THE ELECTROMAGNETIC FIELD OF THE POINT HERTZIAN RADIATOR IN UNIAXIAL MAGNETIC CRYSTAL

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Abstract

Earlier received new analytical solution of the equation of Maxwell are expressed by several wave potentials, one of them is constructed by means of an integrated sine and a cosine. In this work asymptotic solution from the analytical solution of the equations of Maxwell for electromagnetic field strength of the emitter radiator of Hertz in uniaxial magnetic crystal are received.

Keywords: Asymptotic solution, radiator of Hertz, crystal

Introduction

At the solution of many problems of radio engineering, telecommunication and other scientific and technical branches it is much important to know structure of an electromagnetic field in the considered part of space. These issues include, for example, the development of radiating systems, ensuring electromagnetic compatibility of radio devices and telecommunications systems, and etc. For calculation the electromagnetic field in each case it is required to solve the corresponding electrodynamic problem.

In work [1] the method of calculation of an electromagnetic field of an electric dipole in a homogeneous medium with uniaxial anisotropy of electric and magnetic properties with use of vector potential was considered.

Now at a high level the mathematical apparatus of the theory of the generalized functions is developed. The class of the generalized functions contains the juddering and singular generalized functions, in particular, and a class of smooth functions. In work [2] for the solution of such task the method of the generalized functions is used.

In the presented work electromagnetic field of a point radiator of Hertz in a far zone for the uniaxial anisotropic medium, a tensor of magnetic permeability which has a diagonal appearance with various diagonal components is investigated. Elements $\hat{\mu}$ are selected in the form of a diagonal matrix: $\mu_{zz} = \mu_1, \mu_{xx} = \mu_{yy} = \mu$, which corresponds to the magnetic uniaxial crystal. The axis of a crystal is directed along an axis z.

Statement of the problem

In work [2] exact solutions of the equations of Maxwell for electric and magnetic dipoles in uniaxial magnetic crystals are received. Exact solutions are received as the sum of two independent solutions. The first composed one of decisions is defined by the wave potential Ψ_0 and a vector of the moment of the electric dipole $\boldsymbol{p}_0^{\varepsilon}$, operating along an axis of a magnetic crystal:

$$\begin{cases} \boldsymbol{E}_1 = \dfrac{-1}{\varepsilon\varepsilon_0}(\nabla\nabla + k_0^2)(\Psi_0 \boldsymbol{p}_0^{\varepsilon}), \\ \boldsymbol{H}_1 = i\omega \cdot [\nabla, (\Psi_0 \boldsymbol{p}_0^{\varepsilon})], \end{cases}$$

(1)

where ∇ – operator of Hamilton,

$$\nabla = \frac{\partial}{\partial x}\mathbf{e}_x + \frac{\partial}{\partial y}\mathbf{e}_y + \frac{\partial}{\partial z}\mathbf{e}_z, \qquad \Psi_0 = \frac{-\exp(ik_0 r)}{4\pi \cdot r},$$

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad k_0^2 = \omega^2 \varepsilon_0 \varepsilon \mu \mu_0, \qquad \boldsymbol{p}_0^{\varepsilon} = (0,0, p_z^{\varepsilon}).$$

The second composed analytical solutions are defined by wave potentials Ψ_0, Ψ_1^m, Ψ_2^m and a vector of the moment of the electric dipole $\boldsymbol{p}_\perp^{\varepsilon}$, which located perpendicularly to an axis of a crystal of z:

$$\begin{cases} \boldsymbol{E}_2 = -\dfrac{1}{\varepsilon_0\varepsilon}(k_n^{\mu 2}(\boldsymbol{p}_\perp^{\varepsilon}\Psi_1^m) - k_0^2\nabla_\perp(\nabla, (\boldsymbol{p}_\perp^{\varepsilon}\Psi_2^m)) + \nabla(\nabla, (\boldsymbol{p}_\perp^{\varepsilon}\Psi_0))), \\ \boldsymbol{H}_2 = -i\omega(\nabla_\perp \dfrac{\partial}{\partial z}\mathbf{e}_z[\nabla, (\boldsymbol{p}_\perp^{\varepsilon}\Psi_2^m)] + \mathbf{e}_z(\mathbf{e}_z[\nabla, (\boldsymbol{p}_\perp^{\varepsilon}, (\Psi_0 - \Psi_1^m))]) - [\nabla, (\boldsymbol{p}_\perp^{\varepsilon}\Psi_0)]), \end{cases} \tag{2}$$

here the wave potential Ψ_1^m and the radius vector r' for the magnetic anisotropic medium are determined by:

$$\Psi_1^m = -\sqrt{\frac{\mu}{\mu_1}}\frac{\exp(ik_n^{\mu} r')}{4\pi \cdot r'}, \qquad r' = \sqrt{x^2 + y^2 + \frac{\mu}{\mu_1}z^2}. \tag{3}$$

The wave number k_n^{μ} for uniaxial magnetic crystal:

$$k_n^{\mu} = k_0\sqrt{\frac{\mu_1}{\mu}}, \qquad \nabla_\perp = \nabla - \frac{\partial}{\partial z}\mathbf{e}_z, \qquad \boldsymbol{p}_\perp^{\varepsilon} = (p_x^{\varepsilon}, p_y^{\varepsilon}, 0).$$

The function Ψ_2^m is defined by the integral sine and cosine:

$$\begin{aligned} \Psi_2^m = \frac{-i}{8\pi k_0}\big[&\exp(ik_0 z)(\mathrm{Ci}(k_0(r - z)) + i \cdot \mathrm{si}(k_0(r - z))) + \exp(-ik_0 z)(\mathrm{Ci}(k_0(r + z)) + \\ &+ i \cdot \mathrm{si}(k_0(r + z))) - \exp(ik_0 z)(\mathrm{Ci}(k_n^{\mu} r' - k_0 z) + i \cdot \mathrm{si}(k_n^{\mu} r' - k_0 z)) - \\ &- \exp(-ik_0 z)(\mathrm{Ci}(k_n^{\mu} r' + k_0 z) + i \cdot \mathrm{si}(k_n^{\mu} r' + k_0 z))\big]. \end{aligned} \tag{4}$$

The integrated sine and cosine:

$$\mathrm{Ci}(z) = \gamma + \ln(z) + \int_0^z \frac{\cos z - 1}{t} dt, \qquad \mathrm{si}(z) = \int_0^z \frac{\sin z}{t} dt - \frac{\pi}{2}, \tag{5}$$

$\gamma = 0.5772$ – Euler number.

In this part it is required to find the asymptotic solution of the exact solution of vectors of intensity of an electromagnetic field for an electric dipole in the far zone, which is located parallel to and perpendicular to an axis of uniaxial magnetic crystal.

Solution of the problem

To find the strength of electromagnetic field for the point electric dipole operating parallel to an axis of a magnetic crystal in a far zone, it is necessary to consider communication of the Cartesian system of coordinates with spherical system of coordinates [3]:

$$x = r\cos\varphi \cdot \sin\theta, \quad y = r \cdot \sin\theta \cdot \sin\varphi, \quad z = r \cdot \cos\theta . \tag{6}$$

Now I differentiate (1) and using (6), it is possible to obtain an asymptotic solution of intensity of electromagnetic field for a point electric dipole, acting parallel to the magnetic axis of the crystal in the far zone:

$$E_r = E_\varphi = H_r = H_\theta = 0, \tag{7}$$

$$E_\theta = \frac{k_0^2 p_z^\varepsilon \cos\theta cos\varphi \cdot \exp(i(k_0 r - \omega \cdot t))}{4\pi\varepsilon\varepsilon_0 r}, \tag{8}$$

$$H_\varphi = -\frac{\omega \cdot k_0 p_z^\varepsilon \exp(i(k_0 r - \omega \cdot t))\cos\theta\cos\varphi}{4\pi \cdot r}. \tag{9}$$

The received solutions (7)-(9) correspond with solutions for a dipole in the isotropic medium.

Further asymptotic solutions of intensity of electromagnetic field for a point electric dipole in the far zone, which is located perpendicularly to an axis to uniaxial magnetic crystal (2) will be received.

Using (6) in (3), it is easy to convert the radius vector r' and the wave potential Ψ_1^m for uniaxial magnetic crystal of the Cartesian coordinate system in the spherical coordinate system:

$$r' = r\sqrt{\frac{\mu}{\mu_1}\cos^2\theta + \sin^2\theta}, \tag{10}$$

$$\Psi_1^m = -\sqrt{\frac{\mu}{\mu_1}}\,\frac{\exp(i(k_n^\mu r\sqrt{\frac{\mu}{\mu_1}\cos^2\theta+\sin^2\theta}-\omega\cdot t))}{4\pi r\sqrt{\frac{\mu}{\mu_1}\cos^2\theta+\sin^2\theta}}. \tag{11}$$

The asymptotic solution of the integral sine and cosine (5) at $z \to \infty$:

$$\mathrm{si(z)} \approx -\frac{1}{\mathrm{z}}\left(\cos(\mathrm{z})+\frac{\sin(\mathrm{z})}{\mathrm{z}}\right), \qquad \mathrm{Ci(z)} \approx \frac{1}{\mathrm{z}}\left(\sin(\mathrm{z})-\frac{\cos(\mathrm{z})}{\mathrm{z}}\right), \tag{12}$$

By means of (12) it is possible to find the asymptotic solution of function Ψ_2^m (4) in a far zone (Fig.2):

$$\Psi_2^m \approx \frac{\Psi_0 r^2 - \Psi_1^m \cdot r'^2}{k_0^2(r^2 - z^2)}. \tag{13}$$

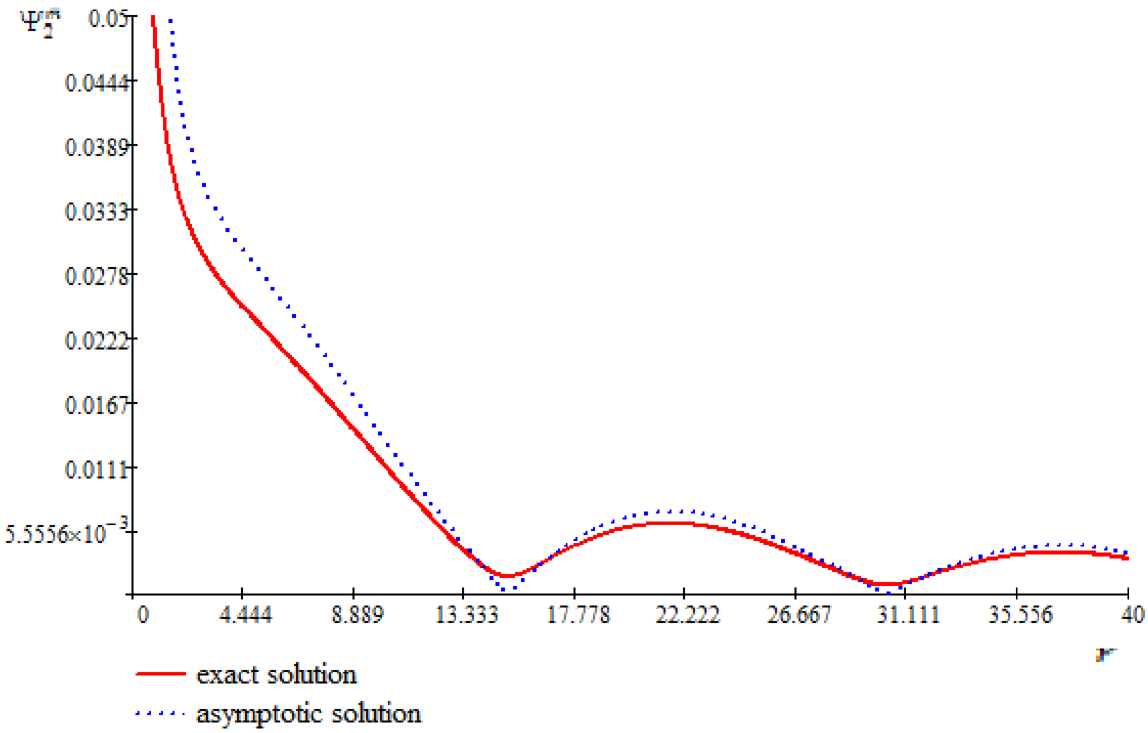


Fig.2. Comparative graph of the exact and the asymptotic solution function Ψ_2^m

The components of the vectors of intensity of electromagnetic field (2) are:

$$\begin{cases} E_{2x} = -\dfrac{1}{\varepsilon_0\varepsilon}(k_{\mathrm{n}}^{\mu 2}\dfrac{\partial^2}{\partial x^2}(p_x^{\varepsilon}\Psi_1^m) + \dfrac{\partial^2}{\partial x^2}(p_x^{\varepsilon}\Psi_0) - k_0^2\dfrac{\partial^2}{\partial x^2}(p_x^{\varepsilon}\Psi_2^m)), \\ E_{2y} = -\dfrac{1}{\varepsilon_0\varepsilon}(\dfrac{\partial^2}{\partial x\partial y}(p_x^{\varepsilon}\Psi_0) - k_0^2\dfrac{\partial^2}{\partial x\partial y}(p_x^{\varepsilon}\Psi_2^m)), \\ E_{2z} = -\dfrac{1}{\varepsilon_0\varepsilon}\dfrac{\partial^2}{\partial x\partial \mathrm{z}}(p_x^{\varepsilon}\Psi_0), \end{cases} \tag{14}$$

$$\begin{cases} H_{2x} = i\omega\dfrac{\partial^3(p_x^{\varepsilon}\Psi_2^m)}{\partial x\partial y\partial \mathrm{z}}, \\ H_{2y} = i\omega(\dfrac{\partial^3(p_x^{\varepsilon}\Psi_2^m)}{\partial y^2\partial \mathrm{z}} + \dfrac{\partial}{\partial \mathrm{z}}(p_x^{\varepsilon}\Psi_0)), \\ H_{2z} = -i\omega\dfrac{\partial(p_x^{\varepsilon}\Psi_1^m)}{\partial y}, \end{cases} \tag{15}$$

As a result of differentiation (14), (15) in the far zone will be the following:

$$\begin{cases} E_{2x} = -\dfrac{k_0^2 p_x^{\varepsilon} x^2}{4\pi\varepsilon_0\varepsilon}(\dfrac{\exp(i(k_0 r - \omega\cdot t))}{r^3} - \dfrac{1}{x^2+y^2}(\dfrac{\exp(i(k_0 r - \omega\cdot t))}{r} - \\ \qquad -\sqrt{\dfrac{\mu_1}{\mu}}\dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{r'}) - \sqrt{\dfrac{\mu_1}{\mu}}\dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{r'}), \\ E_{2y} = -\dfrac{p_x^{\varepsilon} xy}{4\pi\varepsilon_0\varepsilon}(\dfrac{k_0^2\exp(i(k_0 r - \omega\cdot t))}{r^3} - \dfrac{1}{x^2+y^2}(\dfrac{\exp(i(k_0 r - \omega\cdot t))}{r} - \\ \qquad -\sqrt{\dfrac{\mu_1}{\mu}}\dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{r'})), \\ E_{2z} = -\dfrac{p_x^{\varepsilon} x\mathrm{z}}{\varepsilon_0\varepsilon}\dfrac{k_0^2\exp(i(k_0 r - \omega\cdot t))}{r^3}, \end{cases} \tag{16}$$

$$\begin{cases} H_{2x} = -\dfrac{\omega k_0 xy\mathrm{z}}{x^2+y^2}(\dfrac{\exp(i(k_0 r - \omega\cdot t))}{4\pi r^2} - \dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{4\pi r'^2}), \\ H_{2y} = -\dfrac{\omega k_0 y^2\mathrm{z}}{x^2+y^2}(\dfrac{\exp(i(k_0 r - \omega\cdot t))}{4\pi r^2} - \dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{4\pi r'^2}) + \omega k_0 \mathrm{z}\dfrac{\exp(i(k_0 r - \omega\cdot t))}{4\pi r^2}, \\ H_{2z} = -\omega\cdot k_0 y\dfrac{\exp(i(k_n^{\mu} r' - \omega\cdot t))}{4\pi r'^2}. \end{cases} \tag{17}$$

Further to proceed to the solution of the problem, we consider the components of an arbitrary vector moving from a Cartesian coordinate system in the spherical coordinate system [3]:

$$\begin{cases} A_r = A_x \sin\theta cos\varphi + A_y \sin\theta sin\varphi + A_z \cos\theta, \\ A_\theta = A_x \cos\theta cos\varphi + A_y \cos\theta sin\varphi - A_z \sin\theta, \\ A_\varphi = -A_x \sin\varphi + A_y \cos\varphi. \end{cases} \tag{18}$$

Now taking into account (6) and using (18) it is possible to find of intensity of electromagnetic field for the point electric dipole, which is operating perpendicular to the axis of magnetic crystal in the far zone:

$$E_r = H_r = 0, \tag{19}$$

$$E_\theta = \frac{k_0^2 p_x^\varepsilon \cos\theta cos\varphi \cdot \exp(i(k_0 r - \omega \cdot t))}{4\pi\varepsilon\varepsilon_0 r}, \tag{20}$$

$$E_\varphi = -\sqrt{\frac{\mu}{\mu_1}} \frac{k_n^{\mu 2} p_x \sin\varphi \cdot \exp(i(k_n^\mu r' - \omega \cdot t))}{4\pi\varepsilon\varepsilon_0 r'},$$

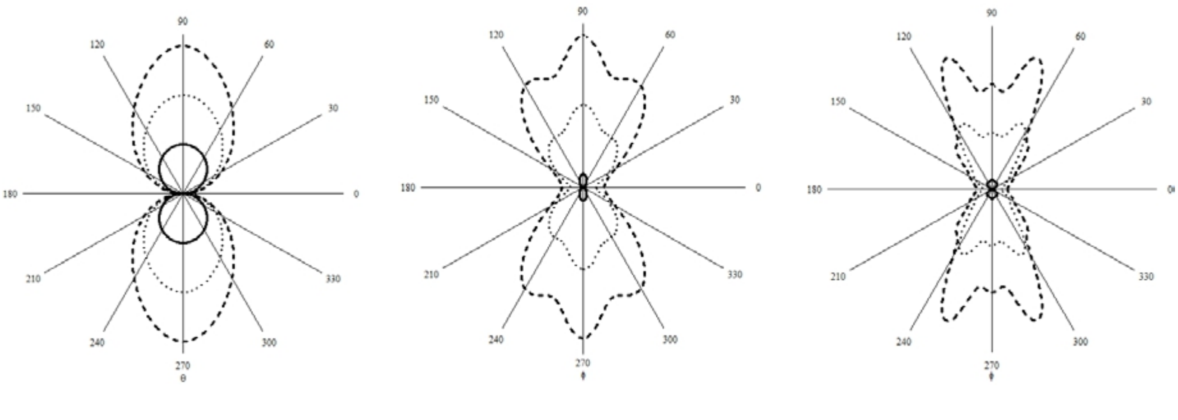
(21)

$$H_\theta = \frac{\omega \cdot k_0 p_x \exp(i(k_n^\mu r' - \omega \cdot t)) \sin\varphi}{4\pi r((\mu/\mu_1)\cos^2\theta + \sin^2\theta)},$$

(22)

$$H_\varphi = \frac{\omega \cdot k_0 p_x \exp(i(k_0 r - \omega \cdot t)) \cos\theta \cdot \cos\varphi}{4\pi \cdot r}. \tag{23}$$

The received asymptotic solutions (19)-(23) when using limit transition $\mu_1 \to \mu$ correspond with solutions of other authors for the isotropic medium. In fig.2 sections of the directional pattern of a dipole of the Hertz, which is located perpendicular to an axis are shown to uniaxial magnetic crystal in a far zone.



а) $r = 1$, $\phi = \pi / 2$ б) $r = 4$, $\theta = \pi / 2$ в) $r = 8$, $\theta = \pi / 2$

— $\mu_1 = 2$, ···· $\mu_1 = 5$, -- $\mu_1 = 8$

Fig.2. Sections of the directional pattern of a dipole of the Hertz, which is located perpendicular to an axis to a magnetic crystal

Conclusion

Thus, in this work asymptotic decisions from the new analytical solution of Maxwell's equations for intensity of an electromagnetic field of a dot radiator of Hertz in a uniaxial magnetic crystal are received.

From drawing it is visible that directional pattern forms at various values of radius of the sphere changes, i.e. the more sphere radius, it is more than local petals. And also the magnetic anisotropy is higher the intensity of radiation of electromagnetic waves on an axis of a crystal of z is maximum. In the direction of the emitter (axis x) doesn't occur the radiation of electromagnetic waves.

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