MATHEMATICAL MODELING OF A DOUBLE PASS UNDERFLOOR HEATING SYSTEM

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Abstract

Toward a green building construction, the underfloor heating by solar energy in multi-apartments buildings was considered. A system of double pass underfloor heating with evacuated tube solar collector was modeled. The used energy is a renewable energy that does not affect the environment. The system modeled consist of an evacuated tube solar collector for heating the water, the underfloor heating system consist of two lined up pipes to help in the case of low inlet water temperatures, the control unit used to manage the heating system and save energy in case of excess solar radiation. The governing equations for the system based on energy conservation were written and simulated iteratively. It has been found that at low temperatures the required mass flow rate of hot water increases dramatically, as a result of insufficient heat to cover the heating load of the space, this system allow the double pass heat transfer mode to be switched on to reduce the flow velocity and allow the workability of the system to be valid at a lower temperatures. Also in the case of low heating loads, the amount of hot water required to cover the heat requirement of the space will be very low. The reduction in the heating load will allow the system to send a part of the hot fluid to the storage tank to store the hot water and achieve a lower temperature water flow. The simulation of such model could give a clear idea about the workability of the underfloor heating system, the required solar collector area, and manage the energy within the system.

A more detail analyses of the DPUFHS is the topic of a current study. The underfloor is considered with multi-pass arrangement, and the control unit will simulate transient conditions of the system. A more realistic system will be considered by accounting for the floor heat losses and the thermal storage in the walls and floor.

Keywords: Battery, Solar Collector, Evacuated Tubes, Underfloor Heating, PLC, Double Pass Pipes, Green Buildings, Energy saving

1. Introduction

The energy is essential for developing countries as it improves the life quality. Factories require energy to work, vehicles require energy to move, and bulbs require energy to illuminate and so on. Energy becomes rare and expensive, the energy bill start causing a big headache for the developing non petroleum countries. Scientists are doing their best to reduce the energy consumption, and try to depend on the renewable energy.

Most of the energy in the developing countries is consumed in the residential buildings. (Beithou, 2012), as a result of the increase in population the people tend to live in multi-apartment buildings, where energy usage is concentrated and the possibility of using passive techniques to reduce the energy consumption is almost impossible. Architects and engineers who are working with the sustainable buildings designs still aim to have the zero heating building, which is possible if the engineering regulations are followed strictly and the renewable energy is implemented correctly. The most famous renewable energy is the solar energy, many scientists worked on calculating the solar intensity on different surfaces under different circumstances.(Islam et. al., 2009, Al-Salaymeh, 2006), where the global daily solar radiation on horizontal surfaces for Amman city is modeled, short formulas were suggested for the monthly mean solar radiation and the duration of the sun shine per day. As solar energy is found to be high in most of the Arab countries (see table 1), it is always proposed to use the solar energy to reduce the hot water and space heating requirements in the residential buildings (A. Yadav, V.K. Bajpai 2011).

Table 1. Monthly mean global radiation (in $Mj/m^2/day$) for cold season months in some Arab countries (Islam et. al., 2009).

State	October	November	December
Amman	17.28	13.68	9.72
Bahrain	17.64	14.04	10.8
Baghdad	15.84	11.88	9.72
Casablanca	13.68	9.72	8.64
State	January	February	March
Amman	9.72	13.32	18
Bahrain	12.96	17.28	17.28
Baghdad	10.8	13.68	17.28
Casablanca	9.72	11.88	16.2

Arora (2011) studied the evacuated tube solar collectors, and analyzed its thermal performance. Evacuated tube collectors are characterized by a high thermal efficiency, and are used for different applications starting from water heating to space heating (M. Belhamel, 2007). Kalogirou (2004) discussed different air and water systems for steam generation, desalination, thermal power, solar furnaces, and other applications that all get use of the available solar energy.

Recently the new development in the evacuated tube solar collectors, and the wide control capability made the use of evacuated tubes for underfloor heating (UFH) feasible. T. Q. Hoang studied the temperature and heat losses characteristics of underfloor electrical resistance and storage system (T. Q. Hoang, 1985). In most of the studies that handled the evacuated tubes for underfloor heating the solar radiation was not enough to be used for UFH purposes. In this study it is intended to reduce the energy consumption in the multi-apartment buildings which are becoming widespread in most of the developing countries and use a double pass underfloor system to over come the extra heat required and increase the efficiency of the evacuated tube collectors.

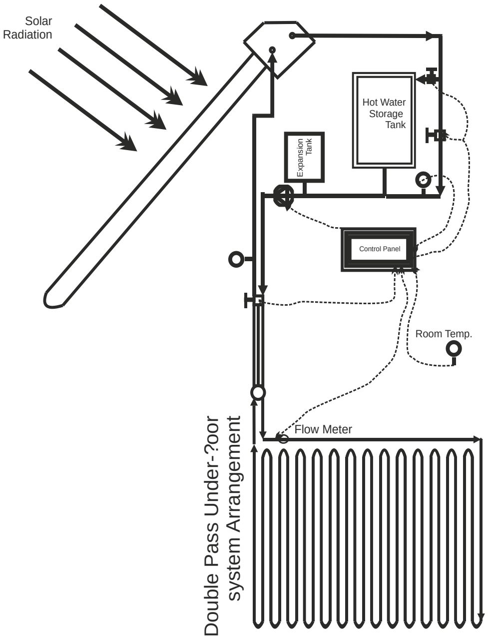

Figure 1. Schematic diagram of the proposed double pass underfloor heating system.

2. SYSTEM WORKING PRINCIPLE

The double pass underfloor heating system (DPUFHS) is a system that allows the variation of the hot water mass flow rate and the underfloor heat transfer area for a quite control of heat transfer to the building see Fig.1

Figure.1 shows a schematic diagram of the DPUFHS with evacuated tubes solar collector as the heating source of the system. The DPUFHS consists of:

- Underfloor Heating Pipes, consisting of two lines of the heating pipes lined up side to side, the first piping line covers the high temperature low heating load case, where the second piping line is used when the temperature of the inlet hot water is not sufficient to cover the heating load of the building.

- Variable Speed Pump, this pump circulates the required amount of water in the UFH pipes to keep the room temperature constant, the pump speed is determined upon the inlet water and exit water thermostat readings from the control unit.
- The Expansion Tank, which allows the fluid to safely expand by compressing the air in the chamber. The size of the expansion tank needed depends on the total volume of fluid and fluid pressures and temperatures.,
- Sensors, a set of thermocouples and flow meters are used to precisely control the heat transfer to the building and reduce the energy losses.
- Storage Tank, if the water temperature is higher than required the controller allow a part of the hot water to flow to the storage tank, this hot water is stored for later use.
- Evacuated tube collector (see Fig. 2).

Evacuated tubes: collector is used to collect the solar energy from the sun. The aim of the tubes is to rise the water temperature in the manifold; tubes are 4 cm internal diameter and 1.5 m long.

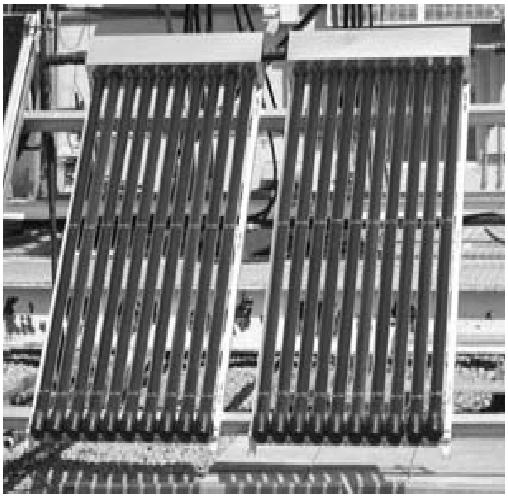

Figure 2: A picture showing evacuated tuve collectors used for underfloor heating.

- Solenoid Valves, a number of solenoid valves 2-way 3-way valves are used to control the flow direction of the working fluid.
- Control Unit, an intelligent programmable logic control system is used to control the management operations of this system, improving its performance,

3. Mathematical Modeling.

As it is mentioned above, the DPUFHS consists of three main sections: evacuated tube collector, house with UFH system and the control unit. The conservation of energy is the base used to develop the mathematical model for the DPUFHS.

3.1 Evacuated Tubes Solar collector

For the cold season which starts from November to April according to Ref. 7. the monthly mean daily values of the global solar radiation for Amman listed in table 1. can be calculated from Ref. 9.

$$G = 3603.7 - 1728.1 * M + 1059.5 * M^2$$
$$- 140.87 * M^3 + 5.3345 * M^4$$
$$\left[kWh / m^2 / day\right] \qquad (1)$$

Taking January as example G=2799.5645 kWh/m^2/day~10.07Mj /m^2/day which is almost the same. This averaged value may be taken as input for the simulation of this model.

The evacuated tube solar collector has the characteristics given in table 2.

Table 2. Characteristics of the Evacuated Tube Solar collector.

Number of tubes	16
Outer diameter	47 mm
Inner diameter	37 mm
Length of the tube	1500 mm
Manifold dimensions	200X200X800
Glass Transitivity τ	0.92

The solar energy transmitted through the glass (Arora, 2011),

$$E_{trans} = \tau * I \qquad (2)$$

Where I is the incident radiation, from the experimental data taken at the Renewable Energy Research Center (RERC) at Applied Science Private University (ASPU), Table 3.

Table 3. Test data taken for the used evacuated tube solar collector on 20 December 2012.

Time	Incident Radiation W/m^2	Heat received =m*C*(T_f-T_i)	Energy Incident $E_{trans} = \tau * I * A * t$	Losses %
11 am	801	80.4	112.027	28.23
12	868	96.69	121.4	20.3
13	960.6	80.58	134.35	40.4
14	960	64.46	134.26	48.01

Thus from the conservation of energy the heat gained by water in the first manifold is

$$E_{w-in} = \eta * \tau * I \tag{3}$$

Soteris A. Kalogirou (2004) noted in his research that the efficiency for the evacuated tube is almost constant as the losses from the tube it self is small compared to other types of solar collectors, even though the over all efficiency is required in our case. By comparing the experimental results with the theoretical results the efficiency reported by F. Struchmann (Lund Univ. 2008) was very close to the realistic case, the efficiency can be written as,

$$\eta = 0.8 - 8.3 * \left[\frac{(T_{in} - T_{amb})}{I} \right] \tag{4}$$

Where T_{in} is the water inlet temperature, and T_{amb} is the ambient temperature. This equation shows that as the inlet temperature approaches the ambient temperature the solar collector reaches its maximum efficiency. For the second manifold the temperature at the end of each evacuated tube will be determined using,

$$E_{in} * A_{ET} = \dot{m} * C * (T_i - T_{i-1}) \tag{5}$$

A_{ET}: Area of a single evacuated tube.

$\dot{m}$: The water mass flow rate.

T_i=the water temperature at the end of the ith evacuated tube.

`It should be noted that the second manifold is used to increase the water temperature to a high temperature in a short time.

The heat gained by water could be found by using the Rayliegh and Nusselt numbers as mentioned by Ref. 8, this method is used for its valuable reduction in calculation time and its satisfactory results as been shown above.

3.2 UFH System

With the knowledge of water inlet temperature to the room, the underfloor heating calculations is performed, assuming the room to be 4X8 m with only one side external, and the other sides are internal (unheated regions), the floor and the ceiling are both insulated as the sitting room is apart of a multi-apartment building using under floor heating system. The outside design conditions are 2^oC, 8^oC and 10^oC.

The heating load for different conditions were calculated using the Carrier E 20-II block load program for early morning load~ 1300 W, Noon Load~726W, after Noon Load ~1016W, all taken with a safety factor 1.1.

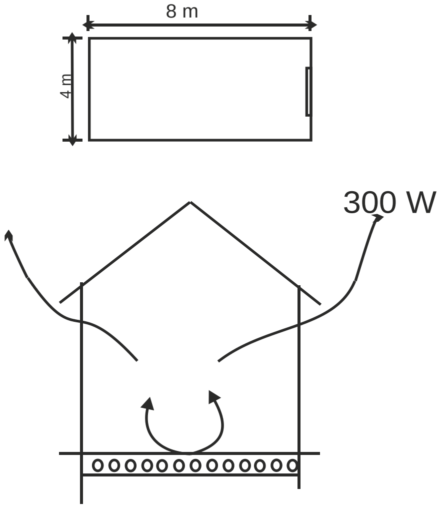

Figure 3. Schematic diagram of the considered house, front and top views.

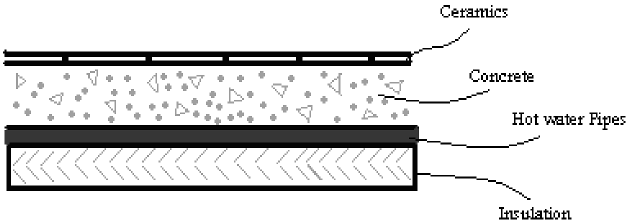

Figure 4. Installation of UFH pipes in the floor structure.

The floor construction is given in table 4.

Table 4. Floor construction dimensions and properties.

Segment	Thickness mm	Thermal conductivity W/m K
Pipe	1 mm	0.35
Concrete	50 mm	1.75
Ceramic	10 mm	1.4

To calculate the inside tube overall heat transfer coefficient *h*, Reynolds number should be calculated,

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{6}$$

The flow comes to be turbulent in the range of the flow, thus the friction factor is

$$f = \left(0.79 * \ln(\mathrm{Re}) - 1.64\right)^{-2} \tag{7}$$

Upon calculating Reynolds number and the friction factor Nusselt number can be found from Ref. 8.

$$Nu = \frac{hD}{k} = \frac{\left(\frac{f}{8}\right)\left(\mathrm{Re} - 1000\right)\mathrm{Pr}}{1 + 12.7\left(\frac{f}{8}\right)^{0.5}\left(\mathrm{Pr}^{\frac{2}{3}} - 1\right)} \tag{8}$$

The overall heat transfer coefficient between water temperature T_w and the room air temperature T_a, can be calculated from figure 4, and table 4., to be

U=8.19 W/m^{2}K

The heat transfer from the water to air is

$$d\dot{Q} = U(T_w - T_a)dA \tag{9}$$

The floor surface temperature can be calculated by taking the inside floor air resistance as 0.08 m^{2o}C/W.

The difference in the heat transfer from water to air and from room air to outside determine the change in the room air temperature.

The radiant temperature T_r is calculated from,

$$T_r = \frac{\sum A_i T_i}{A_i} \tag{10}$$

3.3 Control Unit

The control unit is doing the management job in this system, with the Programmable Logic Control (PLC) it is possible to control the flow rate from the pump by controlling its speed. The solenoid valves are also controlled by the PLC to give on off positions to direct the flow in the required direction.

As it is shown in Figure 5. feed back signals are sent to the PLC from the flow meter, inlet and exit water temperatures and room thermostat. The controller will increase the flow rate until the temperature in the room remains constant, where the heat into the room equals the heating load of the room. If in case the supplied temperature is not enough to cover the heating load, the controller allow the water to flow in the second pass to increase the heat transfer area. If the supplied water temperature is too high for heating the room, the controller send part of the water to the storage tank which is fully insulated, so that this backup hot water may be reused if the temperature of the water supplied to the room decreased.

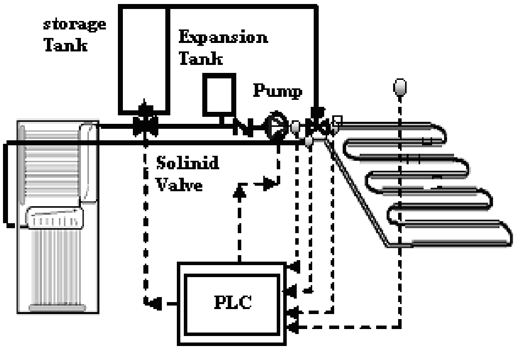

Figure 5. The control and management unit for double pass underfloor heating system.

4. NUMERICAL SOLUTION

For modeling the system given above, the energy equation with the heat transfer correlations were used in a Matlap programme, the sum of the heat transfers should equal to the heating load of the space. If not the space air temperature is recalculated as,

$$T_{an} = T_a + \frac{(Q_{in} - Q_{out})}{\rho_a V_a C_a} \tag{11}$$

The iteration continues until the following convergence criterion is satisfied

$$\left| \frac{T_{an} - T_i}{T_i} \right| \le 0.0001 \tag{12}$$

at all grid points.

5. RESULT AND CONCLUSION

To validate the results of this DPUFHS simulator, the program was run for different conditions, high heating load and low heating load. The results shows the importance of the proposed system in controlling and managing the UFH systems.

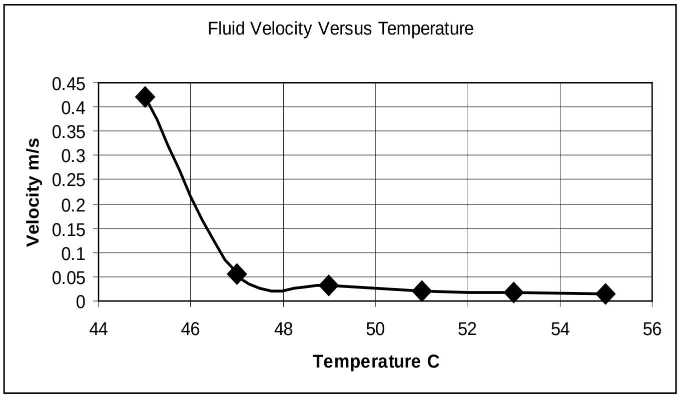

Figure 6. The Required Hot Water Flow Velocity versus Hot Water Temperature.

Figure 6. shows the variation of the hot water flow velocity with the hot water inlet temperature for 1300 W heating load. It is clear that as the hot water inlet temperature increases, the hot water flow velocity decreases, that is because the heat transfer to the heated room increases as a result of the increment in the temperature difference. Figure 6. shows also that there is a point where the flow velocity must be increased dramatically. Actually this point is where the heat transfer become insufficient to cover the heating load of the space, thus the double pass heat transfer mode should be switched on to reduce the flow velocity and allow the workability of the system to be valid at a lower temperatures. If double pass heat transfer mode is switched on the flow velocity will drop to 0.11 m/s. The values of the flow velocities are shown to be small, these values are so because the system is assumed to be fully insulated and the thermal resistance is small, in reality these values will be higher accounting for the room heat losses.

In the case of low heating loads, the amount of hot water required to cover the heat requirement of the space will be very low. The reduction in the heating load will allow the system to send a part of the hot fluid to the storage tank to store the hot water and achieve a lower temperature water flow.

As it is shown above the DPUFHS is modeled for different environmental conditions. The simulation of such model could give a clear idea about the workability of the underfloor heating system, the required solar collector area, and manage the energy within the system. For the more detail analyses of the DPUFHS, the project is going on currently. The underfloor is considered with multi-pass arrangement, and the control unit will simulate transient conditions of the system. A more realistic system will be considered by accounting for the floor heat losses and the thermal storage in the walls and floor.

Acknowledgments

The author is grateful for Applied Science Private University for their financial support

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