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A Comparative Study of Bayesian Portfolio Optimization: Evidence from AI-Related Stocks

Mironov Mikhail, Master's Student
Shanghai University, China

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Abstract

This paper conducts a comparative analysis of portfolio optimization methods with a focus on Bayesian approaches, applying them to a dataset of AI-related stocks from the U.S. market. While the classical Markowitz model relies on fixed estimates of return and risk, the Bayesian framework incorporates parameter uncertainty, allowing for more adaptive decision-making. In addition to portfolio construction, the study applies conditional volatility and beta dynamics as a supplementary tool for Bayesian models' performance analysis, by using the Conditional CAPM model and the DCC-GARCH approach. The performance is evaluated in terms of risk-adjusted returns, particularly the Sharpe ratio, demonstrating the potential advantages of Bayesian optimization in fast-evolving sectors like artificial intelligence. The research finds that although the Markowitz model achieved the highest Sharpe ratio, it also involved the highest concentration risk. Furthermore, the more advanced the Bayesian model, the higher the Sharpe ratio, while conditional volatility and beta levels were simultaneously reduced.

Keywords: Bayesian portfolio optimization; Markowitz Mean-Variance Optimization; AI Stocks; Conditional beta; DCC GARCH

Introduction

1.1. Background

For a long time, the idea of artificial intelligence was imaginable, and a number of people suggested it unachievable. In recent years, artificial intelligence (AI) has undergone rapid development, transformed industries and redefined the way we process information. The emergence of the first generative AI resulted in the boom among companies and a new industry emerged. A new race has started, where the goal is the construction of Artificial Generative Intelligence (AGI). Thus, a number of IT companies started to test new models and create new AI products.

In the field of finance, this evolution has led to the emergence of new tools and methods, including the integration of machine learning models into portfolio management and asset pricing. According to the World Economic Forum (2018), the integration of AI into financial services can significantly accelerate data analysis and lead to more comprehensive decision-making processes.

This research aims to compare different portfolio optimization models, including Bayesian and Markowitz as well, which involve the machine learning in its basis. For these purposes the AI portfolio, which consist of 10 large-cap US AI companies will be constructed by using daily returns. The study's objective is the examination on the model's performances. Additionally, the study will investigate the conditional volatility and market risk exposure of all models by applying the conditional CAPM as the theoretical framework and DCC-GARCH as a supplementary tool. This could bring some significant insights, while it's crucial to consider not only the performance, but the time-varying risk of each model.

1.2 Objective

While the primary objective of this study is the comparative analysis on different Bayesian optimal weight models on the example of USA AI companies, the analysis also aims to examine the AI company index performance, volatility and market risk exposure, by applying conditional CAPM model.

1.3 Significance

This study primarily will help institutional and retail investors, as long it discloses the artificial intelligence topic in the basis of its theoretical framework, by applying machine learning mechanisms for the financial data analysis. Additionally, the conditional CAPM model and DCC-GARCH were implemented into the analysis, while making it more significant for a variety stakeholders who concern about the volatility an market risk exposure under time-varying framework.

1.4 Innovation

This research is innovative in its integration of Bayesian portfolio optimization with machine learning techniques to evaluate AI-related stocks.

While previous studies have explored Bayesian frameworks in finance, the application to AI-sector assets remains underexplored. By combining statistical rigor with a sector-specific focus, the study offers new insights into portfolio construction under uncertainty. The use of updated macro-financial data and AI-driven fundamentals further enhances the novelty of the approach.

2.1 Literature Review

To begin with, the Modern Portfolio Theory (MPT) was initially introduced by Harry Markowitz in his article "Portfolio Selection", which was published in the Journal of Finance in 1952. The theory advocates for a portfolio that is diversified by incorporating assets that are poorly correlated, meaning that they behave differently in different market conditions. Markowitz's most significant contribution lies in his ability to translate the concepts of "risk" and "profitability" into mathematical terms, and the development of the Markowitz Mean Variance Optimization (MVO) model, which is going to be tested in the current research as well.

The foundation of the Bayesian framework can be traced back to the 18th century with the work of Thomas Bayes, whose theorem was later formalized and extended by Pierre-Simon Laplace (Bayes, 1763; Laplace, 1812). In the context of modern statistics and financial modeling, Bayesian methods have gained prominence due to their ability to incorporate prior beliefs and update them with new information. Zellner and Chetty (1965) were among the first to apply Bayesian techniques to econometric models, demonstrating their flexibility and robustness in estimating uncertain parameters. The Bayesian framework treats unknown parameters, such as expected returns and covariances in finance, as random variables with probability distributions. This approach enables analysts to formally incorporate uncertainty and derive posterior distributions that reflect both prior beliefs and observed data, offering a dynamic alternative to classical estimation methods.

Recent advancements in machine learning have significantly expanded the toolkit available for financial data analysis, particularly through the integration of Bayesian methods. Bade, Frahm and Jaekel (2008) applied the portfolio optimization models under the Bayesian framework and compared this methodology with traditional portfolio optimization models, resulting in better performance and finding that prior investor's information has a crucial role on the model's outcomes. Mukeri, Shaikh, and Gaikwad (2020) apply an expert Bayesian framework for bankruptcy prediction, demonstrating that the incorporation of prior knowledge enhances interpretability and reduces false positives compared to traditional models. Pfarrhofer (2024) further advances this approach by using multivariate

Bayesian machine learning models for scenario analysis in macro-financial environments, emphasizing the importance of accounting for nonlinearities and asymmetries in the relationships between economic and financial indicators. In a related line of research, Gonzalvez et al. (2019) explore the application of Gaussian processes and Bayesian optimization in financial contexts, such as interest rate modeling and trend-based investment strategies, highlighting the ability of Bayesian methods to improve forecasting accuracy and decision-making under uncertainty.

Subsequently, the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965), and their papers significantly contributed to financial analysis, by providing a simple yet powerful framework to understand the relationship between risk and expected return. The model introduced the concept of systematic risk, captured by the beta coefficient, and established that the expected return of an asset is determined by its sensitivity to the overall market return, as well as the risk-free rate. Black (1972) presents a model of capital market equilibrium with restricted borrowing, extending the traditional Capital Asset Pricing Model (CAPM) by incorporating constraints on investor borrowing and its impact on asset prices. Fabozzi and Francis (1978) introduce one of the major changes to the standard capital asset pricing model (CAPM) is the replacement of the constant beta by a time-varying beta, which can be named as the emergence of the conditional CAPM.

Nevertheless, there are a various method to estimate the time-varying beta. Engle (2000) proposes the Dynamic Conditional Correlation model, a straightforward enhancement of multivariate GARCH models, which enables for the modeling of time-varying correlations between multiple financial time series, which could aid in the estimation of beta. Jain (2011) proposes the Heterogeneous Autoregressive Beta model to capture time-varying betas, providing a more adaptable approach to modeling dynamic risk exposures by incorporating diverse responses across various assets. Corradi, Distaso, and Fernandes (2013) investigate the connection between conditional alphas and realized betas, emphasizing the significance of time-varying risk measures in explaining asset returns and performance beyond conventional asset pricing models. Engle (2016) develops the Dynamic Conditional Beta model, which expands upon traditional asset pricing models by incorporating time-varying betas to capture dynamic risk exposures in financial markets. Zhang and Choudhry (2016) compare GARCH models and the Kalman Filter in forecasting the daily time-varying beta of European banks during the crisis period, highlighting the advantages and limitations of both approaches in capturing dynamic risk exposures. In conclusion, Aloy et al. (2020) conducted a comparative analysis of various techniques for modeling time-varying conditional betas, focusing on their application to Real Estate

Investment Trusts (REITs), demonstrating the advantages and limitations of different approaches in capturing dynamic risk exposures.

III Empirical analysis

3.1 Methodology

While this study aims to analyze the portfolio optimization approaches, it is crucial to start with the classical way, which is known as the Markowitz Mean-Variance Optimization (MVO), which was proposed by the Harry Markowitz in 1952. In MVO the portfolio is constructed using the sample mean and the sample covariance matrix and the main objective of this model can be expressed as follows:

$$\max\left(w^T\mu - \frac{\lambda}{2}w^T\Sigma w\right) \tag{1}$$

Subject to:

$$\sum_{i=1}^{N} w_i = 1$$

Where $w = [w_1, \dots, w_N]$ is the weight vector, where each w_i corresponds to the percentage of the portfolio that is allocated to an asset, μ is the sample mean, λ is investor's risk aversion coefficient, Σ is the sample covariance matrix.

In purpose of unification of objectives among all models, the current study aims to use MVO with developed objective, which requires to maximize the Sharpe ratio, a commonly used risk-adjusted performance measure. Hence, this condition can be explained as follows:

$$\max\left(\frac{\left(w^T\mu - r_f\right)^2}{w^T\Sigma w}\right) \tag{2}$$

Subject to:

$$\sum_{i=1}^{N} w_i = 1$$

This formulation ensures that the resulting portfolio balances both expected return and volatility in a way that maximizes efficiency. In this study, this adjusted Markowitz model served as a benchmark to compare against Bayesian strategies. Additionally, the current study assumes that the risk-free rate equals zero, as long as it is constant among all models.

Bayesian portfolio optimization extends the classical approach by incorporating parameter uncertainty into the model. Instead of relying on point estimates for expected returns and covariances, the Bayesian approach treats these quantities as random variables with prior distributions. Bayesian statistics is a probabilistic framework that allows us to quantify and update uncertainty about unknown parameters using observed data. In this context, the unknown parameters of interest — such as expected returns (μ) or covariances (Σ) — are treated as random variables. The beliefs about these parameters before observing the data are expressed through prior distributions, and once new data becomes available, these beliefs are updated using Bayes' rule, resulting in posterior distributions. Bayes' rule can be written as:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)} \propto P(X|\theta)P(\theta)$$

(3)

Where θ is model parameters, X is the observed data, $P(\theta|X)$ is the posterior distribution of the parameters, $P(X|\theta)$ is the likelihood function, $P(X)$ is the prior distribution.

In the Bayesian framework the, the analysis begins with the specification of a prior distribution, for example:

$$\mu \sim N(\mu_0, \frac{1}{k_0}\Sigma)$$

(4)

Where μ_0 represents the investor's subjective belief or prior estimate of the expected return for each asset, and k_0 illustrates the prior precision, indicating the confidence level in the prior belief on μ_0.

Furthermore, in purpose to examine the conditional volatility and market risk exposure under the time-varying concept, the conditional CAPM model was implemented. Recall, that the beta coefficient drivers from the classical CAPM where it reflects the stock's exposure to the market risk. The CAPM formula presented below:

$$E(R_e) = R_f + \beta(R_m - R_f),$$

(5)

Where, $E(R_e)$ is the expected return on the equity, R_f – risk-free rate, R_m – return on the market portfolio. β – beta coefficient, which can be calculate as its written below:

$$\beta = \frac{cov(R_m, R_i)}{var(R_m)},$$

(6)

The CAPM is a remarkable framework, crafted by a diverse group of scholars. However, it has a significant drawback — it relies on static variables, which often leads to unrealistic outcomes. To address this issue, a subsequent version of the model was developed, giving rise to the Conditional CAPM. This version incorporates the dynamic nature of financial data, incorporating it into the formula. The model adopts a different perspective:

$$E_t(R_{i,t+1}) - R_f = \beta_{i,t}[E_t(R_{m,t+1}) - R_f], \quad (7)$$

Sometimes, the equation (7) can be expressed more conveniently in the following form:

$$E_t(\tilde{r}_{i,t+1}) = \beta_{i,t} E_t(\tilde{r}_{M,t+1}), \quad (8)$$

Where $E_t(\tilde{r}_{i,t+1}) = E_t(R_{i,t+1}) - R_f$, $E_t(\tilde{r}_{M,t+1}) = E_t(R_{m,t+1}) - R_f$, meaning conditional expectation of the net excess return of asset i and of the market. Here, it is important to say, that the new return is calculated as follows:

$$r = \ln\left(\frac{Price_{t+1}}{Price_t}\right), \quad (9)$$

Thus, the return is calculated as the log division of price change. Furthermore, in the context of a conditional CAPM model based on time-series data, the beta coefficient also becomes time-varying and can be expressed as follows:

$$\beta_{i,t} = \frac{cov_t(R_{m,t+1}, R_{i,t+1})}{var_t(R_{m,t+1})}, \quad (10)$$

There are various methods for calculating the conditional variance and conditional beta, however, the current study will apply GARCH(1,1) and Dynamic Conditional Correlation GARCH (DCC-GARCH) model respectively. The DCC-GARCH model was developed by Engle and Sheppard (2001) and Engle (2002) in order to estimate large, time-varying covariance matrices. It combines dynamic correlation with the GARCH model, allowing it to handle heteroscedasticity as well as large, dynamic covariance matrices. Recall, that the GARCH(1,1) takes the following view:

$$\sigma_t^2 = \omega + a\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \quad (11)$$

Where σ_t^2 is the conditional variance at time t; ω is the constant term; a is the coefficient of the autoregressive term; ε_{t-1}^2 is the squared residual at time t-1; β is the coefficient of the moving average of squared shocks term; σ_{t-1}^2 is the conditional variance at time t-1. Regarding the DCC-GARCH formula, although it is quite complicated, the current study sill aims to briefly explain it.

There is the following set of formulas:

$$H_t = D_t R_t D_t$$
$$D_t = diag(\sqrt{h_{11t}}, \dots, \sqrt{h_{NNt}})$$
$$Q_t = (1 - a - \beta)\bar{Q} + a\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} \quad (12)$$
$$R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1}$$

Where H_t is the conditional covariance matrix; D_t is a diagonal matrix of time-varying standard deviations from univariate GARCH models; h_{NNt} is the conditional variance of asset, which was calculated via GARCH before, R_t is he time-varying correlation matrix; Q_t is an intermediate correlation matrix; Q_{t-1} is the long-run unconditional correlation matrix of standardized residuals; a and β are parameters controlling the dynamics of correlation.

3.2 Research Design

To starts with, the USA AI companies index was constructed, including 10 large-cap stocks, which are connected with AI industry. The index uses daily returns starting from the 10th December of 2020, to the 1st January of 2025. The Nasdaq-100 was chosen as the market representative and as a benchmark. All data were divided into two sections: the first includes the training model period, which is from 10/12/2020 to 1/1/2024, and the second is out-of-sample data, especially for testing models. It starts from 1/1/2024 to 1/1/2025. The Table 3.1 summaries general information regarding the stocks included in AI index constructed for this research and the data were obtained in the middle of May 2025 from the informational platform TradingView for free access.

Table 3.1 The structure of the US AI Index

Company	Capitalization	Main AI Involvement
MSFT	3.42T USD	Investments in OpenAI, Azure AI
NVDA	3.3T USD	GPUs and architecture for AI training
AMZN	2.19T USD	AWS AI/ML services, Alexa
GOOGL	2.1T USD	Leaders in machine learning (DeepMind, Gemini, Bard)
META	1.61T USD	AI Research, LLaMA
ORCL	454.03B USD	AI integration into cloud products
PLTR	291.19B USD	Big data analytics and military AI
IBM	244.65B USD	Watson, enterprise AI solutions
AMD	185.75B USD	NVIDIA alternative: chips for AI
AI	3.09B USD	Pure-play AI company: enterprise AI solutions

Source: TradingView

Furthermore, as long as the main objective of this study is the comparison of different Bayesian portfolio optimization models on the example of AI portfolio, then there are five generations with different approaches and one MVO model as a benchmark were develop. The description on them presented in the Table 3.2.

Table 3.2 All models' description

Generation	Description
Bayesian 1	The initial model with a mean equal sample mean, sigma is 0.01, and weights follow the Dirichlet distribution. All weights almost equally distributed.
Bayesian 2	The developed Bayesian 1 model with increased sigma to 0.05.
Bayesian 3	A model with sigma equals 0.02, the single asset limitation to 0.2 and implementation a penalty multiplied by 1000.
Bayesian 4	A developed Bayesian 3 model with the same sigma, but with asset limitation 0.25 and a penalty multiplied by 10000.
Bayesian 5	A grid search model, with weight limitation = 0.25; sigma = 0.01, 0.02, 0.05; penalty multiplied by 1000, 10000, 100000.
Markowitz	The Markowitz Mean-Variance Optimization model with Sharpe ratio maximization.

Source: Calculated by the Author

3.3 Descriptive Statistics

As the main objective of the study is to compare different Bayesian portfolio optimization models, it is essential to calculate the optimal weighting for securities in the AI portfolio based on the training period. The Table 3.3 shows the share of each stock in the portfolio according to each generation decision. Interestingly, while the first generation used an equally weighted portfolio, subsequent generations preferred to increase the shares of certain companies due to their high profitability. However, the greatest concentration can be seen in the Markowitz (MVO) model, which allocates only three out of the ten companies. While this may be mathematically sound, the lack of diversity is still a risky decision.

Table 3.3 Assets' allocation in the portfolio

Generation	AI	ORCL	AMZN	MSFT	AMD	NVDA	IBM	GOOGL	META	PLTR
Bayesian 1	9,89%	9,92%	9,95%	10,13%	9,95%	10,01%	10,14%	10,04%	9,93%	10,05%
Bayesian 2	8,69%	11,50%	7,68%	11,47%	9,37%	8,63%	16,18%	7,64%	9,70%	9,12%
Bayesian 3	2,84%	22,12%	2,25%	21,32%	5,75%	2,91%	23,41%	2,10%	13,41%	3,90%
Bayesian 4	2,27%	24,51%	1,80%	24,09%	4,61%	2,47%	25,08%	1,72%	10,21%	3,24%
Bayesian 5	0,15%	0,32%	0,20%	33,46%	0,17%	0,19%	64,96%	0,15%	0,22%	0,19%
Markowitz	0,00%	32,04%	0,00%	47,38%	0,00%	0,00%	20,58%	0,00%	0,00%	0,00%

Source: Calculated by the Author

The Table 3.4 provides a summary of the descriptive statistics for six custom-built AI indexes (B1 to B5 and M), each created using different Bayesian portfolio optimization models, compared to the Nasdaq-100 (NDX). The data spans from 2021 to 2024.

Table 3.4 Descriptive statistics of models' performance

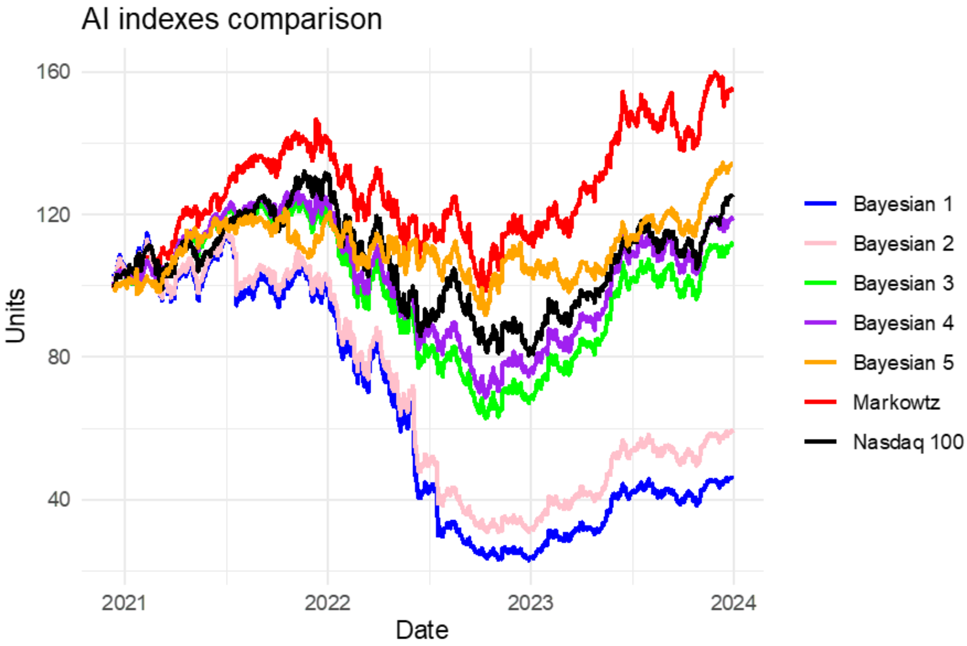
2021-2024	Mean	Median	Sd	Min	Max	Sharpe	Skewness	Kurtosis
Index B1	-0,07%	0,09%	2,57%	-30,11%	9,04%	-0,026	-4,15	48,80
Index B2	-0,04%	0,09%	2,24%	-23,03%	8,38%	-0,019	-2,86	30,43
Index B3	0,03%	0,06%	1,52%	-6,69%	6,38%	0,017	-0,30	4,78
Index B4	0,03%	0,05%	1,44%	-5,70%	5,96%	0,022	-0,22	4,42
Index B5	0,05%	0,06%	1,18%	-6,59%	4,58%	0,038	-0,44	6,01
Index M	0,07%	0,10%	1,33%	-5,81%	6,06%	0,049	-0,03	4,58
NDX	0,04%	0,09%	1,50%	-5,70%	7,22%	0,027	-0,18	4,28

Source: Calculated by the Author

Indexes B1 and B2 have negative mean returns (-0.07% and -0.04%, respectively) and highly negative skewness (-4.15 and -2.86), indicating a significant risk of extreme left-sided events. Their very high kurtosis values (48.80 and 30.43) suggest non-normal distributions with heavy tails. Indexes B3 through B5 demonstrate improved performance, with increasing average returns (up to 0.05%), decreasing standard deviations, and lower skewness and kurtosis values, indicating more stable and symmetric return behavior. The index M, which was created using the traditional Markowitz optimization approach, exhibits the highest average return (0.07%) and the best Sharpe ratio (0.049) among all custom indices. It also demonstrates relatively low-risk characteristics, with a standard deviation of 1.33% and a minimal skewness of -0.03. In contrast, the Nasdaq-100 (NDX) achieved a mean return of 0.04%, with a Sharpe ratio of 0.027 — higher than most Bayesian models, except for B5 and M. This makes B5 and M the most promising models during the training period, with M standing out in terms of overall risk-adjusted performance.

Figure 3.1 illustrates the dynamic of all Bayesian models indexes, based on the USA AI companies portfolio during the training period.

Figure 3.1 Index performances during the training period



Source: Calculated by the Author

The Table 3.5 assesses the out-of-sample performance of the AI-powered indices over the timeframe of 2024 to 2025, using fixed portfolio weights that were determined during the preceding training period. This stage serves to test the models' practicality and their ability to generalize beyond the data they were trained on.

Table 3.5 Descriptive statistics of models' performance

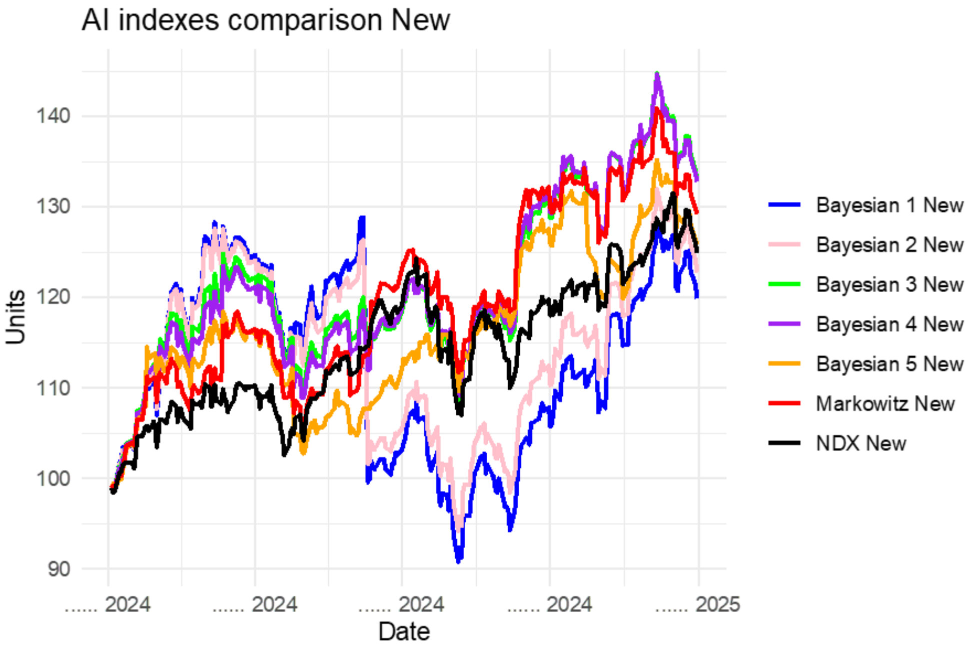
2024-2025	Mean	Median	Sd	Min	Max	Sharpe	Skewness	Kurtosis
Index B1	0,10%	0,16%	2,11%	-22,70%	4,65%	0,046	-5,03	56,22
Index B2	0,10%	0,13%	1,92%	-19,59%	4,43%	0,053	-4,28	45,77
Index B3	0,12%	0,18%	1,35%	-6,54%	4,45%	0,091	-0,49	5,66
Index B4	0,12%	0,16%	1,30%	-5,57%	4,69%	0,093	-0,32	5,05
Index B5	0,10%	0,16%	1,17%	-6,45%	6,08%	0,084	-0,52	9,42
Index M	0,11%	0,16%	1,23%	-4,03%	5,44%	0,089	0,13	6,11
NDX	0,10%	0,15%	1,15%	-3,72%	3,01%	0,083	-0,45	3,95

Source: Calculated by the Author

Remarkably, all six indices (B1 through B5 and M) achieved positive average returns ranging from 0.10% to 0.12%, outperforming their performance during the training period. Notably, Indexes B3, B4, and M continued to exhibit strong Sharpe ratios (0.091, 0.093, and 0.089, respectively), indicating attractive returns adjusted for risk. These findings reinforce the earlier conclusion that Bayesian models with lower variance (such as B3 and B4) and the Markowitz-based Index M remain resilient when applied to new data. Despite the positive mean returns, indexes B1 and B2 continue to exhibit extreme skewness (-5.03 and -4.28) and high kurtosis (56.22 and 45.77), indicating persistent left-tail risk and non-normal distributions. Their Sharpe ratios (0.046 and 0.053) remain the lowest, suggesting that these models may not be suitable for real-world application, despite their improved returns. The Nasdaq-100 (NDX) achieved a mean return of 0.10% and a Sharpe ratio of 0.083, performing similarly to indexes B5 and slightly below M, B3, and B4. This suggests that some of the custom Bayesian and Markowitz-weighted indexes may outperform the benchmark, particularly in terms of risk-adjusted returns.

A Figure 3.2 represents the dynamic of all indexes on the AI company' portfolio during the out-of-sample period, with respect to different optimal weights allocations.

Figure 3.2 Index performances during the out-of-sample period



Source: Calculated by the Author

3.4 Time-varying beta

This part aims to analyze the market risk exposure for the out-of-sample period. While all generations already made their allocation decision, it is essential to examine how risky their strategies on another data sample. The following Table 3.6 presents the results of conditional volatility and market beta estimates for each AI index, calculated using the DCC-GARCH model under the conditional CAPM theory. This approach allows us to analyze the time-varying exposure to market risk (beta) and dynamically estimate portfolio risk (volatility), providing a more accurate picture of portfolio behavior in changing market conditions.

Table 3.6 Descriptive statistics of models' volatility

	Mean	Median	Sd	Min	Max	Skewness	Kurtosis
Vol_B1	2,02%	2,02%	0,05%	1,94%	2,10%	0,06	1,80
Vol_B2	1,85%	1,84%	0,04%	1,78%	1,91%	0,06	1,80
Vol_B3	1,31%	1,31%	0,02%	1,28%	1,34%	0,07	1,81
Vol_B4	1,28%	1,28%	0,01%	1,26%	1,30%	0,08	1,81
Vol_B5	1,13%	1,13%	0,02%	1,09%	1,17%	0,06	1,80
Vol_M	1,22%	1,22%	0,00%	1,22%	1,22%	-0,09	1,81
Vol_NDX	1,14%	1,10%	0,14%	0,96%	1,67%	1,41	4,84
Beta_B1	1,31	1,35	0,35	-0,85	1,85	-2,10	11,24
Beta_B2	1,22	1,27	0,32	-0,77	1,70	-2,13	11,33
Beta_B3	0,94	0,96	0,13	0,08	1,16	-1,81	10,62
Beta_B4	0,91	0,93	0,12	0,21	1,10	-1,50	7,87
Beta_B5	0,58	0,59	0,07	0,38	0,68	-0,82	3,02
Beta_M	0,82	0,84	0,09	0,58	0,95	-0,90	3,12

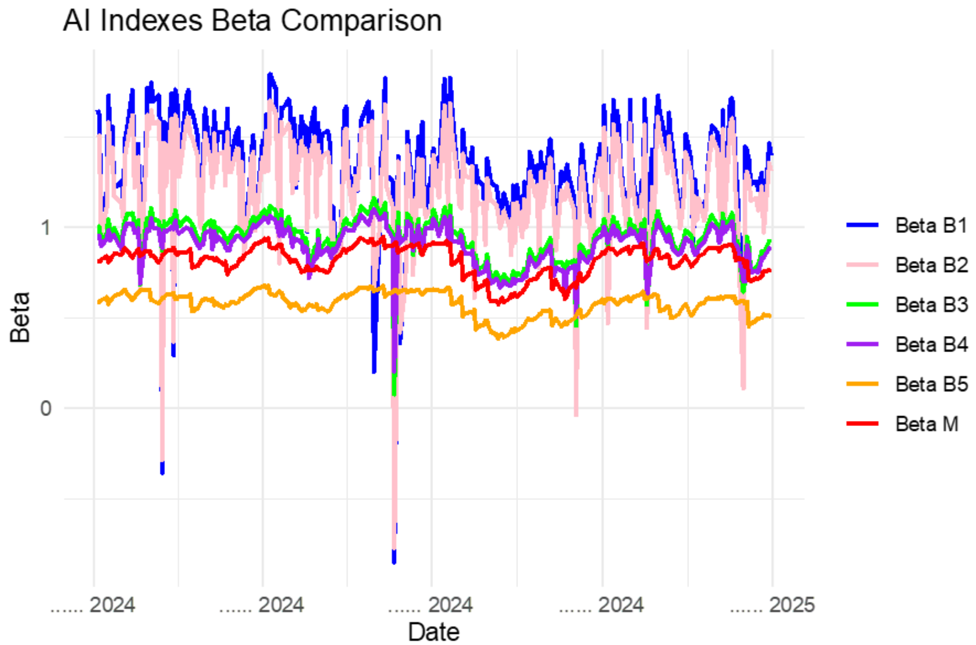
Source: Calculated by the Author

The levels of conditional volatility reveal a clear trend: indexes B3, B4, and B5 consistently exhibit the lowest risk levels (1.31%, 1.28%, and 1.13%, respectively), while indexes B1 and B2 display higher volatility (2.02% and 1.85%). The Markowitz-optimized portfolio (Index M) achieves both low volatility (1.22%) and extremely low dispersion (SD ≈ 0.00%), indicating exceptional stability. Interestingly, while the Nasdaq-100 has a relatively low mean volatility (1.14%), it also has the highest variability (SD = 0.14%) and skewness (1.41), suggesting occasional spikes in risk exposure. Examining the conditional betas, the study notices that B1 and B2 have the highest average market exposure (1.31 and 1.22), accompanied by strong negative skewness and extreme kurtosis, indicating their susceptibility to asymmetric market shocks. In contrast, B5 and M have the lowest average betas (0.58 and 0.82), suggesting limited market dependence and greater potential for diversification benefits. Moreover, the low volatility–low beta profiles of B3, B4, B5, and M support the notion that these indexes can provide more stable performance with reduced exposure to systemic risk.

Overall, the DCC-GARCH results reinforce the findings from previous observations: indexes derived from Bayesian models with lower variance (B3–B5) and the Markowitz-optimized index (M) not only offer favorable return–risk ratios but also exhibit desirable conditional characteristics, such as low volatility and reduced sensitivity to market-wide fluctuations.

A Figure 3.3 illustrates the dynamic of the conditional bets (market risk exposure) of all indexes during the out-of-sample period.

Figure 3.3 Index betas during the out-of-sample period



Source: Calculated by the Author

Conclusion

Current researched aimed to examine different Bayesian portfolio optimization models on the example of AI index of the United States. For this purpose, the AI portfolio was formed consisting form 10 large-cap companies which are connected with AI development. The whole analysis period took last 4 years, including 3 years for training models, and one year for the out-of-sample performance. The study includes 5 generations of Bayesian portfolio optimization models, which started with equally-weighted allocations and ended with models that provide a larger concentration.

According to the results, Markowitz Mean Variance Optimization showed the highest Sharpe ratio and the best performance among other models, however it exhibits high concentration among 3 companies, thus the lack of diversification leads to the increase of unsystematic risk. The first and second generations showed a poor performance, even compared with Nasdaq 100, while being the riskiest models. The models with penalties approach showed a better performance, while exhibit high rate of return under a small risk and relatively small beta (market risk exposure). The last generation, which uses the grid search in its basis showed a great performance during the training period, while during the out-of-sample testing period the model showed performance slightly below Markowitz and B4, but it exhibits the smallest conditional volatility and beta. To sum up, Bayesian portfolio optimization model is a great choice for the allocation shares in the portfolio task, while more complex model exhibits a better performance.

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