RELATIONSHIP BETWEEN SENIOR SECONDARY SCHOOLS STUDENTS' ACHIEVEMENT IN MATHEMATICAL PROBLEM – SOLVING AND INTELLECTUAL ABILITIES TESTS

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Abstract:

The study was aimed at establishing the relationship between the achievement of secondary school students in mathematical problem solving and their achievement in intellectual ability test using factor analysis. The study population consisted of all the students in Senior Secondary Schools (SSS) in Ondo State. Out of this population, a sample of 206 students was purposively selected from three senatorial districts of the State. Seven instruments (six on intellectual abilities and one on mathematical problem solving) were administered on the respondents. The finding of the study showed that a significant relationship existed between mathematical problem solving and intellectual abilities (verbal, induction and numerical abilities) of the students. Thus, teachers should provide avenue for students to improve their intellectual abilities of verbal, induction, numerical, retention and other abilities. This could be done by enriching the learning environment using different instructional techniques, materials and activities in the classroom.

Keywords: Factor analysis, mathematical problem solving, intellectual abilities, achievement, senior secondary students

Introduction:

Surveys of students' attitude towards Mathematics and students' in-class performance and their performance in public examinations provide evidence of students' poor attitude and decline poor performance in Mathematics (Igbokwe, 1997). A number of factors had been identified by various researchers and educators as being responsible for poor achievement in Mathematics such as students' characteristics, instructional/classroom characteristics, teachers' characteristics, societal factors and school factors. Other factors, often cited, include lack of motivation and poor self-image (self-concept) of primary school teachers, lack of innovative teaching methods, lack of teaching facilities, poor school climate, lack of incentives and motivation, poor remuneration, poor condition of service and students' poor problem-solving abilities (Munro, 1979; Ubani, 1983; Aina 1986; Nwoji, 1999)

Problem solving has been observed to be one of the principal causes of scholastic failure in areas of science such as mathematics, chemistry and physics (Perez & Terragosa, 1983). This is so because pupils do not learn how to solve problems but merely memorize solutions explained by teachers in line with the traditional method of teaching. Problem solving transcends all scientific disciplines and it constitutes an integral part of mathematics programmes at all level of studies, almost everything that an individual does involves problem solving which is directed toward achieving a goal. A problem is said to exist whenever there is gap between a present state and an anticipated goal state without any immediate clear picture of how to bridge the gap (Hayes, 1981). Philips and Philips (1991) categorized problems into two namely (1) Generic Problems – These are problems with standard procedure by which they may be solved and the procedure consists of series of steps that are performed in order to accomplish the goal of the problem. (2) Harder problems – These are made of complex problems combining several generic problems or by using more complex language to extrapolate the problem into an unfamiliar solution.

Different authorities have offered different definitions of problem solving as a concept. Ausubel (1968) defines problem solving as a form of discovery learning which bridges the gap between a learner's existing knowledge and solution to problem. Here problem solving will draw heavily on previous/existing knowledge of the learner which must be brought to bear on the situation at hand. However, the task of assessing or keying into previous knowledge is not always easy as this depends on a number of variables existing at the time when the experiences were formed and or when they were being recalled by the learner. Duffield (1989) on his part defined problem solving as a goal directed cognitive learning process that makes use of previously learned knowledge and cognitive strategies. This view of problem solving not only sees it as relying on previous knowledge but also as a learning process involving cognitive controls such as cognitive style and metacognition. However, it tends to ignore the affective and conative elements necessary for problem solving (Jonassen & Tessmer, 1996). The affective elements here include beliefs, attitudes and problem domain while conative ones include the motivational factors, persisting on task, exerting efforts, and making choices.

Padgette (1991) also noted that problem solving is an art, which consists of understanding all the rules and that means all of them, not just the point–by-number type-and then understanding which ones to break in any given problem.Thus, any problem situation contains three important characteristics, the givens, a goal and obstacles. The givens are the elements, their relations and the conditions that compose the initial state of the problem, the goal is a desirable scientific end and the obstacles are the characteristics of the problem solver and the situation that makes it difficult for the solver to know how to transform the initial stage of the problem into the final stage. In order to solve problem successfully the problem solvers need to understand the content and knowledge of the subject matter. Farayola and Salaudeen (2009) also opined that problem solving is a complex mental process that involves visualizing, imagining, manipulating, analyzing, abstracting and associating ideas. They further stated that problem solving is a process which begins with the initial contact with the problem and ends when the solution is reviewed in the light of the given information. From all the definitions given above, problem solving can be defined as a goal directed sequence of cognitive, affective and conative operations geared towards finding an unknown entity for bridging a gap between a present and a goal state.

Several problem solving strategies with different phases or steps have been advanced for use in teaching mathematics and science. Some applicable for both numerical and non-numerical problems while some are useful to numerical problems only. Polya (1945) proposed a four step general framework for problem solving as follows: (i) Understanding the problem; (ii) Devising a plan to solve the problem; (iii) Carrying out the plan and (iv) Looking back. Perez and Torresgrosa (1983) suggested that scientific methodology and problem solving should have some similar conceptions as both are involved in investigative processes. They insisted that problem solving should follow the path of scientific methodology rather than relying on the usual algorithms which merely require mechanical application of clearly defined procedures for each type of problem. According to them, the methodological components of problem solving have the following four characteristics which led to the four stages involved in their strategy:

- A qualitative study of the situation in hand is to be carried out and hypotheses put forward i.e developing a theoretical paradigm.
- Possible strategies for solving the problem have to be devised in the light of the qualitative study carried out and the theoretical knowledge available.
- The problem itself has to be solved with high degree of verbalization being encouraged i.e execution of problem solving.
- The results are analyzed and evaluated.

Another strategy developed by Ross (1988) for problem-solving involves what he calls 4 clusters of skills namely: Problem representation, Information retrieval, Information processing, and Information reporting. Ross (1988) added that out of the four skills outlined, problem representation is the most difficult skill for students. In order to systematically address this problem, there is need to purposefully mount instructional intervention programme that will foster this skill in students. The emphasis here should be on the two main procedures involved in problem representation namely: (i) Focusing the problem (identifying) a particular problem to be solved, and (ii) Developing a frame work (a representation) of the essential elements of the problem in a mental structure conducive to finding a solution. Perhaps what is missing in the strategy is a fifth step which is very important if one must assist the learner discover and learn from his mistakes during problem solving process – the evaluation and feedback stage. The missing gap in Ross (1988) strategy was filled by Alan (1995) who suggested the following steps in problem solving: (i) Defining the problem, (ii) Planning a solution, (iii) Solving the individual part which involves sketches, diagram and grouping, (iv) Putting it all together, and (v) Evaluation.

Research studies on problem solving revealed that problem differed in structure, context, complexity and representation. Gick (1986) and Jonassen and Henning (1999) opined that problem solving depend on conceptual knowledge and procedural knowledge available to the learner. Students without conceptual knowledge of the subject matter will find it difficult to solve problem. Greeno (1978) emphasized the relationship between the conceptual possessed by the problem solver and their knowledge of the procedures that all problem solving is based upon two types of knowledge which are knowledge of the problem solving and conceptual knowledge. In order to solve a problem therefore, a problem solver must develop a framework or representation (model) of the essential elements of the problem in a mental structure conducive to finding a solution after proper focusing of the problem. This implies that every problem solver constructs a mental representation (or mental model) also known as problem space based on prior knowledge (Newell & Simon, 1972). Internal mental models or problem spaces according to Jonassen and Henning (1999) consist of structural knowledge, procedural knowledge, reflective knowledge, images and metaphors of the system and executive or strategic knowledge. Mason–Mason and Tessmer (2000) adds that after mental construction of the problem space, (internal mental representation or external physical representation); a conscious meaning – making effort engaged through activity must follow in order to solve a problem.

Experts in instructional designs and mathematics educators have argued that since different learning experience are provided for achieving different outcomes (Gagne & Briggs, 1978) therefore, instruction to support problem–solving learning outcomes should differ from that used to support concept learning or rote learning for instance. They recommend instructional strategies such as authentic cases, simulations, modeling, coaching and scaffolding (Jonassen, 2000). Different problem solving models have emerged in the last thirty years in a bid to explain the processes involved in problem–solving. The most popular of these models is the information processing and solution construction model e.g. IDEAL which focuses on identifying potential problems, defining and representing the problem, exploring possible strategies, acting on these strategies and looking back and evaluating the effects of these activities (Bransford & Stein, 1984). The information processing theory believes that humans take in information (like any other information processor e.g. computer), translates the problem into an internal representation on which it can operate, operates on it appropriately and output the results. The information processing model further assumes a uniform theory of problems (Smith, 1991). Studies based on this theory tend to find out what knowledge learners already bring to task, what information they key on and how they use these items of information.

Others have argued that problem solving is not a uniform activity and as such cannot be regulated by a uniform theory. This is because problems are different in content, form or process (Jonassen, 2000). This has led to other emerging models vis–a–vis theoretic conceptions of problems solving such as the Schema–Model. This model assumes that different schemas are required in solving different or particular problems. If the learner possesses a complete schema for any problem type, then constructing the problem representation will be a question of mapping an existing schema onto a problem and using the procedure that is part of the problem schema to solve it. Existing problem schemas are said to be the result of previous experiences in solving particular types of problems, enabling learners to proceed directly to the implementation stage of problem solving and try out the activated solutions (Gick, 1986).

The implications of these views are three fold. First, is that problem solving is a byproduct of our knowledge (previous experiences) of the concepts or ideas involved in a problem situation (conceptual knowledge) and also our ability to take the right steps and decisions (procedural knowledge). Second, experts and novices have different problem schemas. Novices do not possess well developed problem schemas and they are not able to recognize problem types and so rely on weak problem strategies such as information processing approaches (Mayer, 1992). On the other hand, experts are good problem solvers as they recognize different problem states thereby reducing the searching through problem space (Sweller, 1988). Third, is the type of problem? Problems vary in their nature or structure, complexity and manner of presentation or representation.

Stewart (1982) concluded that no matter how simple a problem may be, a beginner in problem solving required significant procedural and conceptual knowledge. Conceptual and mathematical knowledge are not enough to determine the success of students in problem solving. Attitude of students toward mathematics is a good predictor in solving problems. Ilemona (2001) asserted that the kind of attitude a child brings to the classroom is very important and that attitudes are fundamental to the dynamics of behaviour and they determine what the student learns. Other factors affecting the learning of mathematics problem solving have been traced to students' intellectual abilities. Lester (1980) reported that reading skills contribute to verbal problem solving abilities and single element among others such as verbal, numerical, memory and computation abilities are associated with and related to success in mathematical problem solving (Falokun, 1981). Akpan (1988) also reported a significant relationship between mathematical problem-solving abilities and the affective behaviour such as motivation and interest while Ojaleye (1996) also reported a significant relationship between the performance of students in intellectual abilities test and mathematical problem solving test among Junior Secondary School students. The purpose of this study therefore is to establish a relationship between intellectual abilities and mathematical problem solving abilities and to subject the items to factor analysis and select items of satisfactory loading to determine the relationship.

Research Questions

The study was designed to answer the following questions:

(i)Among the intellectual ability test which items are of the greater factor loadings?

(ii) Among the mathematical problem solving test, which item are the greater factor loadings

(iii) Which are the common factors that can be inferred?

Research Method

The sample for this study consisted of 206 SS III students in the second term of their final year who are already preparing for Senior Secondary Certificate Examination in Mathematics. Purposive sampling technique was used in selecting the sample from the three senatorial districts of Ondo State. The research instrument used were seven tests of which six were intellectual abilities constructed by the researcher and one obtained from Romberg and Wearne (1979) Mathematical Problem – Solving Test which yielded three scores - Comprehension, Application and Problems Solving. The tests were reviewed and vetted for face and content validity by two experienced secondary school Mathematics teachers and two evaluators with Mathematics background. The Kuder Richardson formula 20 (KR_{20}) was used to establish reliability coefficient estimate from 0.678 to 0.718. Intellectual Ability Tests: Verbal Ability Test [0.682]; Induction Ability Test [0.678]; Numerical Ability Test [0.702]; Retention Ability Test [0.680]; Perceptual Speed Ability Test [0.718]; Spatial Relation Ability Test [0.710] and Problem Solving Tests: Mathematics Comprehension Test [0.702]; Mathematics Application Test [0.675] and Problem Solving Ability Test [0.712].

Results and Discussion

Table 1: Mean and standard deviation of intellectual ability and problem solving tests

	INTELLECTUAL ABILITY TESTS						PROBLEM SOLVING TESTS		
	VAT	IDAT	NAT	RAT	PESAT	SRAT	MCT	MAT	PSAT
Mean	10.88	7.70	9.65	12.27	10.76	8.90	5.54	13.94	10.07
S. D	5.45	4.08	2.75	5.14	5.59	3.62	6.18	4.52	7.71

Table 1 showed the means and standard deviations of intellectual abilities and mathematical problem solving ability tests. From table 1, the mean scores of tests X_1 to X_6 ranged from 7.70 to 12.27 while that of tests X_{7a} to X_{7c} ranged from 5.54 to 13.94. Similarly, the standard deviations for tests X_1 to X_6 ranged from 2.75 to 5.59 while that of tests X_{7a} to X_{7c} also ranged from 4.52 to 7.71.

Table 2: Factor Analysis of Tests Scores and Their Communalities

TESTS	FACTOR I	FACTOR 2	COMMUNALITY
VAT [X_1]	0.2889	0.7530	0.5794
IDAT [X_2]	0.2899	0.7092	0.5162
NAT [X_3]	0.2457	0.8445	0.6611
RAT [X_4]	0.3352	0.3955	0.2289
PESAT [X_5]	0.2585	0.3735	0.0859
SRAT [X_6]	0.7867	0.0382	0.5138
MCT [X_{7a}]	0.3288	0.7612	0.4709
MAT [X_{7b}]	0.6872	0.7965	0.8562
PSAT [X_{7c}]	0.9868	0.0673	0.9848

Number of students = 206; $P < 0.001$

The result of the factor analysis in Table 2 showed that the Spatial Relation Ability Test [X_6], Application Ability Test [X_{7b}] and Problem-Solving Ability Test [X_{7c}] load on factor one while Verbal Ability Test [X_1], Induction Ability Test [X_2], Numerical Ability Test [X_3] and Comprehension Ability Test X_{7a} load on factor two. One can inferred that most of factor one was primarily mathematical problem solving test while factor two was principally intellectual ability test. The communalities obtained in Table 2 revealed that tests X_1, X_2, X_3, X_6 and X_7 had 40 - 60% of their variances explained while tests X_{7b} and X_{7c} had over 70% of their variances explained.

The results of this study revealed that student performance in intellectual abilities tests X_1 to X_6 provide a strong and significant relationship between it and their performance in mathematical problem solving test in secondary schools. This can be explained as the comparable common factors (communalities) found for one factor were over 40% but less that 60% of their variances, loading on that factor principally (intellectual abilities) while two of the tests have over 70% of their variances explained by common factors of mathematical problem – solving. In particular, the application tests appeared to be more highly related to the intellectual ability of secondary school students. These findings agreed with the findings of Meyer (1978), Munro (1979) and Ojaleye (1996) who reported that intellectual abilities are related to mathematical problem solving for various sexes at grade 4 to 7.

Conclusion:

Based on the findings of this study, it is suggested that teachers should take into cognizance the significant intellectual abilities that would improve the teaching/learning of mathematics in the junior secondary schools. Hence, teachers should provide avenue for students to improve their intellectual abilities of verbal, induction, numerical, retention and other abilities and this could be done by enriching the learning environment with different instructional materials and activities which enables the students to participate actively in the classroom. Similarly, government should encourage and motivate the teachers by giving them incentives and remunerations that will in turn propel them to put in their best. Finally, teachers should be exposed to constant and continuous workshop and seminars to learn new strategies in problem solving to meet the demand of the subjects and encourage professionals' mathematics educators to write books which emphasize the use of different strategies of mathematics problem – solving.

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