CONTROL OF HINDMARSH-ROSE MODEL BY NONLINEAR-OPEN-PLUS-CLOSED-LOOP (NOPCL)

Tounsia Benzekri, PhD
University of Science and Technology Houari Boumedien,
Faculty of mathematics, EL ALIA, Bab Ezzouar, Algiers, Algeria

Abstract

In this work, we use the Nonlinear-Open-Plus-Closed-Loop (NOPCL) method to control a nonlinear model: the Hindmarsh-Rose model in which we can exhibit regular and chaotic dynamics. The aim of the NOPCL method is to entrain complex dynamics to arbitrary given goal dynamics, by adding a suitable control term to the system. We use this method to suppress chaos, by entraining chaotic dynamics to a periodic one for the Hindmarsh-Rose model.

Keywords: Control chaos, Hindmarsh-Rose model, nonlinear-open-plus-closed-loop

Introduction:

There have been a great number of studies related to the control of nonlinear dynamical systems (For review see Refs.(1,2,3,4,5,6). These methods have been applied in a wide number of domains including physical and biological systems, robotics, avionics and many other.

Particularly there was a great deal of research to modeling and control mechanism of excitable biological media such as activity of neurons which exhibit chaotic behavior (e.g. Refs.(7,8,9).

The Hindmarsh-Rose model (HRM), which models a neuronal electrical activity, is a three-dimensional model capable of complex dynamics such as bursting oscillations and chaos. Neurons react on injection of a current by a quick, short depolarization of their membrane potential, which is negative in rest.

The activity of neurons consists of series of pulses, alternated by long periods of low activity around rest potential. This is known as an action potential, or spike.

Bursting oscillation is a time evolution consisting of bursts of rapid spikes, alternated by phases of relative quiescence. These series of pulses are considered to carry the information transmitted by neurons.

We use the NOPCL method to show how the Hindmarsh-Rose model can be controlled by driving its output to the desired pattern. The aim of this method is to add a control term, a driving term, to the initial system in order to drive its dynamics from one trajectory into another one. In particular, this method is able to switch chaotic dynamics into a periodic one and vice versa.

Entrainment Control

Let us recall the Entrainment Control as explained in Refs.2.

We denote by u the additive control term, the controlled dynamical system is then given by:

$$\frac{dx}{dt} = F(x,t) + u(t) \qquad (1)$$

The control problem is to find a control function $u \in R^n$, such that the system state x is entrained to arbitrary given goal dynamics g for which the error between x and g satisfies:

$$\lim_{t\to\infty} \|x(t) - g(t)\| = \lim_{t\to\infty} \|e(t)\| = 0 \qquad (2)$$

The basin of entrainment associated with an appropriate time t_s and g is defined by:

$$BE(g, t_s) = \left\{ x(t_s) / \lim_{t\to\infty} \|e(t)\| = 0 \right\} \qquad (3)$$

The goal is to show that the basin of entrainment is not an empty set, that is the error $e = 0$ is asymptotically stable for the error equation, and is independent on the goal dynamics g.

Open-Plus-Closed-Loop control (OPCL)

An (OPCL) strategy was first be proposed by Hubler and Luscher Refs.3 and extended by Jackson and Grosu Refs.2 to control the system (1). The proposed control term u is of the following form:

$$u(t) = S(t)\left\{ \left[\frac{dg}{dt} - F(g,t) \right] - C(g,t)e(t) \right\}, \qquad (4)$$

where the first term of u is called the Huble's open-loop interaction and $S(t)$ is a suitable scalar switching function on time t_s satisfying:

$$S(t) = 0 \ \text{ for } \ t < t_s; \qquad 0 < S(t) \le 1 \ \text{ for } \ t \ge t_s. \qquad (5)$$

The linear closed-loop interaction $C(g,t)$ is given by:

$$C(g,t) = \frac{dF(g,t)}{dg} - A, \qquad (6)$$

where A is an arbitrary matrix whose eigenvalues all have negative real parts.

Jackson and Grosu Refs.2 proved that if the function F is everywhere Lipschitz, with respect of x, then for an arbitrary smooth goal function g, the control u is such that none of basins of entrainment associated to g are empty sets.

Indeed, substituting equation (4) into the control system (1) and letting $S(t) = 1$ yields to the given equation:

$$\frac{de}{dt} = F(e + g, t) - F(g,t) - \left[\frac{dF(g,t)}{dg} - A \right]. \qquad (7)$$

Expanding $F(e + g, t)$ for small e, in the first order, yields the linear approximation equation:

$$\frac{de}{dt} = Ae. \qquad (8)$$

Since all eigenvalues of the matrix A have negative real parts, the asymptotic stability of equation (8) is established.

However, it was shown by Y. Tian et al. Refs.4, that for a certain class of systems the basin of entrainment is rather complicated; it is dependent on the goal dynamics g.

Nonlinear Open-Plus-Closed-Loop control (NOPCL)

The NOPCL control is based on the OPCL control. The control term u is reconsidered as follows:

$$u(t) = S(t)\left\{ \left[\frac{dg}{dt} - F(g,t) \right] - C(g,t)e(t) - N(g,x,t) \right\}, \qquad (9)$$

where C is as defined in (6) and A defined as previously. The nonlinear term $N \in R^n$ is the closed-loop control action whose ith element $N_i(g,x,t)$, is given for sufficiently smooth F , by:

$$N_i(x,g,t) = \frac{1}{2!}\sum_{j,k=1}^{n}\frac{\partial^2 F_i(g,t)}{\partial g_j \partial g_k}e_j e_k + \frac{1}{3!}\sum_{j,k,l=1}^{n}\frac{\partial^3 F_i(g,t)}{\partial g_j \partial g_k \partial g_l}e_j e_k e_l + \ldots + \frac{1}{m!}\sum_{j,k,\ldots,p=1}^{n}\frac{\partial^m F_i(g,t)}{\partial g_j \partial g_k \ldots \partial g_p}e_j e_k \ldots e_p$$

$$m \geq 2, \qquad i = 1,2,\ldots,n \qquad (10)$$

where m is the order of derivative of F called the order of parameter of the function N .

In this case, expanding $F(e+g,t)$, for small e, one obtain:

$$\frac{de_i}{dt} = Ae_i + \frac{1}{(m+1)!}\sum_{j,k,\ldots,p=1}^{n}\frac{\partial^{(m+1)} F_i(g,t)}{\partial g_j \partial g_k \ldots \partial g_p}e_j e_k \ldots e_p + \ldots, \qquad (11)$$

$$i = 1,2,\ldots,n$$

It is easily proven Refs.2 that the basins of entrainment are the whole phase space for systems for which the function F is polynomial of degree m, $m \geq 2$. This is due to the fact that in this case, (11) will be reduced to (8), and e solved from this last equation approaches zero for all initial condition $e(t_s)$.

Control of Hindmarsh-Rose Model

The Hindmarsh-Rose Model was developed by Hindmarsh and Rose (1984) to describe an isolated triggered burst of action potentials observed in a brain cell of a pond snail. The equations are given by:

$$\frac{dx}{dt} = y - x^3 + 3x^2 - z$$

$$\frac{dy}{dt} = 1 - 5x^2 - y$$

$$\frac{dz}{dt} = \varepsilon(4x + K - z) \qquad (12)$$

where x is the membrane potential, y and z represent empirical variables describing the activation and inactivation of the ionics conductance. They describe respectively some fast and slow gating variables for ionics. Slow activation of z is due to the small parameter $0 < \varepsilon \leq 1$.

These equations model the electrical activity of the membrane potential of a single neuron. The external current K is viewed as a control parameter delaying and advancing the activation of the slow current in the model.

Notice that the system is autonome.

Simulation results

In order to illustrate the effect of the driving term, we fix the parameter ε of the HRM to $\varepsilon = 0.006$.

For this system all the fourth order partial derivatives are equal to zero since the function F of HRM is a polynomial of degree 3. Notice that it is easy to see that the function F is everywhere Lipschitz.

The control parameter m is thus taken to be 3 in the NOPCL control.

It follows that the closed-loop control action $N(g,x,t)$ is given by:

$$N(g,x,t) = \left[(-3g_1(t)+3)e_1^2(t) - e_1^3(t), -5e_1^2, 0\right] \qquad (13)$$

For convenience, the matrix A is taken diagonal and the linear closed loop interaction is given by:

$$C_1(g) = \left(-3g_1^2(t) + 6g_1(t)\right)e_1(t) - e_2(t) - e_3(t), \qquad (14)$$

$$C_2(g) = -10 g_1(t) e_1(t) + (1 + a_{22}) e_2(t), \quad (15)$$

$$C_3(g) = \varepsilon (4 e_1(t) - (1 + a_{33}) e_3(t). \quad (16)$$

In our case, the purpose of the control action is to steer the Hindmarsh-Rose model from one of its trajectory to another one. Hence $g(t)$ is such that

$$\frac{dg(t)}{dt} - F(g) = 0, \qquad \forall t \ . \quad (17)$$

The control term u is then the sum of $C(g)$ and $N(g, x, t)$.

The error equation is the same as in (8). Hence, for the Hindmarsh-Rose Model, the basin of entrainment $BE(g)$ is global for all values of g and

$$e_i(t) = \exp(a_{ii}) \qquad \text{for } i = 1,2,3 \quad (18)$$

For numerical analysis, we choose the matrix A as follows:

$$A = diag(-1, -1, -0.01) \quad (19)$$

The goal trajectory $g(t)$ is a bursting periodic motion. Our aim is to steer the HRM from chaotic trajectory to bursting oscillating trajectory.

Numerical simulations shows that the solution turns out to be chaotic for the current $K = 3.15867947$ and periodic for $K = 5.1$.

We depict, in respectively Figure1 and Figure2, the periodic bursting trajectory for $K = 5.1$ and the chaotic trajectory for $K = 3.15867947$ and initial conditions: $(x(t_0), y(t_0), z(t_0)) =$ (-1.1804,-5.809943,0.02212644).

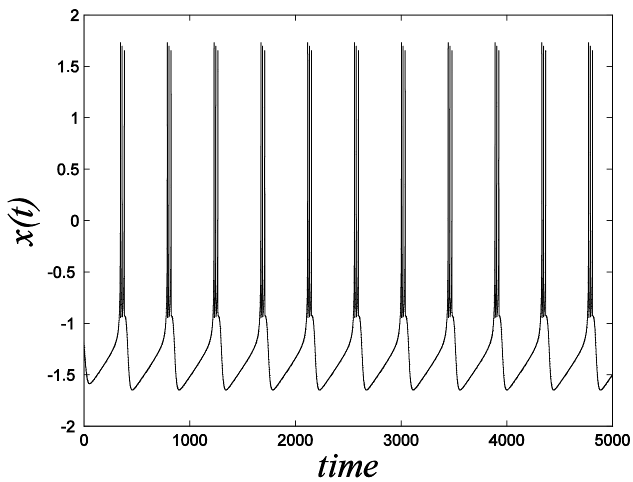

Figure1. Bursting oscillations of the membrane potential $x(t)$ for $K = 5.1$ and initial condition (-1.1804,-5.809943,0.02212644).

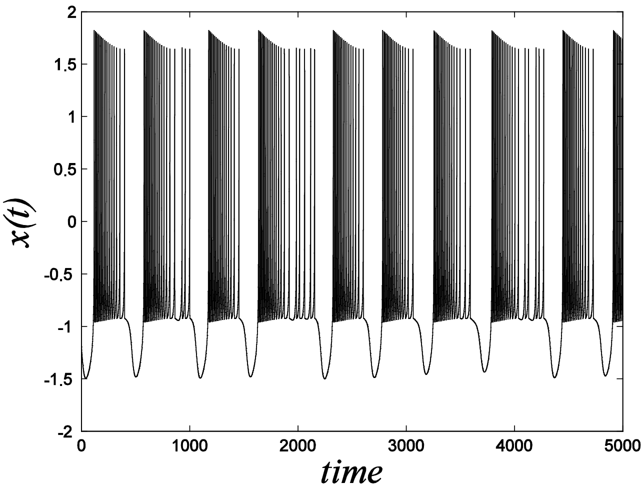

Figure2. Chaotic solution of the membrane potential $x(t)$ for $K = 3.15867947$ and initial condition $(-1.1804,-5.809943,0.02212644)$.

Lyapunov exponents are used to describe the periodic and chaotic dynamics of nonlinear dynamical system.

The time varying largest Liapunov exponent, showing, for $K = 3.15867947$ the chaotic motion, is represented in Figure3.

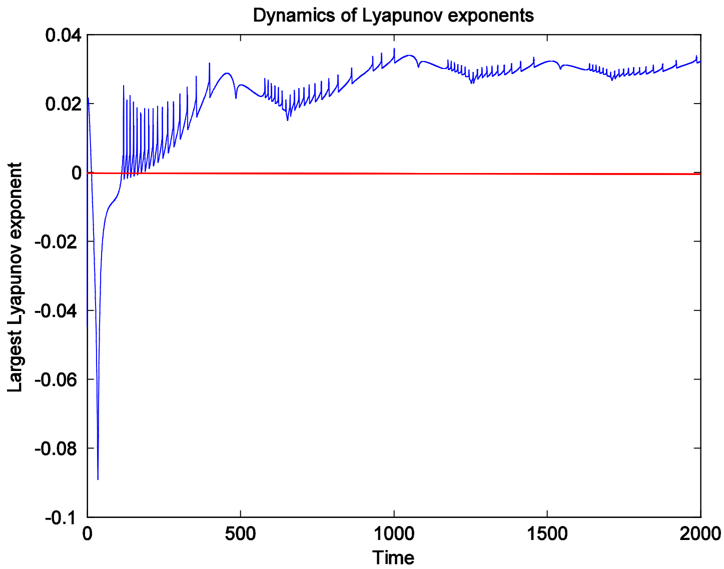

Figure3. The largest Lyapunov exponent for chaotic solution of HRM.

In order to avoid the transition phase of the trajectory, we start control of the chaotic motion at $t_s = 1300s$ for the same initial condition as above.

We observe in Figure4 that at this time t_s, the trajectory is driven to the bursting trajectory, thus removing chaos.

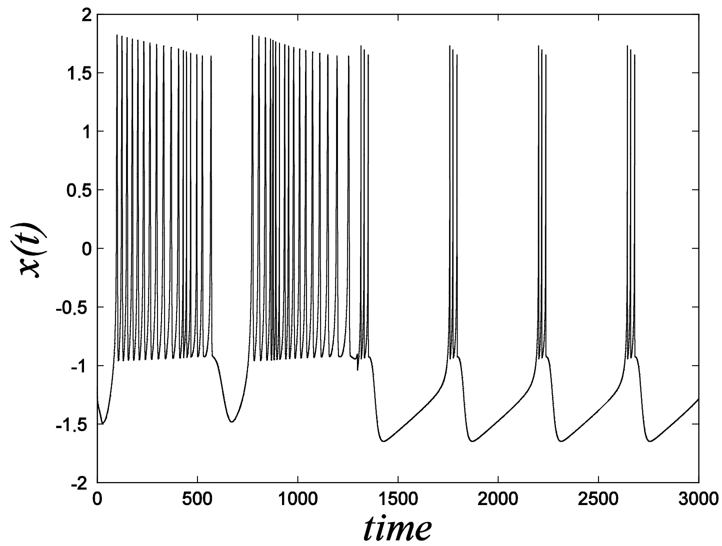

Figure4. The HRM driven from chaotic solution to bursting solution by adding the control term u .

Conclusion:

We considered the Hindmarsh-Rose model. We have shown by using the NOPCL method that it is possible to switch from one trajectory of the system into another one and therefore changing the dynamics of the potential action.

The aim of this method is to add a suitable driving term to the HRM, which forces the controlled system to perform a motion which coincides with a target trajectory of the model. We showed that we can suppress the chaotic dynamics of the HRM.

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