EPIDEMIC CORRUPTION: A BIO-ECONOMIC HOMOLOGY

Salem Hathroubi, Assistant Professor
Al-Imam Muhammad Ibn Saud Islamic University, College of Economics and Administrative Sciences, Department of Economics, Riyadh, Kingdom of Saudi Arabia
Hedi Trabelsi, Professor
University of Tunis, Faculty of Economics and Management, Department of Quantitative Methods

Abstract

This paper aims to study corruption as an epidemic phenomenon using the epidemic diffusion model of Kermack and Mc-Kendrick (1927). We seek to determine the dynamics of corruption and its impact on the composition of the population at a given time. We determine a threshold epidemiological corruption based on the approximation of the honest population.

Keywords: Corruption, epidemiology, SIR model

Introduction

Corruption is a complex social, political, and economic phenomenon that affects all countries at different degrees. Corruption is usually qualified as unethical because it may have unexpectedly harmful effects on economic development, breeding negative work ethics and leading to waste and misallocation of resources, as well as adverse distribution of income and wealth. It is often associated with violence, crime, and in extreme cases may result in popular revolts like the recent Tunisian revolution of freedom and dignity. In recent years a huge of research has been interested to the phenomenon. Researchers have studied many topics. On a macro level, some works have studied the economic, political, and cultural determinants of corruption using econometric approaches (Svensson 2005, Iwasaki and Suzuki 2012). On a micro level some researchers have used the framework of utility maximization under uncertainty (Rose-Ackerman, 1975; Beenstock, 1979) and others have applied game theory models (Macrae

1982, Era Balba 2000) in order to answer to the question: what is the basis for decisions of reasonable men to be corrupt?

In contrast to the large literature on political, social, and economical aspects of corruption there is only a small number of attempts to model the dynamics of corruption in a mathematically quantified way. One of the major problems of corruption is its diffusion what is usually called epidemic corruption.

Becker, Egger and Seidel (2008) deliver an empirical model of perceived corruption which allows for *epidemic effects* or, in other words, the *infection* of other countries with (non-) corrupt behavior or the perception thereof. Using a cross-section of 123 economies for the year 2000, they illustrate that corruption in one country spills over to adjacent economies. This finding implies that institutional changes reducing corruption in one country lead to smaller but qualitatively similar effects in neighboring countries. Blanchard, Krueger and Martin (2005) present a model for the spread of corruption on complex networks in the spirit of epidemiology. Their model describes aspects of the evolution of corruption in a virtual population and incorporates some basic universal features of corruption.

Considering corruption as a nonstandard epidemic process relies on the plausible assumption that corruption rarely emerges out of nothing but is usually related to some already corrupt environment which may ”infect” susceptibles. The aim of this paper is to study the dynamics of corruption in a closed population using the epidemiological SIR (Susceptibles, Infected, Removed) model of Kendrick and MacKermack (1927) and its extensions. In the first section we state the hypothesis and we formulate the dynamics of corrupt population. In the second section we solve the model and we state the threshold epidemiological theorem for corruption. Section three, approximates the theoretical evolution of the honest population which is necessary to approximate the dynamics of the corrupt population. The last section concludes the paper.

Dynamics of corrupt population

Corruption is a substructure of human social interaction. Common sense associates corruption mainly with a deviation from fair play interaction in the development of social relations which depends on the cultural context of a given society. This vague description of corruption is in the spirit of sociology and psychology and differs from the more narrow corruption concepts usually considered in economics or political sciences. There, corruption is mainly seen as a misuse of public power to gain profit in a more or less illegal way. In any case, corruption has many different faces in its concrete appearance and no single model approach will be able to describe the whole picture in an adequate way. But this does not at all imply that mathematical models are useless in this situation. They can provide a substantial improvement in our understanding of corruption as long as one clearly defines the aim and limitations of the taken approach. For the model approach developed in this article we will use the notion of corruption in the more general, first sense. In doing so, let's assume that at a given time a few corrupt individuals appear in a given population taken to be constant and by contamination they can corrupt other people. In a given country, the total population (P_t) can be decomposed, at every moment, in three categories:

$$P_t = SC_t + C_t + H_t$$

SC_t : Is the fraction of the population susceptible to become corrupted (corruptible). It is non-corrupt persons who may become corrupted if they are in contact with individuals initially corrupted (C_t).

H_t **:** is the fraction of the population immunized against corruption. These are honest individuals who do not change their attitude regardless of the situation ($\overline{H}$). These are also people who were corrupt and were caught and pay for their mistakes by imprisonment or if they are removed from work to retirement ($\frac{dH_t}{dt}$) then $H_t = \overline{H} + \frac{dH_t}{dt}$.

The model is also based on the following assumptions:

H_1: we suppose that a corruptible transmits corruption immediately when she/he becomes corrupted.

H_2: corrupt-corruptible exchange is proportional to the statistical average of contacts. The coefficient of proportionality is by definition, the rate of corruptibility (i).

H_3: corrupt-honest relationship is proportional to the corrupt population. The coefficient of proportionality defines the "good repute rate or honesty rate" (equivalent to the cure rate in an infected population usually known as the removal rate) (g). We suppose also that recovered corrupt are immune against corruption.

So the spread of corruption is similar to an infectious disease and can be represented by a dynamic system isomorphic to that of Kermack and McKendrick (1927).

$$(I)\begin{cases} \frac{dSC_t}{dt} = -i\,SC_t.C_t & (1) \\ \frac{dC_t}{dt} = iSC_t.C_t - gC_t & (2) \\ \frac{dH_t}{dt} = gC_t & (3) \end{cases}$$

With initial conditions for t=0: $C_0>0$, $SC_0>0$ and $H_0= \overline{H}$. As in the K-MK model i represents the rate of infection. To investigate the corruption spread under this model, we only need to consider nonnegative solutions for C, *SC* and H. The corruption stops when $C_t = 0$ for the first time. Before we justify the approximation of the general epidemic process, let us look at system (I) more closely. Suppose $C_0 > 0$, $SC_0 > 0$, and $H_0 = \overline{H}$ (this guarantees that $H_t > 0$ for all $t > 0$). The key question is, given parameters i and g and the initial number of corrupt and corruptible, whether the corruption spreads and how it develops with time. Notice that SC_t decreases with *t*, and

$$\frac{dC_t}{dt} = C_t(iSC_t - g) \begin{cases} \leq C_t(SC_0 - g) \leq 0 \text{ for all } t>0 \text{ if } SC_0 \leq g/i \\ > 0 \text{ for some } t>0 \quad \text{if } SC_o > g/i \end{cases}$$

In the case when $SC_0 \leq g/i$, the number of corrupt monotonically decreases with time, that is no epidemic can occur. By an epidemic we mean the situation when $C_t > C_0$ for some $t > 0$. On the other hand, when $SC_0 \geq g/i$, $dCt/dt > 0$ at least initially, and the number of corrupt increases in the beginning. We observe the *threshold phenomena* at $SC_0 = g/i$, or qualitatively different corruption spread above and below this level.

Resolution

The system (I) is block recursive, since the first two equations are independent of H_t. We can then solve the system incorporating only the equations (1) and (2) and then find an approximation to equation (3). Given the system (II):

$$(II) \begin{cases} \frac{dSC_t}{dt} = -i\, SC_t.C_t & (1) \\ \frac{dC_t}{dt} = iSC_t.C_t - gC_t & (2) \end{cases}$$

we can easily see that the orbits of (II) belong to the curve solution of the following equation:

$$\frac{dC_t}{dSC_t} = -1 + \frac{g}{i}\frac{1}{SC_t}$$

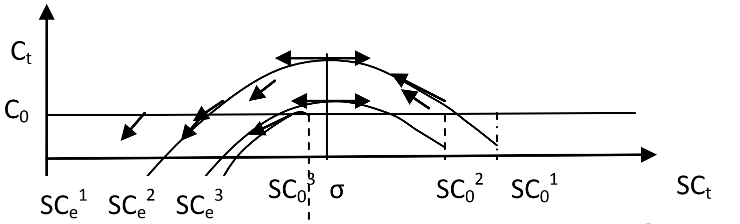
Let $\sigma = \frac{g}{i}$ and integrating, we obtain:

$$C_t = -SC_t + \sigma log SC_t + k$$

This equation describes the law of evolution of the corrupt fraction of the population.

Given the monotonic decrease of the susceptible, for a given C_0, the phase portrait of the system (II) is as follows:

Fig 1: Phase portrait of system (II).



at, the solutions of (II) run th ctories in the opposite direction of clockwise tending toward equilibrium point noted SCe (equilibrium susceptible population).

In this movement, the comparison between the initial number of corruptible and the critical value σ is important in the spread of corruption. SC_0^3 is a corruptible population less than σ, then there is no risk of spreading corruption. The impact of corruption measured by $(SC_0^3 - SC_e^3)$ is reduced. By contrast, for SC_0^1, the number of people likely to be corrupted is much greater than σ, then an epidemic corruption occurs.

It therefore appears that the initial size of the corruptible population (SC_0) and the level of the threshold term (σ) determine the evolution of corruption. Epidemic corruption will occur $\Leftrightarrow SC_0 > \sigma$.

Theorem

If C_0 is low and $SC_0 = \sigma + \varepsilon$ with $\varepsilon > 0$ and $\varepsilon/\sigma \ll 1$, then the impact of corruption is 2ε. In this case, the equilibrium corruptible population is derived from the initial corruptible population SC_0 in a σ symmetry center.

In reality corrupt are unobservable because corruption, by its very nature, is illicit and secretive. We can observe only corrupt individuals who are caught and imprisoned or removed from their work and eventually become honest. So to determine the dynamics of the epidemic corruption, we need a theoretical approximation of the evolution of the honest population (dHt/dt).

Approximation of honest population

Remember that we assumed that the corrupt-honest relationship is proportional to the corrupt population.

$$\frac{dH_t}{dt} = gC_t = g(P_t - H_t - SC_t)$$

So

$$\frac{dSC_t}{dH_t} = \frac{-iSC_t.C_t}{g\ C_t} = -\frac{1}{\sigma}SC_t$$

Then

$$SC_t = SC_0 e^{-\frac{1}{\sigma}H_t}$$

We obtain

$$\frac{dH_t}{dt} = g\left(P_t - H_t - SC_0 e^{-\frac{1}{\sigma}H_t}\right)$$

If we are in conditions close to those of the preceding theorem, then $\frac{H_t}{\sigma}$ is small and $e^{-\frac{1}{\sigma}H_t}$ can be approximated by:

$$e^{-\frac{1}{\sigma}H_t} = 1 - \frac{H_t}{\sigma} + \frac{1}{2}\left(\frac{H_t}{\sigma}\right)^2 + \varepsilon\left(\frac{H_t}{\sigma}\right)$$

$$\lim_{\frac{H_t}{\sigma}\to 0} \varepsilon\left(\frac{H_t}{\sigma}\right) = 0$$

The dynamics of the honest population can be written:

(E) : $$\frac{dH_t}{dt} = g\left(P_t - H_t - SC_0\left(1 - \frac{H_t}{\sigma} + \frac{1}{2}\left(\frac{H_t}{\sigma}\right)^2 + \varepsilon\left(\frac{H_t}{\sigma}\right)\right)\right)$$

Omitting the function $\varepsilon\left(\frac{H_t}{\sigma}\right)$, (E) is a differential equation with separable variables. To integrate, we must break it down into simple elements. The general solution of (E) can be written:

$$H_t = \frac{\sigma^2}{SC_0}\left[\left(\frac{SC_0}{\sigma} - 1\right) + \alpha\, th\left(\frac{1}{2}\alpha g t - \varphi\right)\right]$$

th is the hyperbolic tangent function

$$\alpha = \left[\left(\frac{SC_0}{\sigma} - 1\right)^2 + \frac{2SC_0}{\sigma^2}(P_t - SC_0)\right]^{\frac{1}{2}}$$

$$\varphi = th^{-1}\left[\frac{1}{\sigma}\left(\frac{SC_0}{\sigma} - 1\right)\right]$$

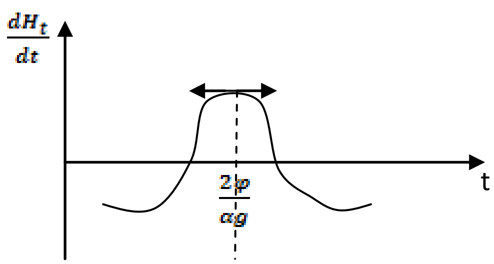
Deriving H_t we obtain a theoretical approximation of the dynamic of the honest fraction.

$$\frac{dH_t}{dt} = \frac{g\alpha^2\sigma^2}{2SC_0}(ch^{-1})^2\left(\frac{1}{2}g\alpha t - \varphi\right)$$

ch is the hyperbolic cosine function.

The symmetrical graph around $t = \frac{2\varphi}{\alpha g}$ defines "the epidemic curve of corruption."

Fig 2: Epidemic curve of corruption



This analysis shows that the number of resistant to corruption (original honest population and the removed population from corrupt to honest) increases monotonically. The speed at which expands the number of those who withdraw from the process of corruption increases and then decreases. The turnaround time corresponds to the inflection point of the evolution of the honest population. While the number of corrupt increases and then decreases. The reversal occurs when the number of corruptible falls at the epidemic threshold.

Conclusion

In this paper we have studied corruption as an epidemic process. The framework of the SIR model of Kermack and McKendrick has allowed the study of corruption dynamics and the statement of a threshold epidemiological theorem for corruption. An epidemic corruption occurs when the number of corruptible surpasses the threshold term. Approximation of unobservable corrupt population is based on the approximation of the honest fraction. Empirical application of the developed theoretical model needs to have an idea on the number of corrupt individuals in a given country at a given time. This kind of data is unavailable because most estimates of corruption are based on surveys of perception (Transparency International, World Bank). In recent years a new type of corruption measure has been developed, the survey-based measure of bribes in which individuals are asked whether any government official in their countries has asked them or expected them to pay a bribe for his services during the previous year. As this type of data becomes more available we will be able to estimate such model.

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