R, MATLAB AND SPSS FOR FACTOR ANALYSIS PURPOSES: SOME PRACTICAL CONSIDERATIONS AND AN APPLICATION

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Abstract

Factor analysis and principal component analysis are two techniques which carry out in a set composed of p-variables as well as large number of data (in multivariable statistical analysis). Both techniques aim to reduce the number of primary variables by calculating a smaller number of them, called factors or principal components(Hair, et al., 2010).In most cases these techniques are applied together, even though they are two different methods. Both aproaches have similarities and differences. A view either oriented on similarities or the differences is reflected in software implementations. In this paper we express some practical considerations on three softwares: SPSS, MatLab and R, related to factor analysis. A real data set is used for this purpose. SPSS does not offer the PCA program as a separate menu item, as MatLab and R. The PCA program is integrated into the factor analysis program. Principal component and maximun likelihood are used to estimate weights, Varimax and Promax are used to estimate weight with rotation while regression technique is used to estimate scores. These are the only estimating procedure available in the base package of R. Finally, we will discuss their solutionswith the mentioned software packages.

Keywords: Principal component, maximum likelihood method, SPSS, MatLab, R**.**

1. Introduction

Let suppose that our sample consists of n subjects and$p$quantitative variables. The rows of the matrix with the data are the vectors

$$X_i^{'} = (X_{i1},\ldots,X_{ip}) , \quad i = 1,\ldots,n . \qquad (1)$$

Let $\mathbf{S}_X$ be the covariance matrix and $\mathbf{R}$ the correlation matrix associated with the random vectors (1).

Describtion of method FA– calculation of factors

The simple model of FA is the one that express each variable X_j as a linear combination of common factors F_j; in the matrix form can be written as (2):

$$X = \mathbf{\mu} + L * F + \mathbf{e} \Leftrightarrow \underbrace{X - \mu}_{p x 1} = \underbrace{L}_{p x q} * \underbrace{F}_{q x 1} + \underbrace{e}_{p x 1} \qquad (2)$$

The elements of the vectors $\mathbf{\mu}' = (\mu_1,\ldots,\mu_p)$ and of the koeficient matrix $L = [l_{ij}]_{i=1:p;\, j=1:q}$ are constants while the elements of the vectors $F' = (F_1,\ldots,F_q)$ and $\mathbf{e}' = (e_1,\ldots,e_p)$ are random variables.

We assume that:

1. $E(e) = 0$,
2. $\operatorname{cov}(e) = E(e * e') = \psi = diag(\psi_1^2,\ldots,\psi_p^2)$ (the covariance matrix of error, is a diagonal matrix)
3. $\operatorname{cov}(e,F) = E(e * F') = 0$ (Uncorrelated between them).

Also it can be shown that $E(F) = 0$, $\operatorname{cov}(F) = E(F * F') = I_q$ (has standard deviation equal to 1 and are uncorrelated).

Especially, we have to estimate:

1. The correlation or covariance matrix.
2. The numbers q of common factors (Elbow criterion, Kaiser criterion, etc.).
3. The loadings matrix of the common factors (Principal component, maximum likelihood, unweight least square, principal axis, etc.).
4. Rotation of the loadings (Varimax, Quantimax, Equamax etc.)
5. The error variance.
6. Factors scores (Regression, Bartlett (Johnson and Wichern, 1982) and Anderson-Rubin).

Communality

Communality is an indicator, expressed in percentage, and related to the contribution of the q factors in explanation of the i-variables:

$$D(X_i) = \underbrace{l_{i1}^2 + l_{i2}^2 + \ldots + l_{iq}^2}_{h_i^2 = communality} + \psi_i \,, \Sigma = LL' + \psi \text{ where } \sigma_{ii} = h_i^2 + \psi_i \qquad (3)$$

So, the i-th communality is the sum of squares of the loadings of the i-th variable on the q common factors (3).

There are several techniques, named together as factor rotation, which make an orthogonal transformation of the matrix **L** (factor loadings matrix), so that the interpretation of factor can be simplified (Johnson, Wichern, 1982).

The rotation affects loadings (big loadings become bigger, and small loadings become smaller) even in eigenvalue, which show how much percentage of variables is explained by each axe (factor). However, the sum of eigenvalue remains unchanged. These techniques are used after extracting the factors. That usage of different techniques brings different results for the same communality of data. Regardless of this, all analysts look towards buildings simple structure that is a model where each variable load only in one factor, so only one factor is used to interpret one variable.

Description of principal component analysis (PCA)

Principal component analysis (PCA) is a method which enables to explain the variance –covariance structure of a set of p-variables through a few linear combinations of these variables (components). In PCA we seek to maximise the variance of the linear combination of these variables. PCA is a technique that applies to data that are not groups of observations and fragmentation of variables in subset Y and X. PCA aim to reduce the dimention of a dataset. One advantage of this method is the simplicity with which we can increase the number of dimensions of space by adding new principal components.

If we note that S_X have the eigenvalue-eigenvector pairs $\{(\lambda_1, a_1), ..., (\lambda_p, a_p)\}$ where $\lambda_1 \geq \lambda_2 \cdots \lambda_p \geq 0$ and the eigenvector have length equal to 1, then the jth principal component is a linear combination

$$Y_j = \mathbf{a}'_j \mathbf{X} \text{ where, } \mathbf{X} = \left(X_1,...,X_p\right)' \text{ and } \mathbf{a}_j = \left(a_{j1},...,a_{jp}\right)' \text{ for } j = 1:p$$

which has the maximum variance among all vectors $\mathbf{a}_j$ which satisfy the conditions:

$$\mathbf{a}'_j \mathbf{a}_j = 1 \text{ and } \mathbf{a}'_j \mathbf{a}_i = 0 \text{ for } i < j.$$

An important indicator for the principal component is the percentage of variance associated to this component. The total variance of X is the sum of variance of components, $\mathrm{D}(\mathrm{X}_1) + ... + \mathrm{D}(\mathrm{X_p}) = \sigma_{11} + ... + \sigma_{pp}$ where σ_{jj}, is j-th element of the diagonal matrix S_X, that is, $tr(S_X)$. Since $\mathbf{tr(S_x)} = \lambda_1 + ... + \lambda_p$, then the proportion of total variance due to the k-th principal component is

$$\frac{\mathrm{D}(\mathrm{Y_k})}{\mathrm{tr}(\Sigma_X)} = \frac{\lambda_k}{\sum_{j=1}^{p}\lambda_j}, \text{ k=1:p}$$

A correct analysis PCA passes through data transformation (standardization) to have much more harmonious dispersions of variables that are studied and that modulate the effect of units differences to which the data are expressed.

The selection of the number of principal copmonents is another important problem of method.It is considered sufficient when at least explains 70%-80% of the total variance. We draw a graphic of pairs (j, λ_j), "scree plot", and we observed the position in which this graphic begin to become "flat" (Cattell, 1966; Jollife I., 2002).Another criterion to address the number of factor problem is the Kaiser criterion (Kaiser 1960).Often $\bar{\lambda} = \frac{1}{p} \sum_{j=1}^{p} \lambda_j$ is calculated and a component j is important when $\lambda_j > \bar{\lambda}$ and otherwise it is not. When the correlation matrix is used, $\bar{\lambda} = 1$, and a component j is important when the eigenvalues is $\lambda_j > 1$. Also this procedure can lead to ignoring the most important information(O'Toole, e et., 1993).For additional details about the PCA, see (Johnson, Wichern 1992).

Softwares:

SPSS: It does not offer the PCA as a separate menu item somewhere in the menu, like MatLab and R offer. The PCA program is integrated into the FA program, which can be found under: Analyze→Data Reduction→Factor.*SPSS* uses always the correlation matrix instead of the covariance matrix for computing the principal components.It offers 7 techniques for the factors extraction (including and PCA), 3 for choosing the number of factors, 5 for estimating loadings and 3 scores. The number of factorscan be explicitly specified or is given for us basen on criterion (eigenvalues >1). We work with the correlation matrix or with the covariance matrix.

MATLAB: This software offers pcacovfunction to determine the principal copmponents of a covariance matrix S, princompfunction to determine PC and eigenvalues of a data matrix X and FA (factoran). The last oneuses maximum likelihood estimation to find factor loadings. The 'Scores'option can specifyvarious scores methods ({'wls'} | 'Bartlett' | 'regression' | 'Thomson') while the 'Rotate'option can specify various rotation methods ('none'| 'varimax'} | 'quartimax' | 'equamax'| 'parsimax' | 'orthomax' | 'promax' | 'procrustes'| 'pattern'). The number of factors must be explicitly specified.

R: R is a free software product which has nowadays reached a high level of maturity and is being increasingly used by many people as a statistical analysis tool.Since it is a programming language with a large number of additional packages it offers several packages for FA:

- Stats (R Development Core Team, 2009) is a standard package installed with R. It offers separate function for PCA (princomp) and for FA (factanal). The last one uses maximum likelihood

estimation to find factor loadings. The number of factors must be explicitly specified. Varimax and Promax are possible.

- Efa (Klinike & Wagner, 2008) is a collection of function to perform for FA like in SPSS. However, it offers less extraction and rotation methods compared to SPSS.
- Psych (W. Revelle, 2003)is a package more used which consist od a collection of function and fa is a function include in this package. The fm option can specify themethod of estimation (ml= for maximum likelihood estimate andpa=for principal axis method). Therotateoptioncan specify various rotation methods (none, varimax, quartimax, bentlerT, geominT, Oblimin,simplimax, bentlerQ, geominQ and cluster)

The real data. Method of PCA: extraction of principal komponents.

During n=63 weeks, the margin of products forp=12departments has been recorded. The variables of the set arereported in Table 1 (Appendix), where V_i, fori=1:12 is cutting labels for 12 departments. Factor Analysis can provide quantitative evidence that departaments within each sector do experience similar week-to-week changes in margin.

Software: 1 Application in SPSS

Initially the PCA method was used with the aim that between 12 independent variables to cover those that effectively influence at the total margin. Our aim is to reduce the number of dimensions (the number of independent variables) that problem will estimates in a space with fewer variables, those that are extracted from PCA and FA.

Table 1 shows the proportion of variability, firstly explained with all factors together and then only with the factors before and after the rotation. The results showthat the first factor explains 59% of the total variance, the second explains 13% and the third 13%. From the other side, the results show the three first common factors, which explain 86% of the total variance (before the extraction), while after the extraction this percentage is reduced (80%). It is a quite good percentage to build the new model which will fit better to the margin model where all variables are considerate. After the rotation method, this percentage does not calculate, but it changes the percentage that explains each factor. This happened because the factors are correlated. These variables will suppose as the important factors for the approximation of the model.

Table 1 Total VarianceExplained by Components before and after Rotation Method

Before the extraction   After the extraction   After the rotation

Factor	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings[a]
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	7,15	59,544	59,544	5,913	49,277	49,277	6,712
2	1,61	13,418	72,962	2,162	18,020	67,297	4,330
3	1,53	12,770	85,733	1,549	12,905	80,202	1,946
4	,642	5,350	91,083				
5	,295	2,461	93,544				
6	,263	2,194	95,738				
7	,222	1,850	97,587				
8	,128	1,067	98,655				
9	,083	,693	99,348				
10	,033	,275	99,623				
11	,025	,212	99,834				
12	,020	,166	100,000				

Table 2, (2a, 2b and 2c) shows the cumulative components matrix and the component factors before and after rotation respectively.Results of the FAenable the initial variables grouped according to the strength of the connection withthe principal components. We show that the correlation between the variableV1 and the first factor is equal to 0.825 while with the others factors have a smaller correlation. In this manner the variables will be grouped like below:

- the variablesV1:V7associated with the first factor(correlation is greater than 0,7),
- the variablesV8, V9 and V10associated with the second factor,
- the two others variables associated with the third factor.

Table 2. Loading and cumulative matrices (Before and After Rotation (ML))

	2a					2b					2c	
	Factor					Factor					Initial	Extraction
	1	2	3			1	2	3		V1	,827	,765
V1	,825	,479	,519		V1	,700	,264	,454		V2	,960	,971
V2	,932	,267	,364		V2	,870	-,08	,454		V3	,965	,999
V3	,945	,476	-,034		V3	,999	-,02	-,021		V4	,968	,964
V4	,933	,768	,254		V4	,886	,413	,086		V5	,928	,895
V5	,944	,572	,267		V5	,901	,202	,205		V6	,950	,924
V6	,960	,546	,307		V6	,907	,180	,260		V7	,863	,793
V7	,833	,713	,180		V7	,805	,380	,020		V8	,828	,781
V8	,511	,874	,058		V8	,493	,683	-,268		V9	,873	,913
V9	,395	,942	,198		V9	,323	,875	-,209		V10	,756	,676
V10	,629	,642	,576		V10	,467	,570	,365		V11	,571	,397
V11	,018	,046	,607		V11	-,171	,276	,540		V12	,619	,548
V12	,278	,114	,733		V12	,064	,255	,692				

$0.765 = .700^2 + .264^2 + .454^2$ (1) $V1 = .700F1 + .264F2 + .454F3$ (2) $V1 = .825F1 + .479F2 + .519F3$ (3)

Note that in general groups of variables that were distinguished in this way are logical while correlations within each group are important (M.Muça& Ll. Puka, 2012).Table2a shows that76% of variable variance V1 is explained by the three factors, this is confirmed by the results presented to

Table2b, for example see (1). Regarding the Meyer-Olkin (KMO) statistical value which in this case is 0.809, see Table 3, we see the Kaiser- certain correlation of data (Hair *et al*., 2010) is reported.

Table 3 KMO and Bartlett's test   **Fig. 1** Scree plot of PC

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		,809
Bartlett's Test of Sphericity	Approx. Chi-Square	982,017
	df	66
	Sig.	,000
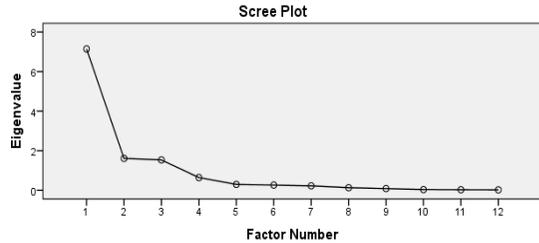
Another analysis (scree plot, see Fig.1) confirms the above conclusions: the graphic representation of couples is in the same order as reported in the Table 1.Separation process of the two eigenvalues greater than 1 in conjunction with those remaining.

Links of variables with factors (the 3 first) appear toFig. 2, at which point coordinates are variables with values of their correlations with the principal components; the latter are axis. In such graphical representation require group of variables which focus on axis. PackagesSPSS, MatLab, Rect., can calculate scores of data to factors and then store them as new variables together with initial. These variables (scores) may be inputs to a multiple regression or cluster analysis.They are important to examine the data structure and the localization of extreme values of the groups.
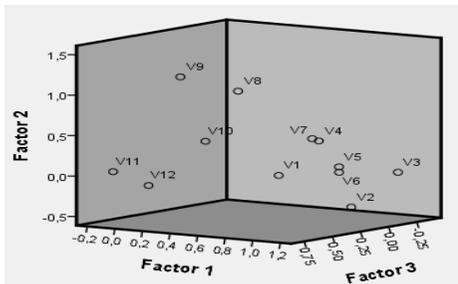
Fig. 2 Graph of loadings   Fig. 3 Graph of scores
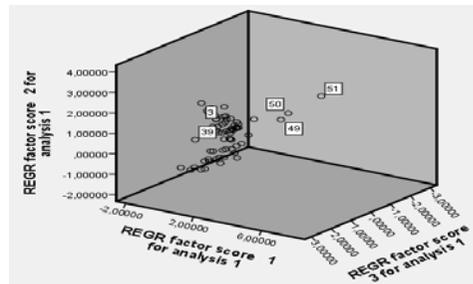
In Fig. 3 is shown the flow of points with their distances from the center. The 49th,50thand 51th scores-week are clearly distinguishedbecause they are far from other scores-week**.**

2) Application inMATLAB

Each row of the loadings matrix (Appendix, Table 1M a)) represents one of the twelve margins per department, and each column corresponds to a common factor. With unrotated estimates, interpretation of the factors in this

fit is difficult because one of 12 margins contain large coefficients for two factors (V10 department).In such cases, it is recommended that the matrix of weight estimated by rotation techniques. Factor Rotation helps to simplify the structure in the Loadings matrix, to make it easier to assign meaningful interpretations to the factors. From the estimated variances, we can see, according to the model that a particular margin varies quite a lot beyond the variation due to the common factors. A specific variance of 1 would indicate that there is no common factor component in that variable; while a specific variance of 0 would indicate that the variable is entirely determined by common factors. These data seem to fall somewhere in between (Appendix, Table 1M b)).

We see that the$p$-value is not displayed. This means that some of the eigenvalues calculated from the correlation matrix, are numbers very close to zero. Thereforethe matrix weight resulted different from what gave SPSS (Table 2b). From the matrix of weight after rotation PROMAX (Appendix, Table 2M) we see the situation becomes clearer. Each margin depends primarily on only one factor,and it is possible to describe each factor in terms of the margin that it affects. This can be illustrated through the graph (see fig. 1M).

Once it is accepted that the number of factors in the model will be 3 and PROMAX technique will be used to estimate the matrix of weight, also might categorize each week in terms of how favorable it was for each of three groups, based on the data from the 12 observed margins.In this case, the factor analysis includes three factors, and so the biplot is three-dimensional. Each of the 12 margins
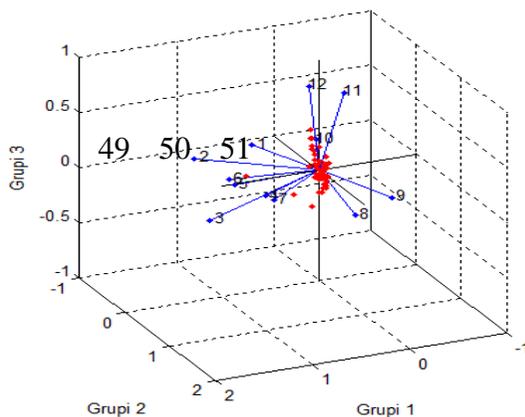
Fig. 1M The graph of weights after rotation and of scores

is represented in this plot by a vector, and the direction and length of the vector indicates how each margin depends on the underlying factors. For example, you have seen that after PROMAX rotation, the seven first margins

have positive loadings on the first factors, and unimportant loadings on the other two factors. That first factor, interpreted as *less frequently purchased products*, is represented in this biplot as one of the horizontal axes. The dependence of those seven margins on that factor corresponds to the seven vectors directed approximately along that axis. Similarly, the dependence of those seven margins on that factor corresponds to the seven vectors directed approximately along that axis. Similarly, the dependence of margins 8 and 9 primarily on the second factor, interpreted as*frequently purchasedproducts*, is represented by vectors directed approximately along that axe. The dependence margins 10, 11 and 12 primarily on the third factor, interpreted as*rarely purchased products*.Each of the 100 observations is represented in this plot (Fig 1M) by a point, and their locations indicate the score of each observation for the three factors. For example, points near thetop of this plot have the highest scores for the third group (*rarely purchased products*). Also, we see three unique points (farther than the flow of points) which correspond to the 49th, 50th and 51th weeks.

3) Application inR
Table 1R
Uniqueness's:

V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
0.236	0.029	0.005	0.036	0.106	0.076	0.209	0.218	0.089	0.319	0.597	0.448

Loadings:

	Grupi1	Grupi2	Grupi3
V1	0.7432	0.1381	0.4393
V2	0.892	-0.2109	0.3617
V3	0.9909	-0.0615	-0.0972
V4	0.9147	0.3455	0.0922
V5	0.9229	0.1157	0.1710
V6	0.9329	0.0826	0.2164
V7	0.8249	0.3309	0.0323
V8	0.5151	0.6979	-0.1730
V9	0.3618	0.8807	-0.0655
V10	0.5225	0.4692	0.4335
V11	-0.1194	0.1768	0.5984
V12	0.1249	0.1161	0.7230

Test of the hypothesis that 3 factors are sufficient.
The chi-square= **99.48**on 33df
The p-value =1.37e-08
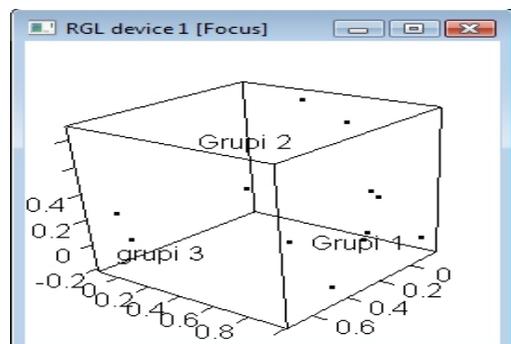
Fig. 1R The graph of weight.

	Group1	Group2	Groupi3
SS loadings	6.2682	1.844	1.5218
Proportion Var	0.5224	0.1537	0.1268
CumulaVar	0.5224	0.676	**0.8028**

The first three factors together explain 80% of total variance. This percentage does not change before and after the rotation (Appendix, Table 2R).

Results And Conclusion

Some of the similarities that appear in both techniques are: 1) Reduction of the number of the variables, 2) They use covariance-correlation matrices, 3) they are based on linear model and 4) PCA and FA exhibit the same results (without rotation).

In SPSS, factor reductionfunction fitsthe factorial model for a specified number of factors or it evaluates on the basis of criteria set out in the above data using different techniques such as: Kaiser criteria, maximum likelihood method, PCA etc.

In Matlab,factoranfunction fit the factorial model for a specified number of factors using maximum likelihood method.

In R, there are used some based libraries such as:foreings, stat andrglto load the data, to evaluate the weights and make graphs respectively.The function factanalis the same as the function factoranin Matlab.

UsingMatlab, the *p*-value and the proportion of variance do not appear. This means that, some of the eigenvalues estimated from the correlation matrix are approximately zero and in the calculation they are considered equal to zero.That is why the loadings matrix is different from the loadings matrix found in SPSS, while using R and SPSS these values are respectively p=1.37e-08 and p=0.00 with chi-square equal to 99.48 and 98.989 respectively. UsingR, the uniquenesses are the same as displayed in Table 2M. The three first factors explain 80% of variance. This percentage is invariablebefore and after rotation (Appendix, Table 2R). While usingSPSSthis percentage is not calculated after the rotation and it is equal to 85% before the rotation.

The results of analysis show that seven of 12 department are classified as*less frequently purchased products* that constitute the first factor,the three following departments (fruit and vegetables, coffee tea and snacks) are classified as *rarely purchased products*that constitute the second factor andthe last two (cigarettes and cans) as *frequently purchasedproducts* that constitute the third factor.

Even that the cigarettes are classified as *frequently purchasedproducts* (they are a daily consumption of a large mass), it happens that it can be purchased in the supermarket as much as the others seven products.

All the experiments and implementation are done using the packages: SPSS, R and MATLAB 6.5.

References:
Klinke, S., &Wanger, C., 2008. Visualizing exploratory factor models.COMPSTAT 2008-Proceedings in Computational Statistics.-18th Symposium held in Porto by P. Brito (ed.). Heidelberg: PhysikaVerlag (CDROM), Software:
http://stiner.wiwi.huberlin.de/mediawiki/mmstatde/index.php/SpezielleThemen-R-Visualisierung von Faktormodelen
R Development Core team 2009. R: A language and environment for statistical computing, Vienna, Austria: R Foundation for Statistical Computing.
*W. Revelle, 2013. p*ersonality-project.org/*r* webpage http://cran.r-project.org/web/packages/psych/psych.pdf
Johnson, R. A., Wichern D. W. (1992): Applied Multivariate Analysis, 3rd ed., Englewood Cliffs, NJ. Prentice-Hall.
Cattell Raymond Bernard, 1966.The scree test for the number of factors.*Multivariate Behavioural Research*.
Johnson R, Wichern D. 1982.Applied Multivariate Statistical Analysis, Prentice Hall, New Jersey, 6[th]ed.
JRHair JF, Black WC, Babin BJ, Anderson RE. 2010. Multivariate data analysis, 7th ed. Macmillan, New York..
ellife I.T., 2002. Principal Component Analysis, 2nd ed. Springer.
O'Toole, A.J., Abdi, H., Deffenbacher, K.A., and Valentin, D. 1993.A low dimensional representation of faces in the higher dimensions of the space. Journal of the Optical Society of America, Series A..
Kaiser, H.F. 1960. The application of electronic computers to factor analysis.Educational and Psychological Measurement.
Mu ç aM, Llukan P., 2012"Analizafaktorialedhepaketat SPSS, MATLAB AND R: konsideratangapërdorimipraktik.Takimii VII vjetorndërkombëtar, Shkup (Maqedoni).InstitutiAlb-shkenca. SeksioniiShkencaveNatyrore

APPENDIX

Table 1Description of the variables.

V1	SAUSAGE PRODUCTION
V2	DRINKS
V3	DRIED FRUITS AND FLOWERS
V4	BY-PRODUCTS FROM FRUIT AND VEGETABLES
V5	MAYONEZE, GOOSE, MARGARINE, MUSTARD, SALAD DRESSES
V6	PREPARED DISHES
V7	CONFECTIONARY
V8	FRESH FRUITS AND VEGETABLES
V9	COFFEE, TEA AND OTHER SNACKS
V10	FOOD
V11	CIGARETTES
V12	CANS

➢ **Implementation in Matlab** FA_MATLAB(7.3.0)**.**

1. Loadings matrix before rotation

Table 1M

a) Loadings			Uniqueness
0.7433	0.1381	0.4393	0.2355
0.892	-0.2109	0.3616	0.029
0.9909	-0.0615	-0.0973	0.005
0.9147	0.3455	0.0922	0.0355
0.9229	0.1157	0.171	0.1056
0.9329	0.0825	0.2164	0.076
0.8249	0.3309	0.0323	0.209
0.5151	0.6979	-0.1729	0.2177
0.3618	0.8807	-0.0654	0.0892
0.5225	0.4692	0.4336	0.3189
-0.1194	0.1767	0.5984	0.5964
0.1249	0.1161	0.723	0.4482

Tabela 2M Loadings matrices before and after**Varimax and Promax.**

Loandings　　　**Varimax**　　　**Promax**

Loandings			Varimax			Promax		
0.7433	0.1381	0.4393	0.7475	0.2396	0.3852	0.696	0.0099	0.331
0.892	-0.2109	0.3616	0.9605	-0.0365	0.2169	11.046	-0.3859	0.147
0.9909	-0.0615	-0.0973	0.9515	0.2195	-0.2039	10.954	-0.0591	-0.3128
0.9147	0.3455	0.0922	0.8113	0.5478	0.0788	0.7237	0.3687	-0.0162
0.9229	0.1157	0.171	0.885	0.3174	0.1026	0.8993	0.0673	0.0143
0.9329	0.0825	0.2164	0.9092	0.2798	0.1382	0.9344	0.0126	0.0519
0.8249	0.3309	0.0323	0.7195	0.5221	0.0261	0.6381	0.3721	-0.0628
0.5151	0.6979	-0.1729	0.3051	0.828	-0.0602	0.0416	0.8843	-0.1332
0.3618	0.8807	-0.0654	0.1311	0.94	0.1	-0.2516	10.711	0.0471
0.5225	0.4692	0.4336	0.4569	0.4962	0.4755	0.2227	0.4125	0.44
-0.1194	0.1767	0.5984	-0.0662	0.0201	0.6315	-0.2459	0.0192	0.6766
.1249	0.1161	0.723	0.2014	0.0024	0.715	0.0591	-0.0982	0.7425

Table 3M Correlation matrix

1	0.5918	3485
0.5918	1	0.2496
3485	0.2496	1

> **Implementation in R** FA_R (2.14.2)

Table 2R Loadings matrices before and after rotationvarimaxandpromax.

Matriva e peshave pa rrotullim

	Grupi 1	Grupi 2	Grupi 3
V1	0.7432	0.1381	0.4393
V2	0.8920	-0.2109	0.3617
V3	0.9909	-0.0615	-0.0972
V4	0.9147	0.3455	0.0922
V5	0.9229	0.1157	0.1710
V6	0.9329	0.0826	0.2164
V7	0.8249	0.3309	0.0323
V8	0.5151	0.6979	-0.1730
V9	0.3618	0.8807	-0.0655
V10	0.5225	0.4692	0.4335
V11	-0.1194	0.1768	0.5984
V12	0.1249	0.1161	0.7230

Matriva e peshave me rrotullim Varimax

	Grupi 1	Grupi 2	Grupi 3
V1	0.7475	0.2396	0.3852
V2	0.9605	-0.0365	0.2169
V3	0.9515	0.2195	-0.2039
V4	0.8113	0.5478	0.0788
V5	0.8850	0.3174	0.1027
V6	0.9092	0.2798	0.1383
V7	0.7195	0.5221	0.0262
V8	0.3051	0.8280	-0.0602
V9	0.1311	0.9400	0.1001
V10	0.4569	0.4962	0.4756
V11	-0.0663	0.0200	0.6315
V12	0.2013	0.0024	0.7150

Matriva e peshave me rrotullim Promax

	Grupi 1	Grupi 2	Grupi 3
V1	0.6960	0.0099	0.3310
V2	1.1046	-0.3859	0.1470
V3	1.0954	-0.0591	-0.3128
V4	0.7237	0.3687	-0.0162
V5	0.8993	0.0673	0.0143
V6	0.9344	0.0126	0.0519
V7	0.6381	0.3721	-0.0627
V8	0.0416	0.8843	-0.1332
V9	-0.2516	1.0711	0.0471
V10	0.2227	0.4125	0.4400
V11	-0.2459	0.0192	0.6766
V12	0.0591	-0.0983	0.7425

	Group1	Group2	Group3
SSloadings	6.2682	1.844	1.5218
Proportion Var	0.5224	0.1537	0.1268
Cumulative Var	0.5224	0.676	**0.8028**

	Grupi1	Grupi2	Grupi3
SSloadings	5.5362	2.6745	1.4233
Proportion Var	0.4613	0.2229	0.1186
Cumulative Var	0.4613	0.6842	**0.8028**

	Grupi1	Grupi2	Grupi3
SS loadings	5.6958	2.5411	1.4588
Proportion Var	0.4746	0.2118	0.1216
Cumulative Var	0.4746	0.6864	**0.808**
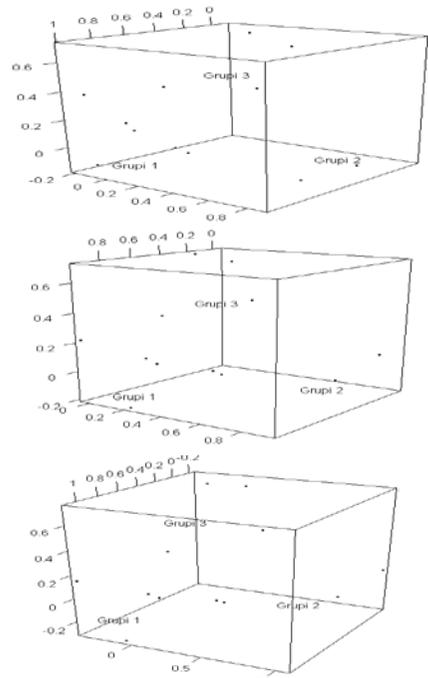
Fig. 2 R The graph of weights before and after rotationvarimaxandpromax.
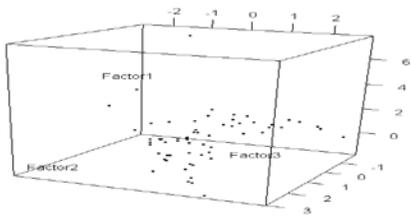
Fig. 3 R Graph of scores

Fig. 4R Graph of weight.