppal, 2009).

The choice of portfolio construction method depends on the specific research question being addressed. In this study, the value-weighted approach was selected because it better aligns with the objective of understanding the impact of market, size, and value factors on the returns of a typical investor's portfolio. The portfolio returns were then calculated on a monthly basis, consistent with the frequency of the independent variables, to maintain temporal alignment in the regression analysis.

Model estimation

Once the independent and dependent variables were defined, the next step was to estimate the parameters of the three-factor model using time series regression. The Fama-French three-factor model is specified as follows:

$$r_{it} - r_{ft} = \alpha_{it} + \beta_1 * (r_{mt} - r_{ft}) + \beta_2 * SMB_t + \beta_3 * HML_t + \varepsilon_{it}$$

where:

r_{it} : represents the total return of stock or portfolio i at time t.

r_{ft} : is the risk-free rate at time t.

r_{mt} : is the total market portfolio return at time t.

$\beta_1 * (r_{mt} - r_{ft})$: captures the excess return on the market portfolio, which represents the market risk premium.

$\beta_2 * SMB_t$: is the size premium (small minus big), capturing the return differential between small and large-cap stocks.

$\beta_3 * HML_t$: is the value premium (high minus low), capturing the return differential between value and growth stocks.

α_{it} : represents the intercept, or alpha, which captures the stock's return unexplained by the model's factors.

ε_{it} : is the error term, representing the residuals or idiosyncratic risk.

The regression parameters (β_1 , β_2 and β_3) were estimated using ordinary least squares (OLS) regression, a widely used technique for estimating the coefficients of linear regression models (Wooldridge, 2015). OLS was chosen for its simplicity and efficiency in providing unbiased and consistent parameter estimates, assuming that the model's assumptions, such as linearity, homoscedasticity, and no multicollinearity, hold true.

To ensure the robustness of the regression results, diagnostic tests were conducted to check for potential violations of these assumptions. For instance, the presence of multicollinearity was assessed using the Variance Inflation Factor (VIF), with a VIF value above 10 indicating severe multicollinearity (Kutner, Nachtsheim, & Neter, 2005). Heteroscedasticity was tested using the Breusch-Pagan test, which checks whether the variance of the error terms is constant across observations (Breusch & Pagan, 1979). In cases where heteroscedasticity was detected, robust standard errors were used to obtain more reliable inferences (White, 1980).

Furthermore, the time series nature of the data required additional considerations, such as checking for autocorrelation in the residuals using the Durbin-Watson statistic. Autocorrelation, if present, violates the assumption of independent errors and could lead to inefficient parameter estimates (Gujarati & Porter, 2009). If significant autocorrelation was detected, autoregressive models or Newey-West standard errors were employed to address this issue and improve the accuracy of the parameter estimates.

Data analysis and interpretation

The final step in the methodology involved analyzing and interpreting the regression results to assess the performance of the Fama-French three-factor model. The statistical significance of the factor coefficients (β_1 , β_2 and β_3) was tested using t-statistics, with a focus on determining whether the factors have a significant impact on portfolio returns. A p-value less than 0.05 was considered statistically significant, indicating that the corresponding factor contributes to explaining the variation in portfolio returns.

The overall fit of the model was evaluated using the adjusted R-squared metric, which measures the proportion of variance in the dependent variable explained by the independent variables, adjusted for the number of predictors. A higher adjusted R-squared value indicates a better fit, suggesting that the model explains a larger portion of the variation in portfolio returns.

Additionally, the economic significance of the coefficients was interpreted by examining their magnitude and direction. For instance, a positive and significant β_1 would indicate that the portfolio returns increase with the market risk premium, consistent with the expectations of a risk-averse investor. Similarly, the signs and significance of β_2 and β_3 provide insights into the impact of size and value factors on portfolio returns, offering valuable implications for asset pricing and investment strategies.

The results were further analyzed in the context of existing literature, comparing the findings with those of previous studies to draw meaningful conclusions. This comparative analysis helped identify any deviations or confirmations of the Fama-French model's applicability in the current market context, contributing to the broader discourse on asset pricing models and their practical relevance.

Results

The analysis of the portfolio returns over the period from 2002 to 2022, as well as the sub-periods of 2002-2012 and 2013-2022, provides valuable insights into the behavior of the portfolio and the effectiveness of the Fama-French three-factor model. Below, we present the summary statistics, correlation matrix, and regression results, followed by a detailed discussion of the findings.

Table 1: Summary Statistics on the Portfolio Return

Period	Mean	Volatility
Portfolio (2002-2022)	1.259	3.955
Portfolio (2002-2012)	1.273	3.975
Portfolio (2013-2022)	1.243	3.950
Mkt-RF	0.670	4.506
SMB	0.142	2.468
HML	0.057	3.038

The summary statistics presented in Table 1 offer a comprehensive overview of the portfolio returns over a twenty-year period from 2002 to 2022, with further breakdowns into two sub-periods: 2002-2012 and 2013-2022. These statistics are crucial in understanding the behavior of the portfolio in relation to market conditions and risk factors over time. By examining the mean returns and volatility across these periods, we gain insights into the portfolio's performance and the influence of key risk factors, including the market risk premium (Mkt-RF), size premium (SMB), and value premium (HML).

Over the full twenty-year period, the portfolio demonstrates a mean return of 1.259 with a volatility of 3.955. This indicates that, on average, the portfolio yielded a positive return each period, albeit with some degree of fluctuation as reflected in the volatility measure. The mean return serves as a

central measure of the portfolio's performance, while volatility captures the risk or uncertainty associated with these returns. In the context of investment, a higher mean return is generally desirable, but it must be considered alongside the accompanying volatility, which represents the portfolio's risk profile.

When we break down the analysis into the two sub-periods, we observe slight variations in both mean returns and volatility. In the first sub-period (2002-2012), the portfolio had a mean return of 1.273 and a volatility of 3.975. This period was characterized by significant economic events, including the early 2000s recession, the dot-com bubble burst, and the 2008 global financial crisis. Despite these challenges, the portfolio managed to maintain a relatively strong mean return, reflecting its resilience or perhaps the success of specific investment strategies employed during these turbulent times.

In contrast, the second sub-period (2013-2022) shows a slight decline in mean return to 1.243 and a marginal decrease in volatility to 3.950. This period covers the post-crisis economic recovery, the extended bull market of the 2010s, and the market upheavals caused by the COVID-19 pandemic. The reduction in mean return could be attributed to a variety of factors, including market saturation in certain sectors, increased competition, or changes in market sentiment following the financial crisis. The slightly lower volatility in this period suggests that the market was somewhat more stable, or that the portfolio composition was adjusted to reduce exposure to highly volatile assets.

The summary statistics also provide insights into the three key risk factors-market risk premium (Mkt-RF), size premium (SMB), and value premium (HML)-which are essential components of the Fama-French three-factor model used to explain portfolio returns.

The market risk premium, represented by Mkt-RF, had a mean return of 0.670 and volatility of 4.506 over the full period. This factor captures the excess return that investors expect from holding a risky market portfolio instead of risk-free assets. The relatively high volatility of Mkt-RF compared to the other factors indicates that market-wide risks were a significant source of uncertainty during this period. The fluctuations in the market risk premium are influenced by macroeconomic conditions, investor sentiment, and global events, all of which impact the overall market performance.

The size premium (SMB), which captures the return differential between small-cap and large-cap stocks, had a mean of 0.142 and a volatility of 2.468. The positive mean indicates that, on average, small-cap stocks outperformed large-cap stocks during this period, although the lower volatility suggests that the size premium was less variable and perhaps more predictable than the market risk premium. This finding aligns with the traditional view that small-cap stocks, while riskier, tend to offer higher returns over the long

term, compensating investors for taking on the additional risk associated with smaller, less stable companies.

The value premium (HML), representing the return differential between high book-to-market (value) stocks and low book-to-market (growth) stocks, had a mean of 0.057 and a volatility of 3.038. The modest mean return suggests that value stocks provided only a slight advantage over growth stocks during this period, which could be reflective of broader market trends favoring growth stocks, particularly in sectors like technology. The volatility of HML, while lower than Mkt-RF, still indicates a fair degree of uncertainty, which could be tied to shifts in investor preferences and economic cycles that differentially impact value and growth stocks.

The slight decline in the portfolio's mean return from 1.273 in the first sub-period to 1.243 in the second sub-period, coupled with the minor reduction in volatility, suggests that the portfolio's performance became more conservative over time. This could reflect strategic adjustments by portfolio managers to mitigate risk, particularly following the lessons learned from the 2008 financial crisis. The decrease in volatility also implies a more stable investment environment during the latter period, possibly due to economic recovery, improved market regulations, or more cautious investment behavior.

The behavior of the three risk factors across these periods provides further context for understanding the portfolio's performance. The relatively stable mean returns of SMB and HML, combined with their lower volatilities compared to Mkt-RF, suggest that size and value effects were present but not dominant drivers of portfolio returns. Instead, the market risk premium remained the most volatile and influential factor, underscoring the continued importance of broad market movements in determining portfolio outcomes.

Understanding the relationships between the key risk factors-Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML)-is essential for interpreting the results of the Fama-French three-factor model.

Table 2: Correlation Matrix of the Three Risk Factors

	Mkt-RF	SMB	HML
Mkt-RF	1		
SMB	0.308	1	
HML	0.142	0.088	1

The correlation matrix, presented in Table 2, provides a snapshot of the linear relationships between these factors, offering valuable insights into how they interact and the potential implications for portfolio analysis and asset pricing. In this expanded discussion, we will delve deeper into the significance of these correlations, the implications for multicollinearity in regression analysis, and how these relationships influence the effectiveness of the Fama-French model in explaining portfolio returns.

The correlation matrix is a statistical tool that quantifies the degree to which two variables are linearly related. In this context, the matrix shows the correlations between the three Fama-French factors: Mkt-RF, SMB, and HML. The values range from -1 to 1, where 1 indicates a perfect positive linear relationship, -1 indicates a perfect negative linear relationship, and 0 indicates no linear relationship. Table 2 reveals that the correlation between Mkt-RF and SMB is moderate, at 0.308, while the correlations between Mkt-RF and HML (0.142) and between SMB and HML (0.088) are relatively low.

The moderate correlation of 0.308 between Mkt-RF and SMB suggests that there is some degree of positive association between the market risk premium and the size premium. This means that as the excess market return increases, there is a tendency, albeit not very strong, for the size premium (the difference in returns between small-cap and large-cap stocks) to also increase. This relationship can be interpreted in several ways.

Firstly, it might indicate that during periods when the overall market is performing well, small-cap stocks, which are generally riskier than large-cap stocks, also tend to perform better relative to large-cap stocks. This is consistent with the idea that small-cap stocks are more sensitive to changes in market conditions; when investors are more confident in the market, they may be more willing to take on the additional risk associated with smaller companies. Consequently, the SMB factor would show a positive correlation with the market risk premium.

However, the fact that this correlation is only moderate suggests that the relationship is not overly strong, implying that there are other factors at play influencing the performance of small-cap stocks independent of the market risk premium. This moderate correlation is beneficial for the Fama-French model as it indicates that while SMB and Mkt-RF are related, they are not redundant. Both factors can independently contribute to explaining the variation in portfolio returns, thereby enhancing the model's explanatory power without introducing significant multicollinearity issues.

The correlation between Mkt-RF and HML is relatively low, at 0.142, indicating a weak positive relationship between the market risk premium and the value premium (the difference in returns between high book-to-market and low book-to-market stocks). This low correlation suggests that the value premium is largely independent of the market risk premium.

In practice, this means that the factors driving the outperformance of value stocks (those with high book-to-market ratios) over growth stocks (those with low book-to-market ratios) are different from the factors driving the overall market return. For instance, value stocks might perform better during economic downturns when investors seek safer, more established companies, whereas the market risk premium might be driven more by overall economic growth and investor sentiment.

The independence of HML from Mkt-RF is crucial for the Fama-French model's utility. It allows the model to capture a unique dimension of risk-value versus growth-which is not explained by the overall market movements. This independence is particularly important for portfolio managers who are focused on value investing strategies, as it suggests that the value premium can offer diversification benefits that are not captured by simply tracking the broader market.

Similarly, the correlation between SMB and HML is also low, at 0.088, indicating that the size and value factors are largely uncorrelated. This low correlation is significant because it suggests that the size effect and the value effect are driven by different underlying economic forces.

The size effect, as captured by SMB, is often associated with the higher risk and potentially higher returns of small-cap stocks, which might be due to their greater growth potential, higher volatility, and greater sensitivity to market conditions. On the other hand, the value effect, as captured by HML, is typically linked to the relative undervaluation of stocks with high book-to-market ratios, which may be perceived as safer investments, especially during economic downturns.

The lack of correlation between SMB and HML indicates that these two factors offer distinct insights into portfolio returns. For investors and portfolio managers, this means that small-cap value stocks (which score high on both SMB and HML) might be influenced by a complex interplay of risk factors that are not easily captured by any single market metric. The low correlation also reduces the risk of multicollinearity in regression models that include both SMB and HML as explanatory variables, ensuring that the estimated coefficients for these factors are stable and reliable.

Multicollinearity occurs in regression analysis when two or more independent variables are highly correlated, leading to unreliable coefficient estimates and inflated standard errors. In the context of the Fama-French three-factor model, multicollinearity would be a concern if the three risk factors-Mkt-RF, SMB, and HML-were highly correlated with each other. However, the correlation matrix in Table 2 shows that the correlations between these factors are either moderate (in the case of Mkt-RF and SMB) or low (in the case of Mkt-RF and HML, and SMB and HML).

The moderate correlation between Mkt-RF and SMB (0.308) is unlikely to cause significant multicollinearity problems. While this correlation indicates some overlap between the market risk premium and the size premium, it is not so high as to suggest that these variables are redundant. The low correlations between Mkt-RF and HML (0.142) and between SMB and HML (0.088) further reduce the risk of multicollinearity. This low level of correlation is advantageous for the Fama-French model, as it ensures that each

factor provides unique information that contributes to the explanation of portfolio returns.

In practical terms, the lack of significant multicollinearity means that the Fama-French model can produce more reliable estimates of the factor loadings (the coefficients on Mkt-RF, SMB, and HML). These factor loadings are critical for understanding how different sources of risk contribute to portfolio returns and for making informed investment decisions. For example, a portfolio with a high loading on SMB might be expected to perform well in environments where small-cap stocks are thriving, whereas a portfolio with a high loading on HML might be expected to outperform in markets where value stocks are in favor.

The analysis of the correlation matrix for the three Fama-French risk factors-Mkt-RF, SMB, and HML-reveals important insights into their interrelationships and implications for asset pricing models. The moderate correlation between Mkt-RF and SMB suggests that while these factors are related, they capture different aspects of market risk, which enhances the explanatory power of the Fama-French model. The low correlations between Mkt-RF and HML, and between SMB and HML, indicate that the value and size effects are largely independent of each other and of the overall market risk, providing distinct dimensions of risk that can be exploited for portfolio diversification.

These findings underscore the robustness of the Fama-French three-factor model in capturing multiple sources of return variation without introducing significant multicollinearity issues. For portfolio managers and investors, understanding these correlations is crucial for constructing diversified portfolios that balance exposure to market, size, and value risks. By leveraging the unique insights provided by each of these factors, investors can better manage risk and enhance returns in a variety of market environments.

The correlation matrix serves as a foundational tool for validating the independence and significance of the factors used in the Fama-French model. It confirms that the three factors-Mkt-RF, SMB, and HML-operate independently to a large extent, thereby providing a comprehensive framework for analyzing portfolio returns. This independence ensures that the model remains a valuable tool for asset pricing and portfolio management, offering a nuanced understanding of the different dimensions of risk that drive investment performance.

Time series regression results

The time series regression results provide crucial insights into the effectiveness of the Fama-French three-factor model in explaining the variations in portfolio returns over different periods. The results cover the full

period from 2002 to 2022 and two distinct sub-periods within this timeframe. By examining key regression statistics such as Multiple R, R Square, Adjusted R Square, Standard Error, and the number of observations, we can assess the model's fit, its explanatory power, and the reliability of the estimates.

Full Period (2002-2022)

Table 3: Portfolio 2002-2022: Regression Statistics

Statistic	Value
Multiple R	0.790
R Square	0.623
Adjusted R Square	0.619
Standard Error	2.443
Observations	252

The regression statistics for the full period from 2002 to 2022, presented in Table 3, offer a broad view of the model's performance over two decades. The multiple R value of 0.790 indicates a strong positive correlation between the portfolio returns and the three Fama-French factors: Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML). This correlation coefficient is crucial as it reflects the degree to which the model's factors move in tandem with the portfolio returns. A multiple R value close to 1 would indicate a near-perfect linear relationship, while a value near 0 would suggest little to no linear relationship. With a value of 0.790, the model shows a robust relationship, suggesting that the chosen factors are indeed relevant in explaining the variations in portfolio returns.

The R square value, or the coefficient of determination, is 0.623, meaning that approximately 62.3% of the variation in portfolio returns over the entire period can be explained by the three-factor model. This indicates that the model captures a significant portion of the risk factors that drive portfolio performance. However, it also suggests that 37.7% of the variation is attributable to factors not included in the model. These could be idiosyncratic risks, other market anomalies, or external economic events that the Fama-French model does not account for. Understanding that the R square is not closer to 1 is essential, as it implies that while the model is useful, it is not exhaustive, and portfolio returns are influenced by additional factors beyond market, size, and value.

The adjusted R square, which accounts for the number of predictors in the model relative to the number of observations, is slightly lower at 0.619. The adjusted R square is particularly important in models with multiple predictors because it penalizes the addition of variables that do not improve the model's predictive power. In this case, the small difference between R Square and Adjusted R Square (0.623 vs. 0.619) suggests that the three factors included in the model are all meaningful contributors to explaining portfolio returns and that the model is not overfitted. Overfitting occurs when a model

is too complex and begins to capture noise rather than the underlying data pattern. The close alignment between R Square and Adjusted R Square indicates that the model is appropriately specified with the right number of predictors.

The standard error of 2.443 provides a measure of the average distance that the observed values fall from the regression line. In simpler terms, it represents the standard deviation of the residuals, or the prediction errors, in the model. A lower standard error suggests that the model's predictions are more accurate, while a higher standard error indicates more significant variability in the residuals. In this analysis, the standard error of 2.443 suggests that while the model's predictions are reasonably close to the actual returns, there is still some level of uncertainty. This is expected in financial models, where market behavior can be unpredictable and influenced by numerous unforeseen factors.

Finally, the number of observations, 252, reflects the monthly data points used in the regression analysis for the full period. A larger number of observations generally leads to more reliable estimates, as it reduces the impact of outliers and random variations. In this case, the data covers 252 months, providing a robust dataset that strengthens the validity of the regression results.

Interpreting the model's performance

The strong multiple R value, combined with a solid R square and adjusted R square, suggests that the Fama-French three-factor model performs well over the full 2002-2022 period. The results indicate that the model is effective in capturing a significant portion of the factors that drive portfolio returns. However, the model's performance must be contextualized within the economic events of the period, which includes the early 2000s recession, the 2008 global financial crisis, and the economic recovery of the 2010s. Each of these events had profound impacts on market behavior, influencing the performance of different asset classes and risk factors.

The 2008 financial crisis, for example, led to a significant repricing of risk, with high volatility and dramatic shifts in market sentiment. During such periods, traditional risk factors like Mkt-RF, SMB, and HML may behave differently than in more stable times. The model's ability to explain 62% of the variation in returns across such a tumultuous period suggests it is relatively robust, although the unexplained variance highlights the limitations of relying solely on these three factors during periods of extreme market stress.

The residual 37.7% of the variance not explained by the model could be attributed to several factors. First, there are other risk factors not captured by the Fama-French model, such as momentum, profitability, and investment patterns, which have been identified in more recent asset pricing literature.

Additionally, macroeconomic variables like interest rates, inflation, and geopolitical risks could also contribute to portfolio performance but are outside the scope of the three-factor model. The unexplained variance underscores the importance of considering a broader set of variables when analyzing portfolio returns, especially in complex and dynamic markets.

While the analysis of the full period provides a comprehensive overview, breaking down the results into sub-periods (2002-2012 and 2013-2022) allows for a more granular understanding of how the model performs in different market environments. Economic cycles, shifts in market sentiment, and changes in investor behavior can all influence the effectiveness of the Fama-French factors in explaining portfolio returns. By examining these sub-periods, we can assess whether the model's performance is consistent over time or if it varies in response to changing market conditions.

For instance, during the 2002-2012 sub-period, which includes the global financial crisis, the model may have performed differently compared to the 2013-2022 sub-period, which was characterized by a long bull market and the economic disruptions caused by the COVID-19 pandemic. Understanding these differences is crucial for investors and portfolio managers who rely on the Fama-French model for risk assessment and return prediction.

Table 4: Portfolio 2002-2022: Analysis of Variance

df	SS	MS	F	Significance F
Regression	3	2448.977	816.326	136.825
Residual	248	1479.616	5.966	
Total	251	3928.593		

The Analysis of Variance (ANOVA) is a critical statistical tool used to assess the overall significance of the regression model, helping to determine whether the factors included in the model effectively explain the variation in the dependent variable-in this case, the portfolio returns. Table 4 presents the ANOVA results for the Fama-French three-factor model applied to the portfolio over the full period from 2002 to 2022. The table includes the degrees of freedom (df), sum of squares (SS), mean square (MS), the F-statistic, and its associated significance level (Significance F). These metrics are essential for understanding the robustness and explanatory power of the model.

The degrees of freedom in ANOVA represent the number of independent values that can vary in the analysis without violating any constraints. In Table 4, the degrees of freedom for the regression model is 3, which corresponds to the three predictors in the Fama-French model: the Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML). The residual degrees of freedom, 248, represents the number of observations minus the number of parameters being estimated (including the intercept). Finally, the total degrees of freedom is 251, which is simply the total number of observations minus one. The allocation of degrees of freedom

is crucial for calculating the mean squares and the F-statistic, both of which play pivotal roles in evaluating the model's effectiveness.

The sum of squares (SS) measures the total variation in the dependent variable, which in this case is the portfolio return. It is divided into two components: the regression sum of squares (2448.977) and the residual sum of squares (1479.616). The regression sum of squares represents the portion of the total variation that is explained by the Fama-French model, while the residual sum of squares represents the unexplained variation or the error term in the model. The total sum of squares (3928.593) is the sum of these two components and represents the total variation in the data.

The regression sum of squares being substantially larger than the residual sum of squares indicates that the model explains a significant portion of the variation in portfolio returns. This is a positive sign, as it suggests that the three factors included in the model-market risk, size, and value-are indeed capturing key elements that drive portfolio performance. The residual sum of squares, while still present, is considerably smaller, indicating that the unexplained variance, while not negligible, is less dominant. This balance between explained and unexplained variance is a hallmark of a well-fitting model.

The mean square is calculated by dividing the sum of squares by the corresponding degrees of freedom. For the regression, the mean square is 816.326 (2448.977 divided by 3), and for the residual, it is 5.966 (1479.616 divided by 248). The mean square for the regression indicates the average amount of variation explained by each of the predictors in the model. A higher mean square for the regression compared to the residual suggests that the model's factors are providing valuable explanatory power relative to the noise or random error in the data.

In this case, the regression mean square is significantly larger than the residual mean square, reinforcing the idea that the model is effectively capturing the essential drivers of portfolio returns. This large difference between the regression and residual mean squares is what leads to a high F-statistic, which is the next crucial component of the ANOVA table.

The F-statistic, calculated as the ratio of the regression mean square to the residual mean square, is a key metric in ANOVA used to test the overall significance of the regression model. In this analysis, the F-statistic is 136.825, which is substantially high. The F-statistic essentially tests the null hypothesis that the coefficients of all the predictors in the model are equal to zero, meaning that none of the predictors have any explanatory power. A high F-statistic, as observed here, strongly suggests that the null hypothesis can be rejected, meaning that at least one of the predictors is significantly related to the dependent variable.

The high F-statistic indicates that the model provides a good fit for the data, and the factors included in the Fama-French model-market risk, size, and value-are statistically significant in explaining the variation in portfolio returns. This result is particularly important because it validates the use of the Fama-French three-factor model in this context. It shows that the model is not only theoretically sound but also practically effective in capturing the dynamics of the portfolio returns over the twenty-year period.

The significance level associated with the F-statistic, often referred to as Significance F, represents the probability of observing an F-statistic as large as the one calculated if the null hypothesis were true (i.e., if the model had no explanatory power). In this case, the significance level is extremely low, well below conventional thresholds like 0.05 or even 0.01, indicating that the probability of observing such a high F-statistic by chance is exceedingly small.

This low significance level confirms that the Fama-French model is statistically significant, meaning that the relationship between the portfolio returns and the factors is not due to random chance. This reinforces the conclusion that the model is well suited to explaining the variation in the portfolio returns over the specified period. The practical implication of this result is that investors and portfolio managers can have confidence in using the Fama-French model to guide their decision-making processes, knowing that the model is underpinned by strong statistical evidence.

The ANOVA results must be interpreted in the broader context of the time period and economic events that characterized the years from 2002 to 2022. This period includes significant events such as the early 2000s recession, the global financial crisis of 2008, the subsequent recovery, and the market volatility associated with the COVID-19 pandemic. Each of these events had profound effects on financial markets, influencing asset prices, investor behavior, and the risk factors captured by the Fama-French model.

The high F-statistic and the corresponding low significance level suggest that despite these turbulent times, the model remained robust, capturing the key drivers of portfolio returns. This robustness across different economic cycles is a testament to the Fama-French model's flexibility and relevance. It suggests that the model is not just a static tool but one that can adapt to varying market conditions, providing valuable insights across different phases of the market cycle.

Moreover, the results underscore the importance of using a multi-factor model like Fama-French rather than relying solely on traditional single-factor models like the Capital Asset Pricing Model (CAPM). The inclusion of size and value factors, in addition to market risk, allows for a more nuanced understanding of portfolio performance, particularly in environments where smaller firms or value stocks may be disproportionately affected by macroeconomic events.

Table 5: Portfolio 2002-2022: Fama-French Three-Factor Model Results

Coefficient	Standard Error	t Stat	p-value	Lower 95%	Upper 95%
Intercept	0.732	0.156	4.708	4.17E-06	0.426
Mkt-RF	0.734	0.036	20.250	1.79E-54	0.662
SMB	-0.409	0.066	-6.221	2.09E-09	-0.538
HML	-0.094	0.051	-1.832	0.068	-0.195

The Fama-French three-factor model is a cornerstone of modern asset pricing theory, expanding on the traditional Capital Asset Pricing Model (CAPM) by incorporating additional risk factors—specifically, size (SMB: Small Minus Big) and value (HML: High Minus Low). Table 5 presents the results of applying this model to portfolio returns over the 2002-2022 period, offering a nuanced view of how these factors interact and influence portfolio performance. The table includes critical statistics such as the coefficients, standard errors, t-statistics, p-values, and confidence intervals for each factor, providing a comprehensive picture of the model’s predictive power.

The intercept, or alpha, in the context of the Fama-French model, represents the portion of the portfolio’s returns that cannot be explained by the three factors—market risk (Mkt-RF), size (SMB), and value (HML). In this analysis, the intercept is 0.732 with a standard error of 0.156, resulting in a t-statistic of 4.708 and a highly significant p-value of 4.17E-06. This positive and significant alpha suggests that the portfolio generated excess returns above what would be expected based on its exposure to the three risk factors.

A positive alpha indicates that the portfolio outperformed the market on a risk-adjusted basis, which could be attributed to factors such as effective stock selection, superior management strategies, or the exploitation of market inefficiencies. However, it is essential to consider that while alpha represents outperformance, it is not guaranteed to persist over time. Market conditions, economic cycles, and changes in investor sentiment can all affect a portfolio’s ability to maintain a positive alpha.

The market risk premium (Mkt-RF) is the difference between the returns of the market portfolio and the risk-free rate. It is a central component of both the CAPM and the Fama-French models, representing the return investors expect for taking on the additional risk of investing in the market versus a risk-free asset. In this analysis, the coefficient for Mkt-RF is 0.734, with a standard error of 0.036, yielding a t-statistic of 20.250 and an extremely low p-value of 1.79E-54. These results highlight the market risk premium as a highly significant predictor of portfolio returns.

A coefficient of 0.734 indicates that for every unit increase in the market risk premium, the portfolio’s return increases by 0.734 units, holding all else constant. This strong positive relationship is expected, as market movements are a primary driver of portfolio returns. The high t-statistic and low p-value further reinforce the robustness of this relationship, suggesting

that market risk is a fundamental factor influencing the portfolio's performance during the 2002-2022 period.

The significance of the Mkt-RF coefficient underscores the importance of market exposure in portfolio management. Investors seeking to maximize returns must carefully consider their portfolio's sensitivity to market movements. However, this sensitivity also comes with increased risk, particularly during periods of market volatility. As such, the relationship between the portfolio and the market risk premium is a double-edged sword, offering potential for higher returns but also greater exposure to market downturns.

The SMB factor captures the size effect, which is the tendency for small-cap stocks to outperform large-cap stocks over time. The coefficient for SMB in this analysis is -0.409, with a standard error of 0.066, resulting in a t-statistic of -6.221 and a highly significant p-value of 2.09E-09. The negative coefficient suggests that the portfolio, which is composed of large-cap stocks, tends to underperform when small-cap stocks are doing well.

This inverse relationship between SMB and the portfolio's returns indicates that the portfolio is more heavily weighted towards large-cap stocks, which are less sensitive to the size premium. Large-cap stocks are typically more established, with stable cash flows and lower volatility compared to small-cap stocks. However, during periods when small-cap stocks outperform, such as in early stages of economic recovery or when market sentiment favors growth over stability, a portfolio heavily weighted towards large-cap stocks may lag behind.

The statistical significance of the SMB factor highlights the importance of market capitalization in portfolio performance. While large-cap stocks provide stability, incorporating small-cap stocks into a portfolio can enhance returns, particularly in favorable market conditions for smaller companies. This finding suggests that portfolio diversification across different market capitalizations can be a valuable strategy for mitigating risk and capturing opportunities across market cycles.

The HML factor measures the value premium, which is the additional return that investors expect from holding value stocks-those with high book-to-market ratios-over growth stocks-those with low book-to-market ratios. In this analysis, the coefficient for HML is -0.094, with a standard error of 0.051, leading to a t-statistic of -1.832 and a p-value of 0.068. The negative coefficient indicates that the portfolio's returns decrease slightly when value stocks outperform growth stocks, but the relationship is not statistically significant at conventional levels (e.g., 0.05).

The lack of statistical significance for HML suggests that the value premium does not have a strong impact on the portfolio's returns during the 2002-2022 period. This could be due to several factors, including the

composition of the portfolio, which may not be heavily weighted towards value stocks, or broader market trends that have favored growth stocks, particularly in sectors like technology. The relatively low t-statistic further indicates that the relationship between the portfolio returns and the value premium is weak and uncertain.

This result is particularly interesting in the context of the 2000s and 2010s, which saw significant growth in technology and other growth-oriented sectors. The underperformance of value stocks relative to growth stocks during this period may have contributed to the weak influence of the HML factor on the portfolio. However, it is essential to note that value investing has traditionally been seen as a long-term strategy, with value stocks often outperforming during market corrections or downturns when investors seek more conservative, stable investments.

The Fama-French three-factor model results for the 2002-2022 period provide valuable insights for portfolio management and investment strategy. The highly significant market risk premium underscores the importance of market exposure in driving portfolio returns. Investors and portfolio managers must be acutely aware of their portfolio's sensitivity to market movements, as this will largely dictate performance, especially during periods of market volatility.

The negative and significant SMB coefficient suggests that large-cap stocks in the portfolio tend to underperform when small-cap stocks are thriving. This finding indicates that while large-cap stocks offer stability, incorporating small-cap stocks into the portfolio could enhance performance during certain market conditions. Portfolio diversification across different market capitalizations becomes crucial in optimizing returns and managing risk.

The non-significant HML coefficient indicates that the value premium was not a major driver of portfolio returns during this period. This could lead portfolio managers to reconsider the weight they place on value stocks, especially in a market environment that favors growth stocks. However, it is also a reminder that market trends can shift, and what underperforms in one period may outperform in another. Thus, maintaining flexibility in investment strategy and being open to adjusting the portfolio composition based on changing market conditions is essential.

The analysis of the Fama-French three-factor model for the 2002-2022 period demonstrates the model's effectiveness in capturing key elements that drive portfolio returns. The strong significance of the market risk premium and the size premium highlights the relevance of these factors in shaping portfolio performance. However, the lack of significance for the value premium suggests that its impact was limited during this period, likely due to broader market trends favoring growth stocks.

These findings underscore the importance of a nuanced approach to portfolio management, where understanding the interplay of market risk, size, and value is crucial. The Fama-French model provides a robust framework for this analysis, but it also emphasizes the need for continuous assessment and adjustment of investment strategies in response to evolving market conditions. By leveraging the insights provided by this model, investors can better navigate the complexities of the market and make informed decisions that enhance portfolio performance over the long term.

Sub-Period (2002-2012)

Table 6: Portfolio 2002-2012: Regression Statistics

Statistic	Value
Multiple R	0.782
R Square	0.611
Adjusted R Square	0.602
Standard Error	2.505
Observations	132

The multiple R value for the 2002-2012 sub-period is 0.782, indicating a strong positive correlation between the portfolio returns and the three Fama-French factors-Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML). This value is only slightly lower than the Multiple R for the full period (0.790), suggesting that the model's ability to explain the relationship between the portfolio returns and the risk factors remained relatively consistent, even during a decade characterized by heightened market uncertainty.

A multiple R value of 0.782 implies that there is a robust linear relationship between the portfolio's returns and the explanatory variables. In practical terms, this strong correlation suggests that the factors included in the model-market risk, size, and value-are relevant and significant in determining portfolio performance during this volatile period. The stability of this correlation across different periods reinforces the reliability of the Fama-French model as a tool for understanding the dynamics of portfolio returns under varying market conditions.

The R square value of 0.611 for the 2002-2012 sub-period indicates that approximately 61.1% of the variation in portfolio returns can be explained by the model. This figure is slightly lower than the R Square for the full 2002-2022 period (0.623), which suggests that the model's explanatory power was somewhat diminished during this sub-period. This is not entirely surprising, given the economic turbulence that characterized the 2002-2012 decade, which likely introduced additional sources of volatility and uncertainty not captured by the three factors in the Fama-French model.

The adjusted R square, which accounts for the number of predictors in the model and adjusts for the sample size, is 0.602. This value is close to the R square, indicating that the model does not suffer from overfitting and that the inclusion of the three factors-Mkt-RF, SMB, and HML-appropriately captures the key drivers of portfolio returns without introducing unnecessary complexity. An adjusted R square of 0.602 implies that the model is robust and reliable, with 60.2% of the variation in portfolio returns being attributable to the factors included in the model. This leaves 39.8% of the variation unexplained, which could be due to idiosyncratic risks, other omitted factors, or random market fluctuations.

The slight reduction in the R square and adjusted R square values compared to the full period suggests that while the Fama-French model remains a strong tool for explaining portfolio returns, its effectiveness may be somewhat constrained during periods of extreme market volatility and economic uncertainty. The unexplained variation could be attributed to factors such as investor behavior during crises, liquidity issues, or the impact of government interventions and fiscal policies that are not directly captured by the market, size, and value factors.

The standard error for the 2002-2012 sub-period is 2.505, which is slightly higher than the Standard Error for the full period (2.443). The Standard Error measures the average distance that the observed portfolio returns fall from the regression line, essentially capturing the model's prediction accuracy. A higher standard error indicates greater variability in the residuals, suggesting that the model's predictions were less precise during this sub-period.

The increase in the standard error during the 2002-2012 period likely reflects the heightened uncertainty and volatility in financial markets during these years. Events such as the global financial crisis introduced significant disruptions in the markets, causing more erratic behavior in asset prices and portfolio returns. This variability would naturally lead to larger residuals, as the model's ability to accurately predict returns based on the three factors alone would be challenged by the extraordinary circumstances of the time.

Despite the increase in Standard Error, the value of 2.505 is still within a reasonable range, suggesting that while the model's predictions were less precise, they were not drastically inaccurate. This indicates that the Fama-French model still provided valuable insights into the factors driving portfolio returns, even in a period marked by extreme market conditions. However, the higher Standard Error also highlights the need for investors and portfolio managers to exercise caution when relying on the model's predictions during periods of significant market stress, as the potential for prediction errors increases under such conditions.

The regression analysis for the 2002-2012 sub-period is based on 132 observations, reflecting monthly data points over the ten-year period. The number of observations is a crucial aspect of any regression analysis, as it affects the reliability and stability of the estimated coefficients. A larger number of observations generally leads to more accurate and stable estimates, as it reduces the impact of outliers and random variations.

In this case, 132 observations provide a robust dataset for the regression analysis, allowing for a reliable assessment of the model's performance during the sub-period. The fact that the analysis is based on monthly data ensures that the model captures the long-term trends and cyclical patterns in portfolio returns, rather than being overly influenced by short-term noise. This is particularly important in a period like 2002-2012, where short-term market movements were often driven by news events and investor sentiment rather than fundamental economic factors.

The 2002-2012 sub-period was a decade of significant economic and financial turbulence, beginning with the aftermath of the dot-com bubble and the early 2000s recession and culminating in the global financial crisis of 2008 and its subsequent fallout. These events had profound impacts on global financial markets, leading to dramatic fluctuations in asset prices, increased volatility, and shifts in investor behavior.

The Fama-French three-factor model's performance during this period provides valuable insights into how different factors influenced portfolio returns in a challenging market environment. The strong correlation between the portfolio returns and the market risk premium (as indicated by the Multiple R value) underscores the importance of market exposure during times of economic uncertainty. However, the slight reduction in the model's explanatory power (as reflected in the R Square and Adjusted R Square values) suggests that additional factors not captured by the model may have played a more significant role during this period.

For example, during the global financial crisis, liquidity risk, credit risk, and systemic risk became more prominent, affecting asset prices and portfolio returns in ways that the traditional Fama-French factors may not fully capture. Additionally, government interventions, such as bailouts, monetary policy changes, and fiscal stimulus packages, also had significant impacts on financial markets, introducing elements of uncertainty and unpredictability that are not directly addressed by the Fama-French model.

Table 7: Portfolio 2002-2012: Analysis of Variance

df	SS	MS	F	Significance F
Regression	3	1264.414	421.471	67.154
Residual	128	803.350	6.276	
Total	131	2067.764		

The degrees of freedom in the ANOVA table represent the number of independent pieces of information used to estimate the variance. In this analysis, the degrees of freedom for the regression is 3, corresponding to the three predictors in the Fama-French model: Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML). The residual degrees of freedom, 128, represent the number of observations (131) minus the number of parameters being estimated (3 predictors + 1 intercept). The total degrees of freedom is 131, reflecting the total number of observations minus one.

The allocation of degrees of freedom is crucial as it impacts the calculation of the mean square and the F-statistic. The regression degrees of freedom indicate how much of the total variation in portfolio returns can be attributed to the three factors, while the residual degrees of freedom capture the variation that remains unexplained by the model. The higher the degrees of freedom for the residual, the more data points are available to estimate the variance of the errors, leading to more robust statistical conclusions.

The sum of squares (SS) measures the total variation in the portfolio returns, which is partitioned into two components: the regression sum of squares (1264.414) and the residual sum of squares (803.350). The regression sum of squares represents the portion of the total variation explained by the Fama-French model, while the residual sum of squares represents the unexplained variation, or the error term. The total sum of squares (2067.764) is the sum of these two components and reflects the overall variability in the portfolio returns during the 2002-2012 sub-period.

The relatively large regression sum of squares compared to the residual sum of squares indicates that the model explains a substantial portion of the variation in portfolio returns. Specifically, the model accounts for 1264.414 units of the total variation, leaving 803.350 units unexplained. This distribution suggests that the three factors included in the Fama-French model—market risk, size, and value—are indeed capturing key elements that drive portfolio performance during this decade.

However, the presence of a significant residual sum of squares also indicates that there is still a considerable amount of variation in portfolio returns that the model does not capture. This unexplained variation could be due to several factors, including idiosyncratic risk, other omitted variables (such as momentum or liquidity), or external economic shocks that are not directly related to the three factors in the model.

The mean square is calculated by dividing the sum of squares by the corresponding degrees of freedom. For the regression, the mean square is 421.471 (1264.414 divided by 3), and for the residual, it is 6.276 (803.350 divided by 128). The mean square for the regression indicates the average amount of variation explained by each of the three predictors in the model. The substantially higher mean square for the regression compared to the

residual suggests that the model's factors are providing significant explanatory power relative to the noise or random error in the data.

This large difference between the regression mean square and the residual mean square is a key indicator of the model's effectiveness. The regression mean square being much larger than the residual mean square implies that the model's predictors-Mkt-RF, SMB, and HML-collectively explain much of the variance in the portfolio returns, rather than the variation being due to random chance or unexplained factors.

The F-statistic, calculated as the ratio of the regression mean square to the residual mean square, is 67.154 for this sub-period. This F-statistic is highly significant, as indicated by the extremely low p-value associated with it (Significance F). The F-statistic tests the null hypothesis that the coefficients of all the predictors are equal to zero, meaning that none of the predictors have any explanatory power. A high F-statistic, such as the one observed here, strongly suggests that the null hypothesis can be rejected, indicating that at least one of the predictors is significantly related to the dependent variable-in this case, portfolio returns.

The significance of the F-statistic confirms that the Fama-French three-factor model provides a good fit for the data during the 2002-2012 sub-period. Despite the economic volatility and market disruptions of this decade, the model remains robust, capturing the essential factors that drive portfolio performance. This result is particularly important because it validates the use of the Fama-French model even in periods of economic uncertainty, demonstrating its adaptability and relevance across different market conditions.

The significance level associated with the F-statistic, often referred to as Significance F, represents the probability of observing such a high F-statistic if the null hypothesis were true (i.e., if the model had no explanatory power). In this case, the significance level is extremely low, well below conventional thresholds like 0.05, indicating that the probability of observing this F-statistic by chance is exceedingly small.

This low Significance F value reinforces the conclusion that the Fama-French model is statistically significant in explaining the variation in portfolio returns during the 2002-2012 sub-period. The practical implication is that investors and portfolio managers can rely on the model's insights when analyzing portfolio performance, even during periods of significant market stress and volatility. The strong statistical evidence provided by the F-statistic and its significance level suggests that the factors of market risk, size, and value continue to be relevant and impactful drivers of portfolio returns, even in challenging economic environments.

The 2002-2012 sub-period was marked by several major economic events that had a profound impact on global financial markets. The early part

of the decade saw the fallout from the dot-com bubble, leading to a recession in the early 2000s. This was followed by a period of recovery and growth, which was abruptly interrupted by the global financial crisis of 2008. The crisis led to unprecedented market volatility, with significant declines in asset prices and a flight to safety by investors.

The ANOVA results for this sub-period demonstrate the resilience of the Fama-French three-factor model in capturing the key drivers of portfolio returns, even during such turbulent times. The high F-statistic and low Significance F indicate that the model's factors remained relevant and significant, despite the extraordinary market conditions. This suggests that the Fama-French model is not only a robust tool for portfolio analysis in normal market conditions but also an effective framework for understanding portfolio performance during periods of economic crisis.

However, the significant residual sum of squares also highlights the limitations of the model in fully capturing the complexity of market dynamics during such periods. While the three factors-market risk, size, and value-are important, they may not be sufficient to explain all the variation in portfolio returns during times of extreme market stress. Other factors, such as liquidity risk, credit risk, and systemic risk, may become more prominent during crises, and these are not directly accounted for in the Fama-French model.

Table 8: Portfolio 2002-2012: Fama-French Three-Factor Model Results

Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.977	0.220	4.436	1.96E-05	0.541
Mkt-RF	0.734	0.053	13.960	1.73E-27	0.630
SMB	-0.253	0.099	-2.567	0.011	-0.448
HML	-0.181	0.090	-2.008	0.047	-0.359

The intercept, or alpha, in the context of the Fama-French model, represents the portion of portfolio returns that cannot be explained by the three factors-Mkt-RF, SMB, and HML. For the 2002-2012 period, the intercept is 0.977 with a standard error of 0.220, yielding a t-statistic of 4.436 and a highly significant p-value of 1.96E-05. This significant positive alpha suggests that the portfolio generated excess returns above what would be expected based on its exposure to the market, size, and value factors.

The presence of a significant positive alpha implies that the portfolio outperformed the benchmarks set by the Fama-French model, potentially due to superior stock selection, effective timing strategies, or the exploitation of market inefficiencies. However, while a positive alpha is desirable, it also raises questions about the sustainability of such outperformance. Investors should consider whether the factors contributing to this alpha are replicable in future periods or whether they were unique to the economic conditions of the 2002-2012 decade.

The market risk premium (Mkt-RF) remains a dominant factor in explaining portfolio returns during the 2002-2012 period. The coefficient for Mkt-RF is 0.734, with a standard error of 0.053, resulting in a t-statistic of 13.960 and an exceptionally low p-value of 1.73E-27. These results confirm that the market risk premium is a highly significant predictor of portfolio returns, consistent with expectations from both the CAPM and the Fama-French models.

A coefficient of 0.734 indicates that for every unit increase in the market risk premium, the portfolio's return increases by 0.734 units, all else being equal. This strong positive relationship underscores the importance of market exposure in determining portfolio performance, especially during a period characterized by significant market volatility. The global financial crisis, in particular, led to dramatic shifts in market returns, making the market risk premium a critical factor for portfolios with substantial market exposure.

The significance of the Mkt-RF coefficient suggests that the portfolio was closely aligned with overall market movements during this period. This alignment could be advantageous during bull markets but poses risks during market downturns, as seen during the financial crisis. The results highlight the dual-edged nature of market risk, offering the potential for higher returns but also exposing the portfolio to greater downside risk during periods of market stress.

The SMB factor, which measures the size premium or the excess return of small-cap stocks over large-cap stocks, shows a negative coefficient of -0.253 for the 2002-2012 period. The standard error for SMB is 0.099, yielding a t-statistic of -2.567 and a p-value of 0.011. The negative and statistically significant relationship suggests that the portfolio, which is likely composed of large-cap stocks, tended to underperform when small-cap stocks were doing well.

The negative SMB coefficient indicates that the portfolio was more heavily weighted towards large-cap stocks, which are typically less volatile and offer more stable returns than small-cap stocks. However, during periods when small-cap stocks outperform-often in the early stages of economic recovery or in more speculative market environments-a portfolio with a large-cap bias may lag behind. The significance of the SMB factor during this period highlights the importance of market capitalization in shaping portfolio returns.

Interestingly, the magnitude of the SMB coefficient is smaller compared to other periods, reflecting the complex and sometimes contradictory market dynamics of the 2002-2012 decade. For instance, during the financial crisis, investors may have sought refuge in larger, more established companies, leading to the relative underperformance of small-cap stocks. This behavior would reinforce the inverse relationship between the SMB factor and portfolio returns, as seen in the model's results.

One of the most striking results from the 2002-2012 sub-period is the negative and statistically significant coefficient for the HML factor, which measures the value premium. The HML coefficient is -0.181 , with a standard error of 0.090 , resulting in a t-statistic of -2.008 and a p-value of 0.047 . This negative relationship suggests that value stocks, typically characterized by high book-to-market ratios, detracted from portfolio performance during this period.

The statistical significance of the HML coefficient indicates that the value premium had a meaningful impact on the portfolio's returns, but in a negative direction. This result is somewhat counterintuitive, as value stocks are often expected to outperform in the long run, particularly during periods of economic recovery. However, the 2002-2012 period was unique in that growth stocks, particularly in the technology sector, regained favor after the dot-com bust and continued to drive market performance leading up to and following the financial crisis.

The negative HML coefficient may reflect the broader market trend where growth stocks, with their lower book-to-market ratios, outperformed value stocks. This shift could be attributed to several factors, including investor preference for companies with strong growth prospects, the rise of technology and innovation-driven industries, and the overall risk aversion during and after the financial crisis, which led to a flight to quality and growth-oriented investments.

The results from the 2002-2012 sub-period underscore the importance of understanding the dynamic nature of risk factors and their impact on portfolio performance. The significant positive alpha suggests that the portfolio managed to outperform the expected returns based on its risk exposures, which is a commendable achievement during a decade marked by economic challenges. However, the results also highlight the risks associated with different factors.

The strong and significant relationship between the market risk premium and portfolio returns reinforces the need for portfolio managers to carefully monitor market exposure, particularly during periods of heightened volatility. The inverse relationship with the SMB factor suggests that a heavy reliance on large-cap stocks might limit the portfolio's upside potential during periods when small-cap stocks are in favor. This finding suggests that incorporating a more balanced approach to market capitalization could be beneficial in optimizing returns.

The negative and significant HML coefficient raises important questions about the role of value investing during this period. While value stocks are traditionally seen as safer, more conservative investments, the results from this decade suggest that they may have underperformed relative to growth stocks. Portfolio managers should consider the broader market

context when evaluating the potential for value versus growth, recognizing that different economic environments can lead to shifts in investor preferences and performance outcomes.

Sub-Period (2013-2022)

Table 9: Portfolio 2013-2022: Regression Statistics

Statistic	Value
Multiple R	0.819
R Square	0.670
Adjusted R Square	0.662
Standard Error	2.300
Observations	120

The multiple R value for the 2013-2022 sub-period is 0.819, indicating a strong positive correlation between the portfolio returns and the three factors-Market Risk Premium (Mkt-RF), Small Minus Big (SMB), and High Minus Low (HML)-included in the Fama-French model. This value suggests that the model effectively captures the linear relationship between these factors and the portfolio returns, providing a reliable basis for analyzing the drivers of performance during this decade.

A multiple R of 0.819 is higher than the corresponding value for the 2002-2012 sub-period (0.782), indicating that the relationship between the portfolio returns and the model's factors became even stronger in the 2013-2022 period. This improvement in the correlation could be attributed to the relative stability of the markets during much of this decade, which allowed the Fama-French factors to more accurately capture the variations in portfolio returns. The higher Multiple R value reflects the model's robustness and its ability to remain relevant across different economic environments.

The R square value for the 2013-2022 sub-period is 0.670, meaning that 67% of the variation in portfolio returns can be explained by the three-factor model. This figure is an improvement over the R Square of 0.611 observed in the 2002-2012 sub-period, suggesting that the model's explanatory power increased during the later period. This enhancement could be due to several factors, including the maturation of the market following the financial crisis, increased investor confidence, and the strong performance of certain sectors, such as technology, which may have aligned well with the factors captured by the model.

The adjusted R square for the 2013-2022 sub-period is 0.662, slightly lower than the R square, but still indicative of a strong model fit. The adjusted R square accounts for the number of predictors in the model relative to the number of observations, providing a more accurate measure of the model's explanatory power, particularly when comparing different time periods. The increase in adjusted R square from 0.602 in the previous decade to 0.662 in

the 2013-2022 period indicates that the model became more effective at capturing the drivers of portfolio returns, with less unexplained variance.

This improved adjusted R square suggests that the three factors-Mkt-RF, SMB, and HML-were more closely aligned with the market dynamics of the 2013-2022 period, making the model a more powerful tool for portfolio analysis. The reduced unexplained variance implies that fewer factors outside of the model were influencing portfolio returns, potentially due to a more stable economic environment or the dominance of certain sectors that the model captures well.

The standard error for the 2013-2022 sub-period is 2.300, which is lower than the standard error of 2.505 observed in the 2002-2012 sub-period. The standard error measures the average distance between the observed portfolio returns and the values predicted by the model, effectively capturing the model's prediction accuracy. A lower standard error indicates that the model's predictions were more precise during this sub-period, reflecting a tighter fit between the model and the actual portfolio performance.

The decrease in standard error suggests that the Fama-French model was better at predicting portfolio returns during the 2013-2022 period compared to the previous decade. This improvement in prediction accuracy could be attributed to several factors, including the overall market stability and the strong performance of certain sectors that were well captured by the model's factors. The lower Standard Error indicates that the model's residuals-the differences between observed and predicted returns-were smaller, suggesting fewer large deviations and more consistent performance.

This improved precision is particularly important for portfolio managers and investors, as it enhances the reliability of the model's predictions and allows for more accurate assessments of risk and return. The tighter fit between the model and actual returns suggests that the factors included in the Fama-French model were well-suited to explaining the variations in portfolio performance during a decade characterized by economic recovery, technological advancement, and unprecedented market growth.

The regression analysis for the 2013-2022 sub-period is based on 120 observations, reflecting monthly data points over the ten-year period. The number of observations is a critical factor in determining the reliability and stability of the regression estimates. A robust dataset with a sufficient number of observations helps to ensure that the model's coefficients are stable and that the results are not overly influenced by outliers or random variations.

In this case, the 120 observations provide a solid foundation for the regression analysis, allowing the model to capture long-term trends and patterns in portfolio returns. The use of monthly data ensures that the model is sensitive to both short-term fluctuations and longer-term cycles, providing

a comprehensive view of the factors influencing portfolio performance. The consistency of the results across the observations suggests that the model's findings are reliable and that the coefficients for Mkt-RF, SMB, and HML are stable estimates of the true relationships between these factors and portfolio returns.

The 2013-2022 sub-period was a decade of significant economic recovery and growth, following the turmoil of the global financial crisis. The period began with the continuation of the bull market that started in the aftermath of the crisis, driven by low interest rates, quantitative easing, and improving economic indicators. The decade also saw the rise of technology and innovation as dominant forces in the market, with sectors like information technology, healthcare, and consumer discretionary leading the way in terms of performance.

The market dynamics of this period were characterized by relatively low volatility for much of the decade, with significant growth in equity markets and strong investor confidence. This environment was favorable for the Fama-French factors, particularly the market risk premium, as broad market indices like the S&P 500 and Nasdaq saw substantial gains. The low interest rate environment also supported the performance of growth stocks, which may have influenced the model's fit and the relationships observed between the factors and portfolio returns.

The COVID-19 pandemic, which emerged in the final years of this sub-period, introduced significant volatility and uncertainty, leading to a sharp market downturn in early 2020 followed by a rapid recovery. This event tested the resilience of the market and highlighted the importance of understanding the factors driving portfolio performance. Despite the pandemic's impact, the model's strong fit during this period suggests that it was able to capture the key drivers of returns even in the face of unprecedented challenges.

Table 10: Portfolio 2013-2022: Fama-French Three-Factor Model Results

Coefficient	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.398	0.216	1.841	0.068	-0.030
Mkt-RF	0.750	0.050	15.138	3.1E-29	0.652
SMB	-0.569	0.086	-6.626	1.1E-09	-0.739
HML	-0.062	0.060	-1.039	0.301	-0.181

The intercept, or alpha, in the Fama-French model represents the portion of portfolio returns that is not explained by the three factors-Mkt-RF, SMB, and HML. For the 2013-2022 period, the intercept is 0.398 with a standard error of 0.216, resulting in a t-statistic of 1.841 and a p-value of 0.068. This positive alpha suggests that the portfolio generated some excess returns above what would be expected based on its exposure to the market, size, and value factors, although the p-value indicates that this result is not statistically significant at the conventional 0.05 level.

The near-significant alpha raises interesting questions about the sources of these unexplained returns. While the alpha is positive, suggesting potential outperformance, its lack of statistical significance implies that the excess returns might not be consistent or robust across different periods. This could be due to a variety of factors, including market timing, sectoral allocation, or other idiosyncratic elements that are not captured by the Fama-French model. For portfolio managers, this result highlights the importance of considering other risk factors or strategies that might contribute to portfolio performance, beyond those captured by the traditional Fama-French factors.

The market risk premium (Mkt-RF) continues to be the most significant factor influencing portfolio returns during the 2013-2022 period. The coefficient for Mkt-RF is 0.750, with a standard error of 0.050, resulting in a highly significant t-statistic of 15.138 and an extremely low p-value of $3.1E-29$. This strong positive relationship indicates that the portfolio's returns were closely tied to the overall market movements, with each unit increase in the market risk premium leading to a 0.750 unit increase in portfolio returns.

The slightly higher coefficient for Mkt-RF compared to earlier periods suggests that market exposure became even more crucial in driving portfolio performance during this decade. This finding aligns with the broader economic context of the 2010s, which was characterized by a prolonged bull market, driven by low interest rates, quantitative easing, and strong corporate earnings, particularly in technology and growth sectors. The significance of the market risk premium underscores the importance of market timing and sectoral allocation in achieving strong portfolio returns during this period.

For investors and portfolio managers, the dominant role of Mkt-RF highlights the need to carefully monitor market trends and economic indicators that influence the overall market risk premium. The strong dependence on market movements suggests that portfolios with high beta stocks—those that are more sensitive to market fluctuations—would have performed well in the bullish environment of the 2010s but may also be exposed to greater risks during market downturns.

The SMB factor, which captures the size premium or the excess return of small-cap stocks over large-cap stocks, shows a notably stronger inverse relationship with portfolio returns during the 2013-2022 period. The coefficient for SMB is -0.569, with a standard error of 0.086, leading to a t-statistic of -6.626 and a highly significant p-value of $1.1E-09$. The negative and statistically significant coefficient indicates that the portfolio, which appears to be weighted towards large-cap stocks, underperformed relative to small-cap stocks during this period.

The increased magnitude of the negative SMB coefficient compared to previous periods suggests that the underperformance of large-cap stocks relative to small-cap stocks became more pronounced in the 2013-2022

decade. This trend could be attributed to several factors, including the resurgence of small-cap stocks as investors sought higher growth opportunities in a low-interest-rate environment or the increased volatility in large-cap stocks, particularly those in mature industries facing slower growth prospects.

The strong inverse relationship between SMB and portfolio returns implies that portfolios heavily invested in large-cap stocks may have missed out on the higher returns offered by small-cap stocks during this period. This finding highlights the importance of considering market capitalization as a key factor in portfolio construction, particularly in environments where small-cap stocks are well-positioned to outperform due to favorable economic conditions or investor sentiment.

The HML factor, which measures the value premium or the additional return from holding value stocks (high book-to-market ratios) over growth stocks (low book-to-market ratios), remains statistically insignificant during the 2013–2022 period. The coefficient for HML is -0.062 , with a standard error of 0.060 , resulting in a t-statistic of -1.039 and a p-value of 0.301 . The negative but insignificant coefficient suggests that the value premium did not have a substantial impact on portfolio returns during this decade.

The insignificance of the HML factor is consistent with the broader market trends of the 2010s, which saw growth stocks, particularly in the technology sector, outperform value stocks. The low interest rate environment, combined with technological innovation and disruption, favored growth-oriented companies with strong future earnings potential. As a result, value stocks, which are typically seen as more conservative and stable investments, may have lagged behind, leading to the weak influence of the value premium on portfolio returns.

For portfolio managers, the continued insignificance of the HML factor during this period suggests that a value-focused investment strategy may not have been as effective as growth-oriented approaches. However, it is essential to recognize that market conditions can shift, and what underperforms in one period may outperform in another. The insignificance of the value premium in the 2010s may reflect a temporary market phase rather than a permanent shift in the dynamics between value and growth investing.

The Fama-French three-factor model results for the 2013–2022 sub-period provide several important takeaways for portfolio management and investment strategy. The strong significance of the market risk premium (Mkt-RF) reinforces the critical role of market exposure in driving portfolio returns, especially during a decade of sustained economic growth and market expansion. Investors and portfolio managers must remain vigilant in monitoring market trends and adjusting their portfolios accordingly to optimize returns and manage risk.

The strengthened inverse relationship with the SMB factor suggests that large-cap stocks may have underperformed relative to small-cap stocks during this period, emphasizing the importance of diversification across different market capitalizations. Incorporating a mix of small-cap stocks into a portfolio could have provided a performance boost during the 2013-2022 period, particularly as investors sought growth opportunities in a low-interest-rate environment.

The continued insignificance of the HML factor indicates that value investing may not have been as effective during this period, as growth stocks dominated the market. However, this finding also serves as a reminder that market dynamics are constantly evolving, and what underperforms in one period may become more relevant in the next. Portfolio managers should maintain flexibility in their investment strategies, being prepared to shift focus between value and growth as market conditions change.

Discussion

This analysis signifies the results of three non-overlapping periods: 2002 to 2022, 2002 to 2012, and 2013 to 2022, in explaining variations in the return on the sample stocks over shifting market conditions through the Fama-French three-factor model-market risk premium, size premium, and value premium.

Full Period (2002-2022)

The multiple R of 0.80 for the full period is a reasonably good correlation of portfolio returns with the three factors. The R square of 0.64 implies that the model explains 64% of the variance in returns.

The findings support the robustness of the Fama-French model in explaining returns that arise in different market environments-from economic crises through growth phases. It thereby supports the grounding provided by Fama and French's original 1993 research, which demonstrated the model's strength in capturing return variation across multiple market cycles and environments.

It is also in line with strong explanatory power over the full period, something which is supported by various studies (Harvey, Liu and Zhu, 2016). Evidence was seen that the market risk premium remains a dominant role both in the crisis and recovery stages. It is on the premise that Harvey's findings support the view that beta-or the sensitivity of market movements-remain a dominant force in explaining returns at times of large economic transition.

Sub-Period (2002-2012)

With the multiple R of 0.782 and R square of 0.611 for the 2002-2012 period, the model seems to have a fairly reasonable fit. In comparison with the

result from the period 2013-2022, the lower R square might imply other influences on returns in such a high-volatility, crisis-prone decade.

The economic instability, such as that seen during the 2008 financial crisis, serves to weaken the explanatory power of the three-factor model as returns start to depend on additional factors, such as momentum or volatility (Hong, Jeremy, and Yu, 2009).

With an adjusted R square value of 0.602 in this period, which was below that in the period of 2013-2022, additional factors other than the three-factor model have a significant influence, such as macroeconomic shocks and volatility. So, similar to Daniel and Titman's work, beyond market beta, size, and value, other major influential factors become stronger in economically turbulent periods.

Sub-Period (2013-2022)

The period of 2013-2022 had the highest multiple R of 0.819 and R square of 0.670, hence having the strongest explanatory power of the model. It could be contributed to economic recovery and lower market volatility, generally favoring the Fama-French factors, with an emphasis on the market risk premium.

The standard error of 2.300 was smaller compared to 2.505 in 2002-2012. Therefore, the model performed with higher accuracy within a low-volatility environment (Asness et al. 2015), where they indicated its greater efficacy in tranquil, expansionary environments.

Also, the market conditions of this period favored growth over value (Bali et al., 2016), which indicates the value premium has become muted in recent times as growth stocks outperformed amidst technology-driven markets.

Limitations of the Study

The Fama-French three-factor model omits other relevant factors that comprise the extended five-factor model as suggested by Fama and French, 2015-opCit. These could, therefore, provide more explanatory powers, especially for highly volatile years between 2002-2012, at which additional factors besides Mkt-RF, SMB, and HML may come into play.

The general structure of the model does not take into account sector-specific dynamics or how recent years have been so auspiciously disposed to high growth in technology industries. A study pointed out (Daniel and Titman, 1997) that sector exposure to return is a critical factor; thus, for example, the lack of sectoral adjustments in this study may have partly influenced the poor fitness of the model during times of exceptionally strong sectoral growth, such as during 2013-2022.

Each period also bears the signature of different macroeconomic factors, ranging from crises to recoveries. In fact, literature such as Harvey et al. 2016 has shown that such exogenous shocks to the economies can drain model fit because a linear model, such as Fama-French, cannot effectively capture sudden breaks in the market, like the 2008 financial crisis or the COVID-19 pandemic, which greatly affected the 2002-2012 and 2013-2022 periods under consideration, respectively.

This reliance on historical data immediately opens up the possibility of survivorship bias, wherein companies that did not survive would not be included, and that could affect the results. Moreover, according to Fama and French (1993), historically based, static portfolios are not representative of actual rebalancing for the investor and hence may not serve as a good indicator of the model's performance in reality.

Conclusions

The primary aim of this study was to assess the effectiveness of the Fama-French three-factor model in explaining the variability of returns for a portfolio of five U.S. stocks. To evaluate the model's performance under varying market conditions, the analysis covered the period from 2002 to 2022, with a focused comparison of two sub-periods: 2002-2012 and 2013-2022. This approach allowed a comparative assessment of the model's explanatory power across distinct economic environments, particularly examining the model's relevance in the more recent period.

The regression analysis demonstrated that the market risk premium (Mkt-RF) was consistently the most influential factor in explaining portfolio returns throughout the study period. This finding underscores the pivotal role of market risk in driving stock performance, particularly during favorable market conditions such as the prolonged bull market of the 2010s. The significance of the market risk premium indicates that portfolio returns were closely aligned with overall market trends, emphasizing the dominant influence of broad market movements on performance.

The size premium (SMB) exhibited a negative relationship with portfolio returns, reflecting the portfolio's focus on large-cap stocks. This negative correlation was especially pronounced during the 2013-2022 period, a time when small-cap stocks outperformed large-cap stocks. The negative SMB coefficient indicates that the portfolio, with its emphasis on large-cap stocks, underperformed relative to small-cap stocks during times when smaller companies gained market favor. This finding underscores the importance of considering market capitalization in portfolio construction and suggests that a diversified mix of both large-cap and small-cap stocks could enhance returns.

In contrast, the value premium (HML) was found to be largely insignificant across the analyzed periods, suggesting that the distinction

between value and growth stocks had a minimal impact on the portfolio's performance during the study timeframe. The insignificance of the HML factor reflects broader market trends of the 2010s, during which growth stocks particularly in the technology sector-often outperformed value stocks. This reduced the relevance of the value premium as a driver of returns in the observed context.

Overall, this study confirms that while the Fama-French three-factor model provides a valuable framework for understanding the drivers of portfolio returns, its effectiveness can vary based on the prevailing economic environment and market conditions. The model's reliance on the market risk premium underscores the critical importance of market exposure in determining portfolio performance. Meanwhile, the context-dependent impact of the size and value factors suggests that these elements should be carefully considered when applying the model. Investors and portfolio managers are advised to recognize the limitations of the Fama-French model and to complement it with additional factors or adjustments, such as integrating macroeconomic analysis, sector-specific insights, and behavioral considerations, all tailored to current market dynamics to achieve optimal investment outcomes.

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