

Theoretical Reliability Analysis of a Segmented 2-out-of-3 Particulate Filter System for Industrial Chimneys

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Abstract

Industrial chimney particulate filters may experience clogging, leakage, structural damage, and other failure modes that interrupt their intended operation. This study proposes a theoretical segmented-filter configuration comprising three parallel modules with a 2-out-of-3 system-success criterion. The configuration is analysed using the standard k -out-of- n reliability model under assumptions of identical modules, statistically independent failures, equal airflow distribution, and successful isolation of a failed module. For an illustrative module reliability of $R = 0.80$, the predicted segmented-system reliability is 0.896, compared with 0.800 for a conventional single-filter configuration, representing a 12% relative increase. A sensitivity analysis across different assumed module reliabilities demonstrates the mathematical behaviour of the proposed configuration. A Monte Carlo simulation using 100,000 trials for each module-reliability value agreed closely with the analytical model, with a maximum absolute error of 0.00057 and a maximum percentage error of 0.0636%. These results describe theoretical system reliability and do not represent experimentally measured filtration efficiency. Common-cause failures, airflow redistribution, pressure drop, particle-removal efficiency, dust loading and clogging behaviour require experimental or computational investigation. The framework provides a basis for evaluating modular segmentation as a potential reliability-enhancement strategy for industrial particulate-filtration systems.

Keywords: Segmented particulate filter; industrial chimney; reliability engineering; *k*-out-of-*n* system; 2-out-of-3 configuration; Monte Carlo simulation; sensitivity analysis; fault tolerance

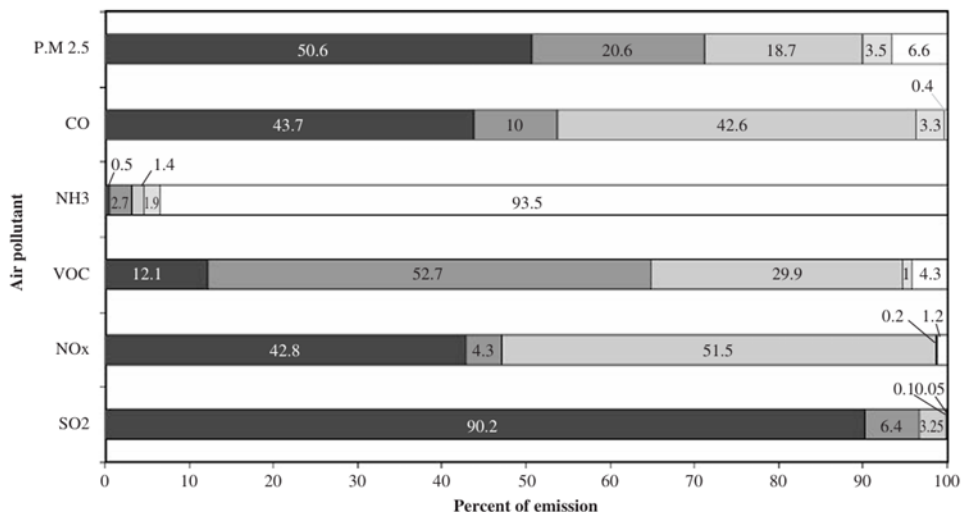
Introduction

Industrial chimney particulate filters are designed to remove suspended solid particles, such as dust, ash and soot, from exhaust gases before they are released into the atmosphere. These systems differ from technologies used for carbon dioxide capture or the absorption of gaseous pollutants, which rely on distinct physical and chemical mechanisms. The issue of environmental pollution caused in part by toxic particles from industrial factories and power plant chimneys is urgent. Since the Industrial Revolution in the 19th century, concerns about air pollution resulting from factories and the emission of toxic smoke from chimneys across Europe have grown. Engineers, town planners, and lawmakers are focused on this problem (Scorer & Barrett, 1962). Industrial facilities and power plants remain important sources of atmospheric particulate and gaseous emissions. A study by the US Environmental Protection Agency (2026) reports that electricity production and industry in the U.S. account for 25% and 23% of CO₂ and other greenhouse gas emissions, respectively. Burning hydrocarbon fuels like coal has increased atmospheric CO₂ and other greenhouse gas concentrations, altering the atmosphere's natural composition. This change affects global radiation and heat balance by increasing sensible heat in the air. Solar radiation reaches Earth's surface, while outgoing terrestrial long-wave radiation is partly trapped, leading to shifts in weather patterns and climate at global and regional scales. These changes can severely impact plant and animal life, resulting in ecological imbalances.

Exposure to ambient air pollution, particularly fine particulate matter, contributes to cardiovascular and respiratory diseases and was estimated to cause approximately 4.2 million premature deaths worldwide in 2019 (World Health Organisation, 2024). Industries use end-of-pipe technologies, such as filters and scrubbers, to capture pollutants and waste before they are released into the environment (Li et al., 2021). The performance of end-of-pipe technologies depends on both the selected treatment method and its operating conditions. For example, Jayadi et al. (2021) experimentally investigated a water-spray scrubber installed on an incinerator chimney. Increasing the water flow from 3.2 to 5.6 L/min was associated with lower measured outlet concentrations of particulate matter, SO₂, NO_x and CO. However, particulate matter and NO_x remained above the applicable emission limits, demonstrating that installing an emission-control device does not necessarily guarantee compliance for every pollutant.

Different technologies can be used to separate solid pollutants from exhaust gases, including mechanical separators, wet separators, electrostatic precipitators and filters. Among these technologies, filters are particularly effective for capturing fine particulate matter and may achieve removal efficiencies exceeding 90%. However, their performance depends on the filter material, particle-size distribution, pressure loss, technical design and regeneration method (Hornická et al., 2023).

In the United States, federal air pollution control was established by the Clean Air Act of 1963 and was substantially strengthened by the Clean Air Act of 1970 and its subsequent amendments (U.S. Environmental Protection Agency, 2025). Nonetheless, further advancements are needed. One promising solution involves multi-layer filters that target harmful pollutants, reducing particles released into the atmosphere. Effective particulate-control systems can reduce atmospheric emissions, while collected material can be safely managed and, where technically feasible, recovered or reused.



From left to right:

- Stationary sources: Combustion processes
- Stationary sources: Industrial, non combustion processes
- Mobile sources: Road and nonroad origin
- Waste disposal
- Miscellaneous (including agriculture)

Figure 1: Distribution of EU-27 total emission estimates for different pollutants, by source category, in 2006 (Kennes & Veiga, 2013)

The practical feasibility of modular particulate filtration is supported by Baumgarten et al. (2023), who developed and experimentally evaluated a

metal-mesh filtration system comprising two filter modules to remove particulate matter from biomass-boiler flue gas. The study examined pressure drop, gas velocity, separation efficiency, leakage and filter regeneration under realistic operating conditions. The authors further proposed a commercial configuration consisting of multiple modules, in which regeneration could be performed sequentially while the other modules continued to filter the exhaust gas. This provides practical support for modular filtration, although it does not directly validate the specific 2-out-of-3 reliability criterion adopted in the present theoretical model.

This study proposes a theoretical reliability framework for segmented particulate filters used in industrial chimneys. Segmentation is investigated as a reliability strategy rather than a demonstrated method for increasing particle collection efficiency. Segmentation is an intrinsic reliability strategy because it modifies the system architecture to limit the consequences of individual module failure (Todinov, 2018). In the proposed configuration, segmentation is applied by dividing the conventional filter into three modular filter units arranged in parallel while maintaining the same total filter material. The concept is investigated solely as a theoretical reliability-enhancement strategy, while its effects on filtration performance require experimental validation.

Study objective

The objective of this study is to theoretically evaluate whether dividing a conventional particulate filter into three parallel modules, each operating under a 2-out-of-3 success criterion, can improve system reliability while maintaining the same total quantity of filter material.

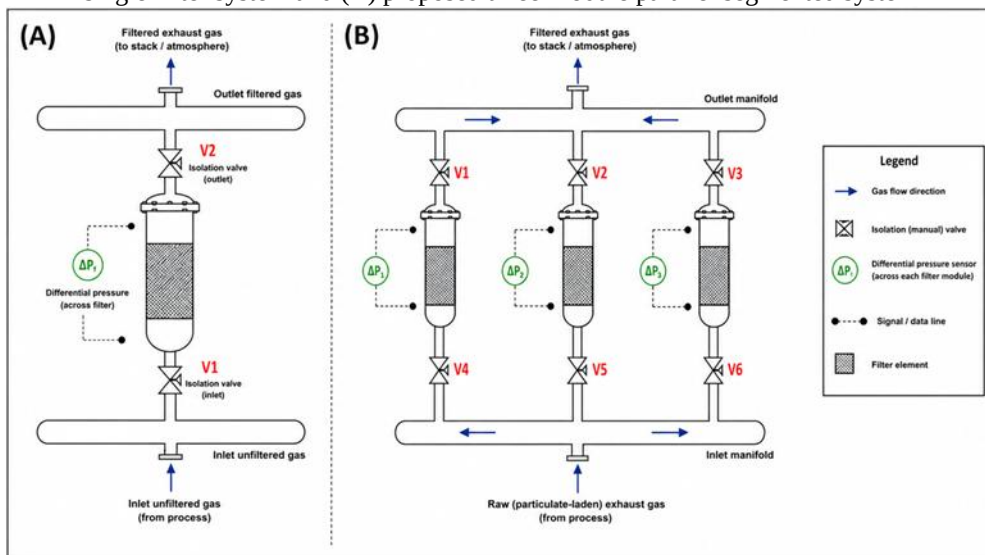
Methodology

Conceptual Filter Configurations and System Boundary

In the present theoretical model, the system boundary is limited to the particulate filter installed within the industrial chimney, as illustrated in Figures 2(A) and 2(B). The system consists of the filter medium and its supporting structure, while the proposed configuration divides the filter into independent segments. The filter is assumed to remove solid particulate matter from the exhaust gas, irrespective of its specific chemical composition. System integrity is maintained when the filter provides the required flow path without leakage, structural damage, or excessive clogging. A segment is considered to have failed when it can no longer provide its intended filtration function because of clogging, excessive pressure drop, leakage, structural damage, or loss of filtration capability. Specific tolerance values are not assigned in this theoretical study because they depend on the filter material, particulate composition, operating conditions, and industrial application.

In this study, reliability is defined as the probability that the filter system continues to provide its required particulate-filtration function throughout a specified reference operating interval without excessive clogging, unacceptable pressure drop, bypass leakage, structural damage, or loss of filtration capability. Because no experimental or field failure-time data are available, neither a numerical mission duration nor quantitative failure thresholds are assigned. Therefore, the reliability values used in this study are dimensionless illustrative probabilities evaluated over the same assumed reference operating interval.

Figure 2: Conceptual configurations considered in the theoretical analysis: (A) conventional single-filter system and (B) proposed three-module parallel segmented system



Note: The proposed configuration consists of three modular filter units arranged side by side in parallel flow paths. For the theoretical comparison, the total quantity of filter medium is assumed to remain equal to that of the conventional filter and to be distributed equally among the three modules. This assumption does not mean that the conventional and segmented configurations necessarily have identical media thickness, frontal area, residence time, pressure drop, airflow distribution, particle-collection efficiency or dust-holding capacity. These physical parameters depend on the detailed module geometry and must be established through design calculations and experimental testing.

Potential Failure Modes of the Filter

The gas filter system includes various components, each with potential failure modes. Common issues include:

1. Seal and gasket deterioration, which can cause gas leaks and reduce filter efficiency. [**Solution:** Upgrade sealing technology by replacing seals and gaskets with high-quality materials to improve durability and resistance to gas exposure.]

2. Malfunctioning pressure sensors. [Solution: Regularly calibrate sensors to ensure accuracy, and review software settings and configurations for sensor interfaces.]
3. Erosion of filter media. [Solution: Use durable media resistant to damage from harsh fluids and abrasive particles.]
4. Extreme temperatures affecting O-rings, leading to sealing failures. [Solution: Implement heat insulation to protect O-rings from temperature extremes.]
5. Filter clogging or blocking. [Solution: Apply periodic filter regeneration using methods such as shaking, air blowing, vibration or backwashing to remove accumulated particles. Appropriate airflow control should also be maintained to limit dust-layer formation, pore blockage and excessive pressure loss (Hornická et al., 2023)].

Figure 3 below illustrates typical gas filtration and gas-clogging mechanisms.

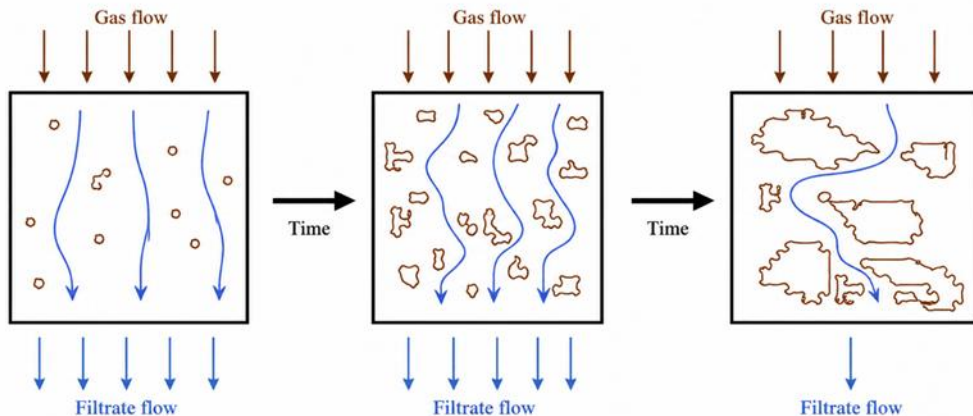


Figure 3: Typical filtration and gas clogging mechanism (Komariah et al., 2018)

Enhancing Gas Filter Reliability through Segmentation Technique

Segmentation is a reliability-enhancement strategy that divides an entity, system, process, or assembly into smaller, manageable parts to isolate failures and reduce overall system vulnerability (Todinov, 2023; Todinov, 2026). In the present study, this principle is applied by dividing the conventional single filter into three modular filter units arranged in parallel, forming the basis of the proposed 2-out-of-3 reliability configuration.

Reliability model assumptions

The theoretical reliability analysis is based on the following assumptions:

- The three filter modules are identical.
- Failures occur independently.
- Exhaust flow is equally distributed among the filter modules.

- A failed module can be isolated without significant bypass leakage.
- The remaining modules continue to satisfy the required exhaust-flow rate and filtration requirements.
- The analysis evaluates system reliability only and does not represent experimentally verified filtration performance.

The 2-out-of-3 criterion is adopted as a theoretical system-success condition rather than an experimentally established operating threshold. Under this assumption, failure of one module does not constitute a system failure because the failed module is assumed to be isolated and the remaining two modules continue to operate. The ability of the two remaining modules to maintain the required airflow, acceptable pressure drop, and particulate-removal performance has not been experimentally verified and therefore requires future validation.

Reliability Analysis and Results

Analytical reliability model

The reliability analysis uses the standard (k)-out-of-(n) method, where the system functions as long as at least (k) of its (n) independent components work. In the proposed three-segment setup, at least two segments must be operational, resulting in a 2-out-of-3 configuration (Arulmozhi, 2002). The reliability of the segmented system is given by:

$$R_{system} = \sum_{i=2}^3 \left(\frac{3}{i} \right) R^i (1 - R)^{3-i} \quad (1)$$

For the proposed configuration, $n = 3$ and $k = 2$.

Therefore, Equation (1) becomes:

$$R_{system} = 3R^2(1 - R) + R^3 = 3R^2 - 2R^3 \quad (2)$$

where (R) represents the assumed reliability of an individual filter segment.

An individual-module reliability of $R = 0.80$ was selected as a benchmark case to demonstrate the calculation transparently and permit a direct comparison between the conventional single-filter and segmented 2-out-of-3 configurations. This value was not derived from experimental data, field failure records, or a particular industrial filter specification. To ensure that the conclusions are not dependent on this single assumed value, the analytical and Monte Carlo analyses were conducted across module-reliability values ranging from $R = 0.60$ to $R = 0.95$. Accordingly, $R = 0.80$ should be interpreted only as an illustrative calculation point within the investigated range. The resulting reliability calculation is summarised in Table 1.

Table 1: Theoretical reliability analysis of the proposed three-segment filter system at a reliability of 80%

Operating condition	Reliability expression	Calculation ($R = 0.8$)	Reliability
All three segments are operational.	R^3	$(0.8)^3 = 0.512$	0.512 (51.2%)
Exactly two segments are operational.	$3R^2(1 - R)$	$3(0.8)^2(1 - 0.8) = 0.384$	0.384 (38.4%)
Overall segmented-system reliability (at least two segments remain operational)	$R_{system} = 3R^2 - 2R^3$	$0.512 + 0.384 = 0.896$	0.896 (89.6%)

Assigning the same illustrative reliability value to the conventional single filter and to each segmented module is a modelling assumption adopted solely to enable a theoretical comparison. It is not a demonstrated physical consequence of dividing the same total amount of filter material into three modules.

The relative increase in theoretical system reliability compared with the conventional single-filter configuration is calculated as:

$$\frac{R_{segmented} - R_{single}}{R_{single}} \times 100 = \frac{0.896 - 0.80}{0.80} = 12\%$$

Thus, under the adopted theoretical assumptions, the segmented 2-out-of-3 configuration provides a 12% relative increase in system reliability compared with the single-filter reliability of 0.80.

Sensitivity analysis and Monte Carlo verification

A Monte Carlo simulation was conducted in OriginPro 2021 to verify the reliability of the analytical model numerically. For each assumed module-reliability value, $N = 100,000$ trials were evaluated. In each trial, three independent random numbers uniformly distributed over $[0, 1]$ were generated to represent the three filter modules. A module was classified as operational when its random number was smaller than the assigned module reliability, R ; otherwise, it was classified as failed.

The operational status of module i in trial j was defined as:

$$X_{ij} = \{1, U_{ij} < R, 0, U_{ij} \geq R, \quad (3)$$

where U_{ij} is a uniformly distributed random number and $X_{ij} = 1$ represents an operational module. The system was considered operational when at least two of the three modules remained operational:

$$S_j = \{1, X_{1j} + X_{2j} + X_{3j} \geq 2, 0, X_{1j} + X_{2j} + X_{3j} < 2. \quad (4)$$

The Monte Carlo estimate of system reliability was calculated as:

$$\hat{R}_{MC} = \frac{1}{N} \sum_{j=1}^N S_j. \quad (5)$$

Module-reliability values of $R = 0.60, 0.70, 0.80, 0.90$ and 0.95 were examined. The same sets of generated random numbers were retained across the different reliability levels to ensure a consistent numerical comparison. The resulting Monte Carlo estimates were compared with the analytical values obtained from Equation (2), as shown in Table 2 and Figure 4.

Table 2: Comparison of analytical and Monte Carlo reliability estimates for the segmented 2-out-of-3 system using 100,000 trials at each module-reliability value

Module reliability (R)	Single-filter reliability	Analytical segmented reliability	Monte Carlo reliability	Absolute error	Percentage error
0.60	0.60000	0.64800	0.64826	0.00026	0.0401%
0.70	0.70000	0.78400	0.78369	0.00031	0.0395%
0.80	0.80000	0.89600	0.89543	0.00057	0.0636%
0.90	0.90000	0.97200	0.97162	0.00038	0.0391%
0.95	0.95000	0.99275	0.99273	0.00002	0.0020%

As shown in Table 2, the Monte Carlo estimates closely agree with the analytical results across all investigated module-reliability values. For $R = 0.80$, the analytical system reliability was 0.89600, while the Monte Carlo estimate was 0.89543, corresponding to an absolute error of 0.00057 and a percentage error of 0.0636%. Across all five cases, the maximum absolute error was 0.00057 and the maximum percentage error was 0.0636%. This close agreement numerically confirms the correct implementation of the analytical 2-out-of-3 reliability equation under the assumptions of identical modules and statistically independent failures.

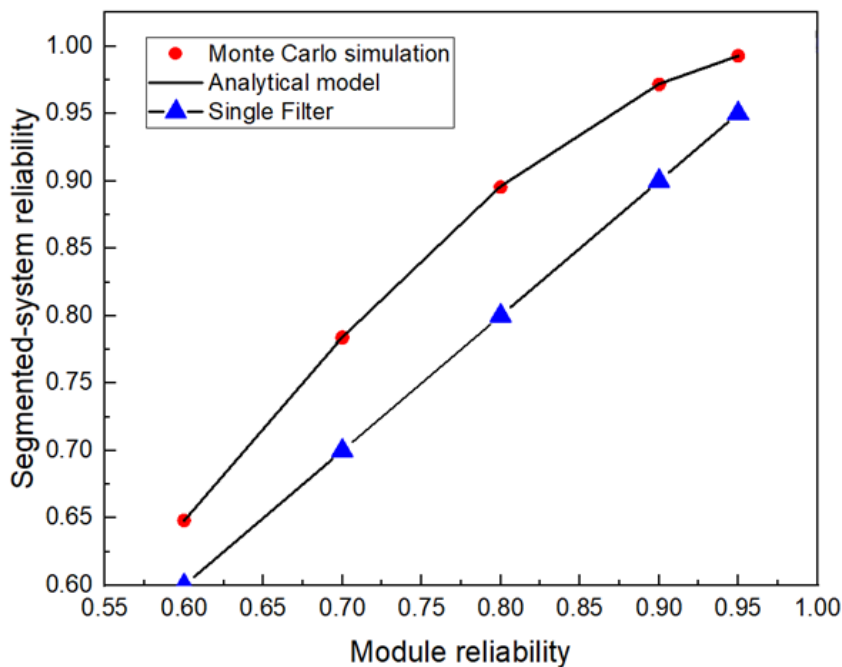


Figure 4: Comparison of conventional single-filter reliability, analytical reliability of the segmented 2-out-of-3 system, and Monte Carlo estimates obtained using 100,000 trials at each module-reliability value

Figure 4 illustrates the increasing reliability advantage of the segmented 2-out-of-3 configuration over the conventional single-filter configuration as module reliability increases. The advantage becomes more pronounced between $R = 0.60$ and $R = 0.90$, before both configurations approach the theoretical upper limit of 1.00. The numerical simulation evaluates the same idealised assumptions as the analytical model and therefore does not establish actual filtration efficiency, pressure-drop behaviour, clogging resistance or industrial operating reliability.

Discussion

The proposed 2-out-of-3 configuration provides a theoretical reliability advantage because failure of one module does not immediately cause system failure. However, when one module is isolated, each surviving module must handle approximately one-half rather than one-third of the total airflow—an increase of about 50%. This behaviour resembles a load-sharing k-out-of-n system, in which failure of one component increases the load on the surviving components (Rykov et al., 2022). The increased airflow may raise face velocity and pressure drop, accelerating particle accumulation and clogging. Therefore, the two surviving modules must have sufficient reserve capacity, which should be verified experimentally or through CFD analysis.

The independent-failure assumption is also idealised because all modules may experience common dust loading, temperature, moisture and vibration. Such common-cause failures could reduce the predicted improvement in reliability. Moreover, system reliability should not be confused with filtration performance; the present model does not demonstrate improved particle-removal efficiency. Nevertheless, the modular filtration system investigated by Baumgarten et al. (2023) supports the practical feasibility of modular operation, although it does not validate the specific 2-out-of-3 criterion proposed here.

Practical Design and Maintenance Considerations

Practical implementation would require adequate reserve capacity, reliable module-isolation mechanisms, pressure-drop monitoring, durable sealing, suitable filter media and periodic regeneration. These measures should be evaluated under realistic operating conditions.

Limitations

This study presents an idealised theoretical reliability framework and does not include laboratory experiments, CFD simulations or industrial operating data. The model assumes identical modules, statistically independent failures, equal airflow distribution, successful isolation of a failed module and sufficient capacity in the two remaining modules. These assumptions may not hold in industrial chimneys because shared dust loading, temperature, chemical exposure, vibration, and maintenance practices can lead to common-cause failures. Furthermore, isolation of one module may redistribute airflow and increase the face velocity, pressure drop and clogging rate of the remaining modules. The model does not define quantitative failure thresholds, mission duration, bypass-leakage limits or emissions-compliance criteria. Therefore, the calculated reliability values should be interpreted as illustrative analytical results rather than as verified predictions of industrial performance.

Conclusion

This study presented a theoretical reliability framework for improving the reliability of industrial chimney particulate filters through segmentation. The proposed configuration divides the filter into three modules while maintaining the same total filter material. It adopts a 2-out-of-3 operating criterion based on the standard (k)-out-of-(n) reliability model. The theoretical analysis demonstrated that system reliability depends on the assumed reliability of the individual filter modules. For an illustrative module reliability of ($R=0.80$), the segmented system achieves a reliability of 0.896 (89.6%), compared with 0.800 (80%) for the conventional single-filter system,

corresponding to a 12% relative improvement. The Monte Carlo simulation, based on 100,000 trials at each investigated module-reliability value, closely reproduced the analytical results. The maximum absolute and percentage errors were 0.00057 and 0.0636%, respectively, providing numerical verification of the analytical equation under the assumptions of identical modules and statistically independent failures.

The broader theoretical comparison also demonstrates the segmented configuration's reliability behaviour across different assumed module reliability values. These findings represent analytical predictions and should not be interpreted as experimentally verified improvements in filtration performance. Future work should experimentally evaluate pressure drop, airflow distribution, particle collection efficiency, dust loading, clogging behaviour, and long-term operational performance. Overall, the proposed framework provides a theoretical basis for further investigation of filter segmentation as a reliability-enhancement strategy for industrial particulate filtration systems.

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Data Availability: All data are included in the content of the paper.

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References:

1. Scorer, R. S., & Barrett, C. F. (1962). Gaseous pollution from chimneys. *International Journal of Air and Water Pollution*, 6, 49–63.
2. World Health Organisation. (2024, October 24). Ambient (outdoor) air pollution. [https://www.who.int/news-room/fact-sheets/detail/ambient-\(outdoor\)-air-quality-and-health](https://www.who.int/news-room/fact-sheets/detail/ambient-(outdoor)-air-quality-and-health).
3. Todinov, M. (2018). *Methods for reliability improvement and risk reduction*. John Wiley & Sons.
4. Todinov, M. T. (2023). *Management, reliability, energy and sustainability (Lecture notes)*. Oxford Brookes University, Oxford, UK.
5. Todinov, M., 2026. A Domain-Independent Method for Risk Reduction Based on Reverse Engineering of Sub-Additive Algebraic Inequalities. *Mathematics*, 14(11), p.1974.
6. U.S. Environmental Protection Agency. (2026). Electric power sector emissions. Retrieved from <https://www.epa.gov/ghgemissions/electric-power-sector-emissions>

7. Kennes, C., & Veiga, M. C. (2013). Introduction to air pollution. In *Air pollution prevention and control: Bioreactors and bioenergy* (pp. 1–18). Wiley.
8. Komariah, L. N., Hadiah, F., Aprianjaya, F., & Nevriadi, F. (2018). Biodiesel effects on fuel filter: Assessment of clogging characteristics. *Journal of Physics: Conference Series*, 1095(1), 012017.
9. Arulmozhi, G. (2002). Exact equation and an algorithm for reliability evaluation of k-out-of-n: G system. *Reliability Engineering & System Safety*, 78(2), 87–91.
10. Li, J., Zeng, J., Ye, Z., & Huang, X. (2021). Are clean technologies more effective than end-of-pipe technologies? Evidence from Chinese manufacturing. *International Journal of Environmental Research and Public Health*, 18(8), Article 4012. <https://doi.org/10.3390/ijerph18084012>.
11. U.S. Environmental Protection Agency. (2025, October 22). *Evolution of the Clean Air Act*. <https://www.epa.gov/clean-air-act-overview/evolution-clean-air-act>.
12. Hornická, K., Ďurčanský, P., Holubčík, M., & Nicolanská, M. (2023). Separation of solid pollutants transported from small heat sources with chimney filters. *Transportation Research Procedia*, 74, 785–790. <https://doi.org/10.1016/j.trpro.2023.11.543>
13. Jayadi, H., Hendrarinata, F., Suyanto, B., & Sunaryo. (2021). Chimney filter model wet scrubber to reduce air pollutant emissions on the incinerator. *Health Notions*, 5(2), 41–45. <https://doi.org/10.33846/hn50201>
14. Rykov, V., Ivanova, N., & Kochetkova, I. (2022). Reliability analysis of a load-sharing k-out-of-n system due to its components' failure. *Mathematics*, 10(14), 2457. <https://doi.org/10.3390/math10142457>
15. Baumgarten, B., Grammer, P., Ehard, F., Winkel, O., Vogt, U., Baumbach, G., Scheffknecht, G., & Thorwarth, H. (2023). Evaluation of a metal mesh filter prototype with wet regeneration. *Biomass Conversion and Biorefinery*, 13, 6007–6022. <https://doi.org/10.1007/s13399-021-01716-2>