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THE P-Δ-DUCTILITY EFFECT : OVERVIEW THE EFFECT OF THE SECOND ORDER IN THE DUCTIL STRUCTURES

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Abstract

Building behavior is a function of a number of factors and their interactions versus external action chosen by us. This discussion rises above the geometry of the structure, its hardness and their connections functions. The main parameters of the connections functions are loads and the calculation stage. With loads we mean static and dynamic ones, while the calculation stage I refer to the phase behavior of the material and structure, the elastic stage or post elastic stage , without deformed or deformed elements. Each stage is realized or lets say , approximate in calculations methods and modifying some standard procedures. In this discussion we will address in a concise way , two key factors in the design of structures, which are second order effects (P-Δ) and the ductility of structures. Both factors have emerged as a necessity of approximating real complex behavior in a calculation methodology. Will hope to better understand the interaction of these two design factors, studying different methodology and approach to the problem by comparing the design requirements. What we stand for discussions , is to increase the accuracy of structural analysis for structures at the design stage of internal forces. It is known that a nonlinear analysis is more accurate than a linear analysis, but on the other hand is an inefficient analysis in terms of time consuming with calculations and computer memory. The solution in our case would require a small memory and fast time.

Keywords : p-δelta , ductility , linearity , nonlinearity , geometric stiffnes

Introduction

In short, we give the two concepts mentioned above, on which the following material will be discussed. Because of their complexity and treatments in different fields, as we can mention: the Stability of structures, Building constructions, Dynamics of structures, Building materials etc. We do not undertake, within this material that could provide treatment to solving every problem, but will concentrate on the design aspect of structures.

P-δ effect

P-Δ effect in structure mainly arises from the direct action of lateral forces and expiry the structure in a state of equilibrium where the deformed structure shape is a more determining factor. This kind of effect is made in the analysis of second order, where the geometry of the elements is come from their warped condition. Gravitational loads (especially in high buildings, they reach a very high order of their values) on their way through the construction elements, where this one are deformed they produce additional forces, which are not taken into account during calculations of structures in undeformed shape. The given gravitational loads are the loads , more precisely defined, in the group of action forces in a structure, we can not say that their change from project values ,will be the determining factor in the effect of P-Δ, but in defining order remains the geometry of the structure. More precise the geometry is defined as the correct second order effects could be considered in structures.

Ductility

Ductility is one of the excellent properties of composite materials reinforced concrete (but not only) and is one of the properties which is paid more attention during design . In short , ductility will be defined as the ability of concrete and steel material (after a correct design) to give an element with excellent featured in post- elastic performance , becoming a better energy absorer . Ductil design refer mainly seismic loads as cyclic loads . The realization of an element managing ductility would bring , by the use of plastic properties of iron materials , mainly to achieve the same element performace . This also applies to a complex structure. When we manage to realize a shape with ductil elements and hierarchy formation of plastic hinges , the same performance of the structure will be realized as a non ductil design. The whole essence of the design is the replacement of elastic forces with elasto - plastic forces to get the same performance , simply by exploiting structure elasto - plastic properties of the elements .

Conclusion

"A structure included in linear elastic stage would have the same performance with a structure included in inelasto-plastic bilinear stage , reducing the force acting but at the same time we consider plastic deformation.

In practical design ductility is estimated by the coefficients of ductility , determined by the specifications of different codes. This topic is not part of this discussion. Despite different methods and empirical formulas, ductility values will be used to reduce the elastic spectra into the elasto-plastic spectra (with other words reducing the value of the aplaied forces). For programs SAP2000, ETABS etc. ductility have been incorporated in the analysis by the response spectra 'RESPONSE SPECTRUM', or static analysis of seismic loading cases 'CASE QUAKE'. Implementing automatically the reduced spectrum.

Deformed geometry of the structure, which is calculated from these programs must be modified to the value of the structure ductility. In order to simplify we say that if we have the reduced spectrum with value q (indirectly have reduced operating forces), deformation from linear analysis , should multiply the q value. Naturally, the above conclusion is valid for systems with one degree of freedom, but for systems with multi degree of freedom, deformations are product of a number of factors and the distribution of the ductility throught the height of structure , so that deformation it would not be a real product of elastic deformation with a single factor ductility. In the elastic phase deformations are proportional to the force, which means that if we reduce deformation forces will be reduced to the same extent and vice versa, but in elasto-plastic phase , it does not apply , even you can not apply the principle of superposition .

To realize a non-linear analysis in elastik phase taking into account the P-Δ effect and the lateral loads, a far more efficient way to reduce computer memory , is a combination of a linear elastic analysis of lateral forces with the rigidity matrix of a nonlinear elastic analysis with P-Δ effect of gravitational forces. Thereby we reduce the use of computer time and taking the same result, also nonlinear analysis P-Δ of gravitational forces can also be used in combination with other analysis. Completion of the above derived by the principle of superposition and independence of action forces. This principle is not valid for the plastic phase because is not maintained the proportion of force-deformation and also can not be determined accurately the impact of various factors action for accepted deformations.

Discussion

"in designing structures of every significance or every type , engineering and science of engineering intends to use more accurate calculations methods with maximum efficiency in time and material. Everything in function of better recognition of real structure deformation and the inner actions. Linear static and dynamic linear analysis offer efficiency in time but no efficiency in precision. Therefore any usage of nonlinear analysis increases accuracy. We ask : how to increase the accuracy by acting with to paralel analysis , one linear and another nonlinear. Specificially how to correct the actions in function of ductility.

Elastic systems.basics principles of mechanics

The principle of superposition

According to the principles of mechanics of deformable bodies in their elastic phase , can apply the principle of superpositing the different actions. Basically, about our task we can say that a structure under the action of lateral and vertical forces, has the same behavior with a structure in which initially applied horizontal forces 'H' and in the deformed shape of this structure we apply the vertical forces 'P' . The figure below presents the deformation shape to the different calculation phases .

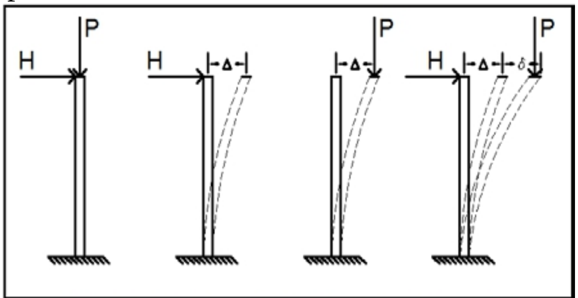


FIGURE 1. SDOF. MECHANIC PRINCIPLES

By applying the finite elements method, this structure can also be addressed vice versa. Engineering programs offers in their application , different accounting options, which of them is P-Δ effect. Referring SAP2000, P-Δ nonlinear analysis of the above shape is equivalent to a linear analysis of lateral forces 'H' structure applied in the above figure, which would have a stiffness equal to the stiffness of the structure that is result from the application of P-Δ effect in nonlinear analysis with vertical loads 'P'. By that , what we take advantage of this procedure is the use of the rigidity of the structure taking into account the effect of the second order, the geometric nonlinearity of the structure, in every analysis that we want , any kind of force which would cause us to lateral displacements of structures.

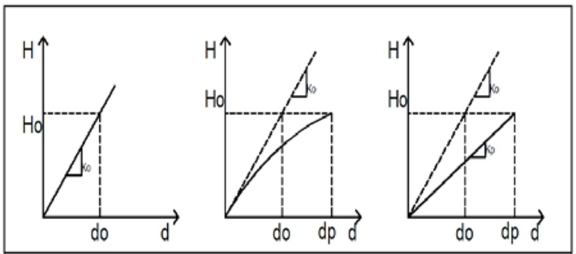


FIGURE 2. SDOF. FORCE-DEFLECTION a)linear no P-D b) non linear with P-D c) linear with P-D

Based on the Finite Element Method for the three cases above, we would have:

a) $\{F\} = [K_0]\,\{U\}$

b) $\{F\} = \{[K_0] + [K_P]\,\}\,\{U\}$

c) $\{F\} = [K_{\mathrm{P}-\Delta}]\,\{U\}$

With the index 'o' we have symbolized, in which case the influence of axial loads on the hardness of the element is zero. With the index 'p' elements have marked rigidity under the influence of axial force. With the index 'P-Δ' the total rigidity of the structure taking into account as lateral loads and normal loads, the axial one. By applying the principle of superpozimit easily get that:

$$[K_0] + [K_P] = [K_{\mathrm{P}-\Delta}]$$

Phase of elastic behavior of the element, and the material does not have any doubt at this point, but the problem that exists is the fact that: Superposition principle is not valid in the plastic phase behavior of the structure or material; most important lateral forces are directly linked to the rigidity of the structure obtained by the analysis. In both design trend above , factors must not be neglected.

Linearization of the problem

Effects of the second orders have always been under study construction shapes, for all kinds of analysis of any structural system type. Analyzes of buildings (mainly buildings and flexible structures against lateral loads) lateral displacement of a mass in a deformed position generates an supplementary moment. Since this additional momentum in the building is equal to the weight of the floor 'P' multiplied by the displacement of the floor 'Δ', this second order behavior (which calculates the balance of deformed shape) was baptized as *'P-Δ ' effect*. Many techniques have been proposed for assessing the behavior of secondary effects. Rutenberg proposes a simple way to include this effect in calculations. Almost most methods consider the problem as a geometric non linearity and propose techniques for solving the successive approximations (iterative method), which essentially remain inefficient in time. Also, these analysis are not suitable for dynamic analysis where we consider axial forces in lateral rigidity of the structure , it increases the natural period of free vibrations. With Rutenbergut proposal, the problem could be linearized and the results can be obtained directly and accurately without the need of successive iterations. This method is based on two assumptions.

- The weight of the structure remains constant during the deformation process due to lateral forces.
- The total displacement of the structure is received in a smaller amount compared with the structural dimensions.

This method does not require repetition of actions (successive approximation technique) while the total axial force at a floor level is equal to the weight of the building above the level of the floor and that does not change through the action of lateral force. Therefore all terms associated with geometric rigidity against lateral forces are not taken into account but only those axial forces under the action of the weight of the structure involved in the drafting of the rigidity of the building. With this method effect 'P-Δ' implemented basic analytical formulation, and is therefore included in both tests, static and dynamic.

$$[K]\{U\} = [F] + [L]\{U\}$$
$$[K]\{U\} - [L]\{U\} = [F]$$
$$([K] - [L])\{U\} = [K]^{*}\{U\} = [F]$$

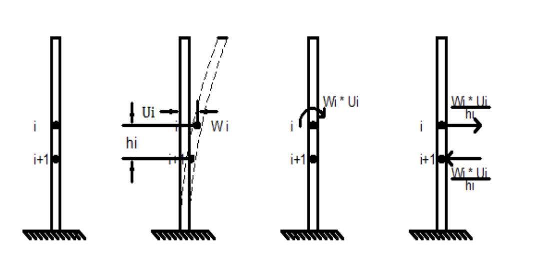


FIGURE 3. MDOF. RUTENBERG MODEL OF P-D

$$\begin{Bmatrix} f_i \\ f_{i+1} \end{Bmatrix} = \frac{w_i}{h_i} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ u_{i+1} \end{Bmatrix}$$

{F}- lateral forces ; [L]- factor matrixes ‘Wi/hi’ of th P-Δ momentums ; [K]* - linear matrix of the system including P-Δ. [K]- linear matrix of the system excluding P-Δ ; {U}-lateral displacements.

The above method of linearization of the problem is in its essence exactly the method we mentioned to entry, for implementation in the application of SAP2000. Nonlinear analysis of the second order effect for temporary usage loads and permanent ones, in buildings takes a constant value, and does not take into account the effect of lateral forces changing the vertical forces (axial forces) in the elements of the structure. While linear analysis of lateral loads with the stiffness matrix, the matrix obtained from nonlinear analysis above, principally linearize the problem of second-order effect.

What mentioned above regarding the effect of axial forces of columns can be illustrated with a very simple example of a portal frame. By portal frame analysis described above assumed as a system with a single degree of freedom , which happens to overthrowing the moment balanced by lateral forces ' H ' , will produce a pair of forces in structure side columns. In one of the columns is a pulling force and as a result the column will be less loaded , while the right column arises wherein compresive forces , the element will be overloaded. The rigel of the frame is taken absolutely rigid only reason to study the case with a single degree of freedom (rotation is not taken into account for its small values of displacement compared with translative) while we do not influence at all in the concept of the problem , because an elastic Rigel will only cause the redistribution of the moments in other elements of the structure.

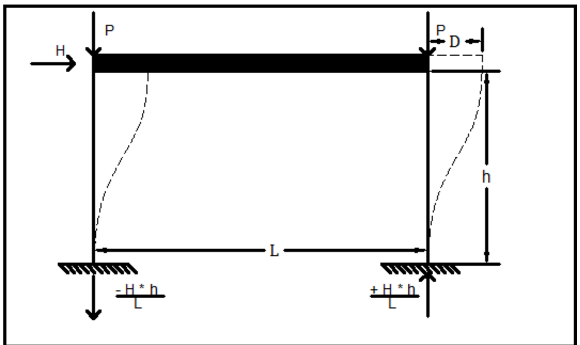


FIGURE 4. PORTAL FRAME

This distribution of the moment to the beam and vice versa will retain the same proportion , whether this moment will arise from the lateral loads or it comes from the P - Δ effect. Any internal force that arises in the structure arises as a backlash of external action, with the same size regardless of what would be the nature of this action.

If at first sight this shape will not boost interest to an engineer to study the effect of the second order, then I'm reminisce previously, that this portal frame is a small element of a multi-floor structure which under the action of earthquake or other lateral actions, would really be under adverse conditions of these effects, both as an localized element or the entire structure. Following expressions enhance us this idea:

$$H_i = k * m * S_a$$

$$V_i = \sum_{top\ story}^{level\,index\ story} H_i$$

For earthquake lateral forces above formulas are valid, and for vertical forces, the own weight of the structure will be able to write:

$$P = \sum_{top\ story}^{level\,index\ story} P_i$$

Given that the law of progression of axial forces (from own and temporary weight of usage) is almost linear, while for lateral forces (seismic action calculated with simple methods) is roughly triangular, so the ratio Vi / Pi will come increasing for the lower floors, favoring severe second effects due to increased progressive lateral displacement.

Bilinearization of the problem

Ongoing treatment of the system with a single degree of freedom and the concept of the formulas given above on the impact of the P-Δ effect in a given structure, the following are some basic concepts on the treatment of the second order effects of which will require us in following discusions. With the concept of bilinearization , we just do enter the first step of non linearization the problem.

Some of the basic mechanical parameters associated with P-Δ effects are illustrated in the figure below for a structure with a single degree of freedom. This consists of a mass, m, with a weight P, associated with a rigid column with a flexible connection to the base. To include viscous dumping we have connected a dumper. Rigidity associated with lateral displacement of the mass, under the action of lateral forces V, is Ko and it does not take into account P-Δ effect. With this model, the overturnig moment M, which act in the elastic connection , is given with the following expression:

$$M = Vh + P\delta$$

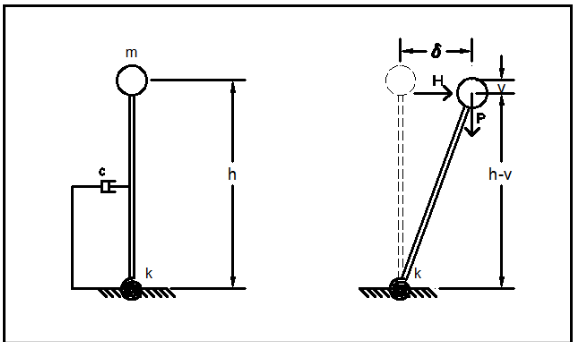


FIGURE 5. SDOF. NON LINEAR MODEL

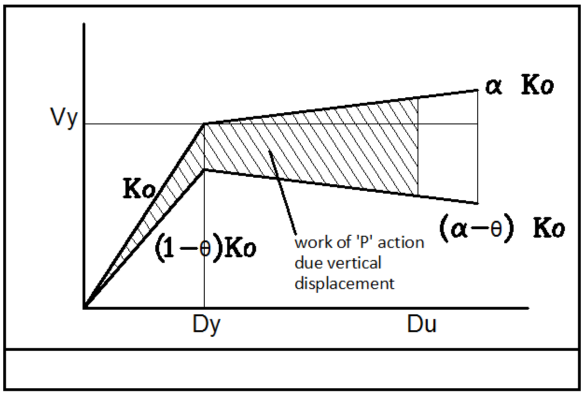
With h is marked the height of the column, and with δ is marked the horizontal displacement of the mass. The second term of the formula expresses , additional moment induced by the P-Δelta effect. A parameter that is used more to evaluate the sensitivity of the structure of P-Δelta effect, is the stability factor, θ. The numerical value of which is defined as the ratio of the induced current overturning moment from the effect of axial load P, with the value corresponding to the moment of the lateral seismic loads. The coefficient is a result of receiving linear elastic behavior of the structure. For a model with a single degree of freedom, the stability factor is calculated :

$$\theta = \frac{P\delta}{Vh} = \frac{P}{K_o h}$$

The influence of P-Δ effect, in a system with a single degree of freedom, with a bilinear histeretic behavior is illustrated in Figure 6, for the case of a monotonous increase of displacement. Omitting P-Δ effect, a horizontal force Vy limit is reached for the yielding , and yielding plastic phase have a small increase of resistance with the rigidity of the structure , αsr Ko. Considering the effect of axial load P, effective rigidity against lateral forces is Ko (1-θ) for elastic deformation stage, and Ko (α-θ) for post-elastic deformation stage. The lined area between the force-displacement graph without P-Δ effect, and the P-Δ effect gives the work done by the axial force for reduction in height.

A consequence of reducing the rigidity effected from the inclusive of P-Δ effect in an analysis, is that the normal period of free oscillation increases from T to T` by the following expression:

$$T^{'} = T\frac{1}{\sqrt{1-\theta}}$$



For the possible values of the coefficient of stability that extends from 0 to 0.2 increase in the period of free vibrations is not very significant in terms of design. However, in view of the study of P-Δ effect has a significant impact. Comparisons are made between the behavior of response spectrum, and as well as land tremblings are invariabel in nature , a small change in period, brings itself to be a difference between the two analysis evaluated.
Reducing rigidity of a structure in inelastic phase, during inclusion of P-Δelta effect means that for every structure in the inelastic behavior will have a tendency to increase the deformation. To offset the effect of P-Δ can be used two estimates:

❖ Increasing the rigidity of the structure. In this way, we manage to reduce the deformation and at the same time the impact of P-Δelta effect. However, it is generally not practical and economical to increase the rigidity of the structure in an amount necessary to eliminate , as taking into account the second order effects. Also the increase of rigidity associated with increased seismic actions, since it would be reduced self period of the structure.

❖ Increasing resistance of the structure. The effect of increasing the resistance brings reducing the 'P-Δ' effect, as a result of the reduction of deformations by reducing plastic deformation. The downside of such a solution reduces plastic deformation in the structure.

The result of P-Δ effect, in a bilinear system can be estimated by the following two factors:

1. The first factor is known as the amplification factor α, which is the ratio of the yielding resistance of the structure including P-Δelta effect with the yielding resistance without P-Δelta effect for the same level of ductility. M' is required yielding resistance for given ductility, taking into account the effect of the second order, while M is yielding resistance for the same ductility without the effect of the second order.

$$M^{'} = \alpha\, M$$

2. The second factor is the amplification factor of P-Δ, β, which takes into account plastic deformation from the ductil behaviour and P-Δ. δ_{max} are the maximum displacements calculated for a given ductility without considering P-Δ action.

$$M^{'} = M + \beta(P\,\delta_{max})$$

Multi story buildings

A number of questions can be raised about the application of the above method for pervious systems with a single degree of freedom, in multistory structures. One of them is : which of the deformed shapes should be used to calculate the P-Δelta effect. In systems with a single degree of freedom , the model was built for a bilinear behavior had to do with a deformed form, where all elastic deformation occurred at the base.

However, as long as there will be multi degree of freedom systems, consequently will have some shapes of deformations, and consequently that the deformed structure behavior will be a function of the warped shape itself. A number of analysis have shown that the deformed shape according to the response spectrum analysis provides a reasonable basis for calculations of P-Δ effect. This envelope with a small deviation to the first mode of the structure. Multiplying the top of the structure deformation of the response spectrum analysis with the ductility factor of the structure, is proven to provide a reasonable estimate of the deformation of this point. But the use of the same scale ductility to the lower levels could lead to huge underestimation of the deformations, compared with a nonlinear dynamic analysis 'Time-History'. Discrepancy increases with ductility factor and basic period of structure.

From the results of the seismic analysis performed by the response spectrum , for determining the displacement of the structure, drifts between floors and demanded ductility , the conclusion drawn exactly what noted above. Let us take an example from the book of A. Chopra, Dynamic of Structures, Theory and Aplications to Earthquake Engineering. If we had:

$$\Delta_{im} = u_i$$
$$\Delta_{iy} = \frac{V_{iy}}{k_i} = y_i$$
$$\mu_i = \frac{u_i}{y_i}$$

Δ_{im} – magnitude of the drifts to the structure , Δ_{iy} – corresponding drift in the early yielding phase , V_{iy} – shear forces corresponding with the early yielding of the elements , k_i strength of i-th floor , μ_i – demanded ductility in i-th floor. By applying the above formulas in a specific example will receive the following graphs.

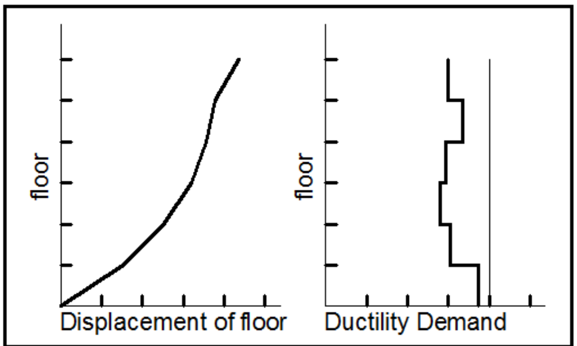


FIGURE 7. DISPLACEMENT / DUCTILITY

The demanded ductility in a floor varies in its height of the structure and the demanded ductility differs from the definition used in the design of spectrum and its calculation from the yielding resistance phase of the structure. The above shape in which the demanded ductility do not exceed the allowed ductility not always happen. The demanded ductility , in most cases depends on the relative stiffness of different floors. To demonstrate this concept, let refer to structures with "soft story", which can be floors with smaller stiffness compared to the upper floors, or floor with yielding to less than the floors above.

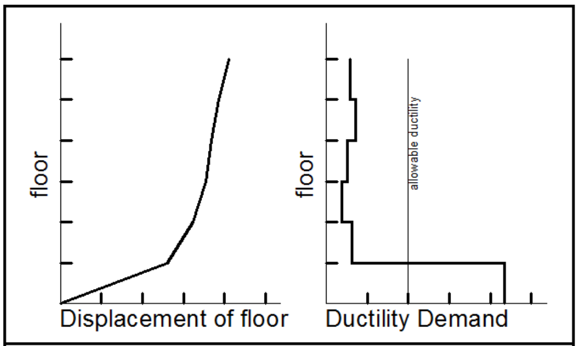


FIGURE 8. "SOFT STORY" BEHAVIOUR

What is important to note from these results is the fact that the structural deformations computed as the product of the displacement obtained by the response spectrum analysis (with the designed spectrum corrected with the allowed ductility) multiplied by allowed ductility does not give us the correct results, for the reason that each floor featuring a certain ductility, and in many cases it's demanded ductility passes the allowed ductility (which means that the structure will colapse because the internal forces do not correspond with the designed forces and exceeds the required performance). Deformation of such involved shape would not be available for further calculations of the second order effect.

Another fact that matters mentioning at this point is the concept of "soft story". As mentioned there are two cases: 1 - floor stiffness lower than the other floors above, 2 – the resistance is lower than the other floors above. Only referring figure 6. we reach the conclusion that axial forces simultaneously reduce stiffness and the yielding resistance of the element and consequently the total floor. Respectively values: $K_{(P)} = K_o(1\text{-}\theta)$; $V_{y(P)} = V_y\ (\Delta_y\ \theta)$. It seems that the presence of normal force, if there is considerable value then we would have a substantial effect, that a floor of a building is called a "soft story". According bilinear model, evaluation factor, is the coefficient of stability θ, which depends directly on the size of the vertical force and lateral displacement, both these two in a symbiotic relationship with seismic analysis

Modeling

Application sap2000. P-δ effect in different multi story structures. P-δelta nonlinear analysis. "quake" linear analysis.

Below we give the geometric rigidity matrix element which takes into account the influence of axial forces in one element.

$$K_{P-D} = K_0 + K_\sigma$$

$$K_0 = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad K_\sigma = -\frac{P}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{5}{6} & -\frac{L}{10} & 0 & -\frac{5}{6} & -\frac{L}{10} \\ 0 & -\frac{L}{10} & \frac{2L^2}{15} & 0 & \frac{L}{10} & \frac{L^2}{30} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{5}{6} & \frac{L}{10} & 0 & \frac{5}{6} & \frac{L}{10} \\ 0 & -\frac{L}{10} & \frac{L^2}{30} & 0 & \frac{L}{10} & \frac{2L^2}{15} \end{bmatrix}$$

Rigidity matrices presented above can be applied in FEM (finite element method). Below we give an application in SAP2000 for comparing methods outlined above for seismic calculations considering the effect of the second order.

Geometric property

The model is a ten story planar frame with three spaces. The floor height is 3 m while spaces are 6 m. It is assumed that the same space includes other direction (for the distribution of load in the beam). Beams are accepted size 40x50 cm and are not updated in different variants. The columns vary for each variant of experiment in a close diapozon of their rigidity characteristic , however, differences in behavior are evident.

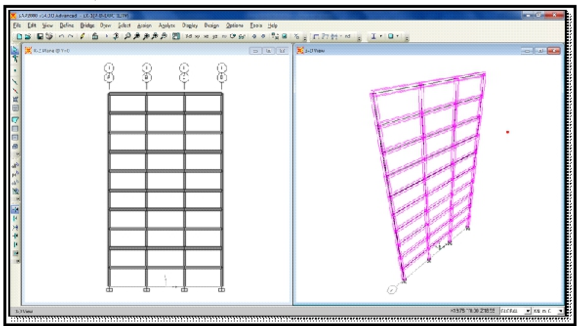


FIGURE 9. EXAMPLE MODEL

Loads

i. Permanent loads and temporary loads, included in the first group of building load from which we take the vertical loads and the mass of the structure, which will serve in seismic calculation.

ii. Earthquake load type 'quake' will serve as a generator of earthquake lateral forces which will used in the analysis. These values are accepted :

g^n=7 kN/m^2 ; p^n=7 kN/ m^2

The conversion factor of vertical load to mass is , 1 for dead loads and for live loads 0.8. The earthquake loads is received from design spectra according to EC-8 2004 with peak ground acceleration equal to ag = 0.22 and ductility factors q = 1 and q = 4. P-Δ effect can be taken with a direct nonlinear analysis taking into account P-Δ effect from vertical load , and the use of the stiffness in the earthquake analysis. Results from eartquake loadings , type "quake" will be obtained from two vertical load combinations with the horizontal , in two cases. Case A with a ductility factor q = 1 (we supose to take real deformation of the structure) and Case B coresponding ductility factor, q = 4 (we supose to take real internal forces in elements).

columns	50X50	55X55	60X60	40X70	65X65	40X70	40X80	70X70
M'	197.21	224.07	255.13	259.39	289.78	259.39	305.22	328.90
M''	186.72	216.25	249.28	254.22	286.18	254.22	302.71	327.13
ΔM %	5.32	3.49	2.29	1.99	1.24	1.99	0.82	0.53

The example above shows a comparison of internal forces arising in structure between the results obtained from the calculation of P-Δ effect, as production of axial force in the column with the displacement obtained by linear static seismic analysis "Quake" without changing the rigidity $M' = M + P * \Delta_{q=1}$, and linear static seismic analysis "Quake" with reduced stiffness structure for P-Δ effect, M". For hand calculations the P-Δ effect (M '), deformations are not accepted as deformations of the structure multiplied by ductility of the structure, since the paragraph just above it this acceptance was not correctly accurate, but we get it by linear analysis without reducing the design spectrum. The principle used is the principle of equalization of spectral displacements , where spectral displacements in elasto-plastic phase are equal to the elastic phase shifts spektarle.

Application sap2000. Nonlinear analysis type quake. Linear analysis type quake

From the results of the above example we take as substantiated: - for more accurate estimates need that column stiffness must be greater than the stiffness of the beams, thereby redistributing the second order moment M = PxΔ, within beam will be smaller, while within columns will be grater.

The second example application will be based on the example above with some minor modifications:

- Geometric properties have been saved , beam 40x50 cm , column 70x70 cm.
- Structural loads are increased, to increase vertical forces and their effect on structure.
- Applied analysis :

a. PDELTA K=1 (scale factor = 1) type Nonlinear Static

b. PDELTA K=4 (scale factor = 4)
type Nonlinear Static

c. QUAKE
type Linear Static

d. QUAKE PD
type Nonlinear Static

e. QUAKE K-PD1
type Linear Static
start from PDELTA K=1

f. QUAKE K-PD4
type Linear Static
start from PDELTA K=4

The purpose of this analysis is to compare between their results and derive conclusions on the analysis which would have more value for design engineering.

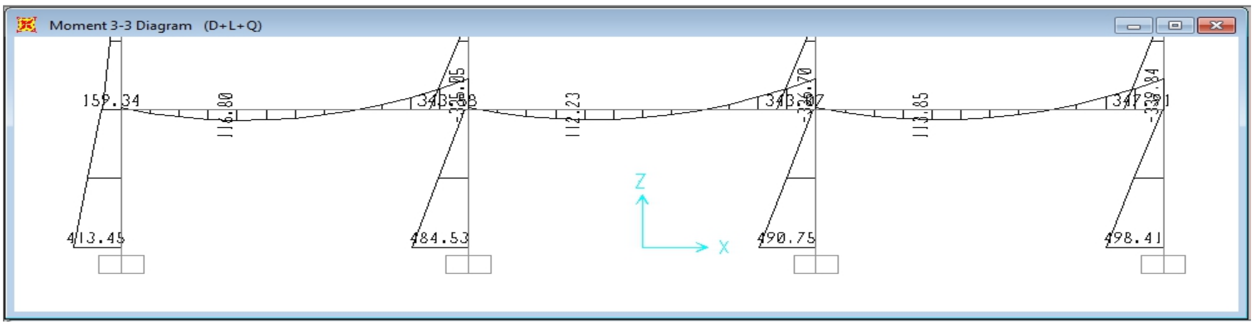


FIGURE 10. DIAGRAM OF MOMENTS FROM QUAKE ANALYSIS : 413.45 ; 484.53 ; 490.75 ; 498.41

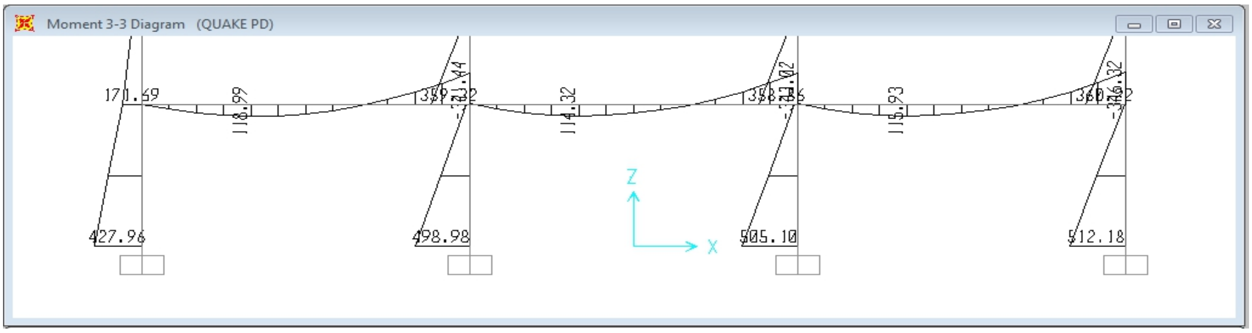


FIGURE 11. DIAGRAM OF MOMENTS FROM QUAKE NONLINEAR PD ANALYSIS : 427.96 ; 498.98 ; 505.10 ; 512.18

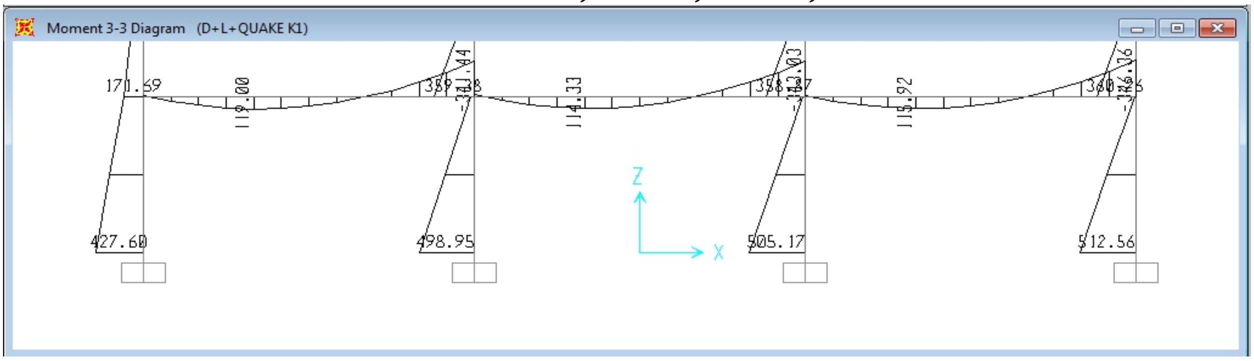


FIGURE 12. DIAGRAM OF MOMENTS FROM QUAKE LINEAR WITH REDUCED STIFFNESS ANALYSIS : 427.60 ; 498.95 ; 505.17 ; 512.56

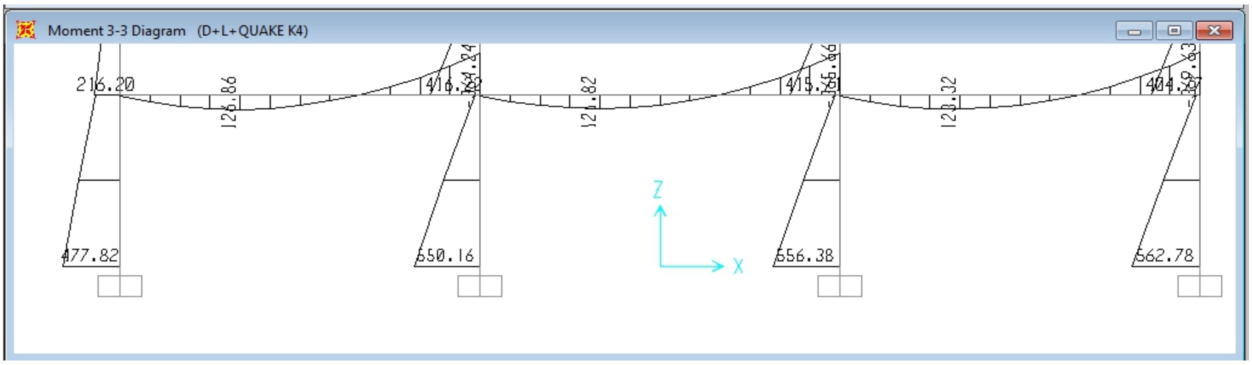


FIGURE 13. DIAGRAM OF MOMENTS FROM QUAKE LINEAR WITH REDUCED STIFFNESS (WITH A DUCTILITY FACTOR) ANALYSIS: 477.82 ; 550.16 ; 556.38 ; 562.78

First, we add that because the analysis type 'QUAKE PD', takes into account the values of internal forces and vertical forces, to compare the same conditions of the same structure , we have combined analysis described above with vertical loads 'Dead' and 'LIVE'. By the above application it is clear that for engineering design aspects , the design of a structure is very convenient to develop an analysis of linear lateral forces, in a structure with reduced stiffness. Although the recent trend is to design according to the design displacement control , and such analyzes use nonlinear methods, we must admit that such analysis require previously a preliminary modeling. If in the theoretical part we sought to avoid interative method and voluminous analysis , what we would think to make an designe by repeating the nonlinear analysis in different proposed structures.

Results of the analysis above are also valid for application analysis, 'RESPONSE SPECTRUM', which are more accurate for seismic calculations.

Conclusion

Sections of this paper, in which we modestly tried to make a submission to the P-Δ problem, but not only. The essence of this study is the relationship P-Δ-DCTILITY. Ductility is the most important parameter on seismic design, but without wanting to get out of topics we do not want to treat the dynamical problems, we are conscious that Seismicity remains fundamental premise of sustainable and safe design. The scope of an engineer remains, analyzing a framework for recognition of its real behavior and controling destruction mechanism of the structure.

The question that arises is: how can design the reality, without knowing the behavior of the structure itself ?

In analyzing the effect of the second order , the design is based on the recognition of the structure deformation shape, which shape is complicated by post-elastic phase of the structure. After determining the deformation shape and application of external forces , we take internal forces, which we use to design stiffness, resistance and factic ductility of the structure.

In trying to increase the accuracy of the analysis of application of the above examples, we can apply the following methodology:

- Apply vertical forces in the building by construction loads and of utilization.
- Apply further a nonlinear analysis of vertical forces , by amplifying their action with the ductility we want to accomplish structure accuracy.
- Apply lateral loads, by using nonlinear analysis stiffness.
- With internal forces obtained from the analysis, we design each elements.

With non-linear analysis of vertical forces achieve these benefits:

- Save time in analyzing the structures, by using linear analysis .
- Realize a more accurate estimate of the structural rigidity.
- Internal forces respond to a reality, and create more safety for elements.
- Consideration of P-Δ effect reduces the possibility of creating soft storys in buildings from non estimated actions and stiffness.

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