EXAMINATION OF THE EIGENVALUES LORENZ CHAOTIC SYSTEM

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Abstract

In this paper, Classical Lorenz Equations are simulated using MATLAB/Simulink, by getting the graphical outputs performances of the equations are studied. The butterfly effect was seen in the expected output graphs. Classical Lorenz Equations were linearization and then Jacobian matrix was obtained by MATLAB software in embedded system and eigenvalues calculated. The numereous eigenvalues poles were seen an the rigth axis side of the complex plane. These poles are causes the unstable behaviors.

Keywords: Lorenz Equation, Chaos, Stabilite, MATLAB, Simulink

Introduction

Chaos is a well-known nonlinear phenomenon, and it is the seemingly random behavior of a deterministic system that is characterized by sensitive dependence on initial conditions. Besides, chaos is occasionally preferable, but usually intrinsically unpredictable as it can restrict the operating range of many physical devices and reduce performance (Hsiao, 2013).

Chaos theory was appeared as a scientific discipline at 1960’s with Edward Lorenz, who has studied to model meteorological systems with Lorenz Equations in computer environment by using the data he collected to estimate the weather forecast (Uyaroğlu, 2009).

Chaos, a nonlinear phenomenon, has been intensively investigated in medical science, biology, and physics (Ma, 2011).

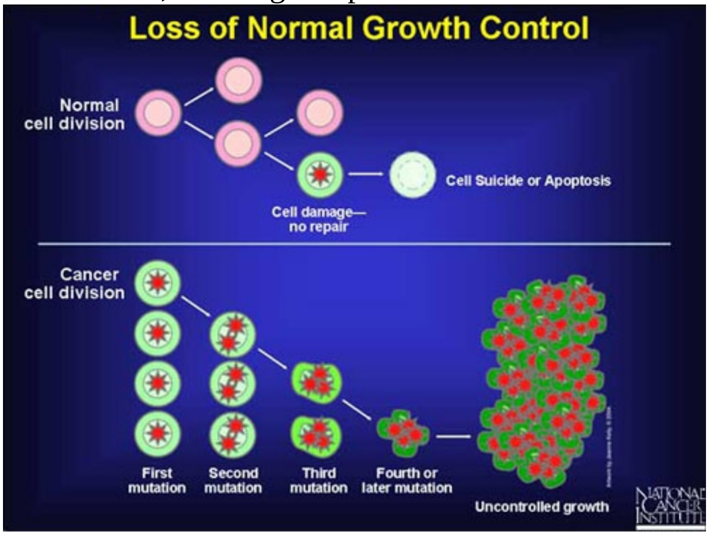
In these days, Chaos theory has successful applications in the fields such as secure communication, automatic control systems, laser physics and financial modeling (Uyaroğlu, 2009).

The problem of controlling chaos has recently attracted the attention of many investigators. Though chaos is a beneficial feature in some chemical or heat and mass transport processes (Ottino, 1989).

Many researchers have endeavored to find new ways to suppress and control chaos more efciently. So far, many researchers have presented diferent types of controllers and control methods, e.g., linear state error feedback control (Uyaroğlu, 2013) (Wu, 2007), sinusoidal state error feedback control (Chai, 2007), variable substitution control (Wu, 2006), variable structure control (Yan, 2007), nonlinear feedback control (Chen, 2004), active control (Thang, 2009), and adaptive control (wei, 2007) have been successfully applied to chaotic systems (Uyaroğlu, 2013).

For the study of chaos theory and application of new or existing chaotic attractors reveal the chaotic systems and improve the dynamics of topological structure, besides the current system has an important place in terms of applicability (Uyaroğlu, 2009).

The chaos studies consist of the processes to improve the terminated chaotic behaviors or chaos behaivor of systems to stabilize the structure, systems and methods for more efficient use of areas. For example, the behavior of cancer cells shows a chaotic structure in a shown Picture 1. The disease cancer occurs due to a group of cells exist differentiatly in the body, as a result of excessive and uncontrolled proliferation. Normally, the growth and proliferration of cells is a normal regime. Accordingly, tissues and organs are able to do their duties as normal. However, these cells start to grow and replicate abnormally, this process give rise to the formation of masses called tumors (novartisonkoloji). Here, the chaotic behavior should be terminated and the cell structure, showing cell proliferation must be stabilized.



Picture 1: Cancer Cell Division (Cancer)

In another example, a dishwasher water distribution nozzles enganging movement in the structure behaves as a dynamical chaos ensuring more efficient washing of the dishes has been provided (Chau, 2011).

In this study, taking advantage of the chaotic behavior of Lorenz equations with the help of MATLAB/Simulink simulation studies were obtained. The changes of variables was examined by obtaining the eigenvalues.

Lorenz Systems

The Lorenz systems is described by the following nonlinear differential equations where x, y and z are state variables, and a, b and c are positive constant parameters when the typical parameter values for a Lorenz System are a=10, b=8/3 and c=28;

$$\begin{aligned} \dot{x} &= a \cdot (y - x) \\ \dot{y} &= x \cdot (c - z) - y \\ \dot{z} &= x \cdot y - b \cdot z \end{aligned} \tag{1}$$

The Lorenz systems is described by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ -28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + x \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{2}$$

Using a MATLAB/Simulink model, as shown in Figure 1., the time series of the Lorenz chaotic system are shown in Figure 2, Figure 3 and Figure 4., the xy, xz, yz and xyz phase portraits are showing in Figure 5, Figure 6, Figure 7 and Figure 8 When x_0=0.001, y_0=0.001 and z_0=0.

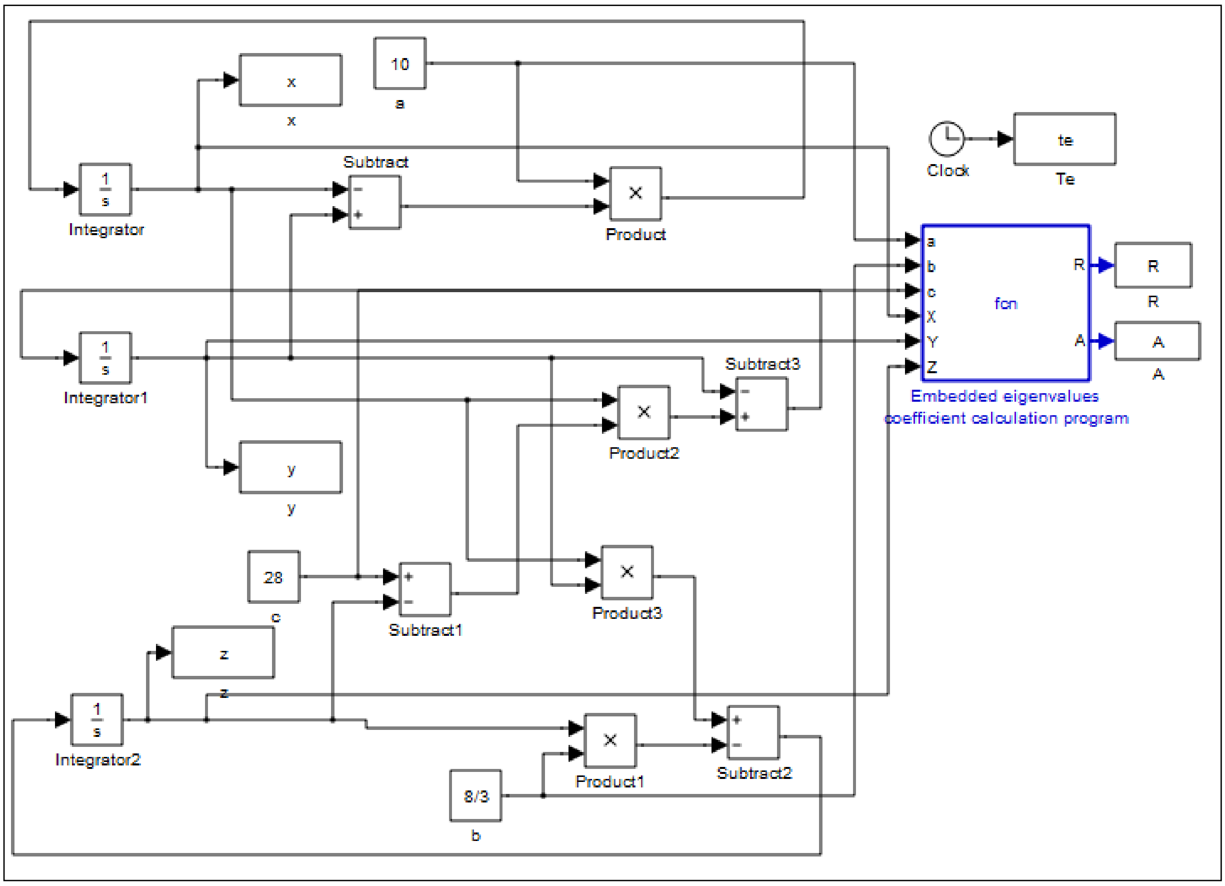


Figure 1: Lorenz MATLAB/SIMULINK chaos model

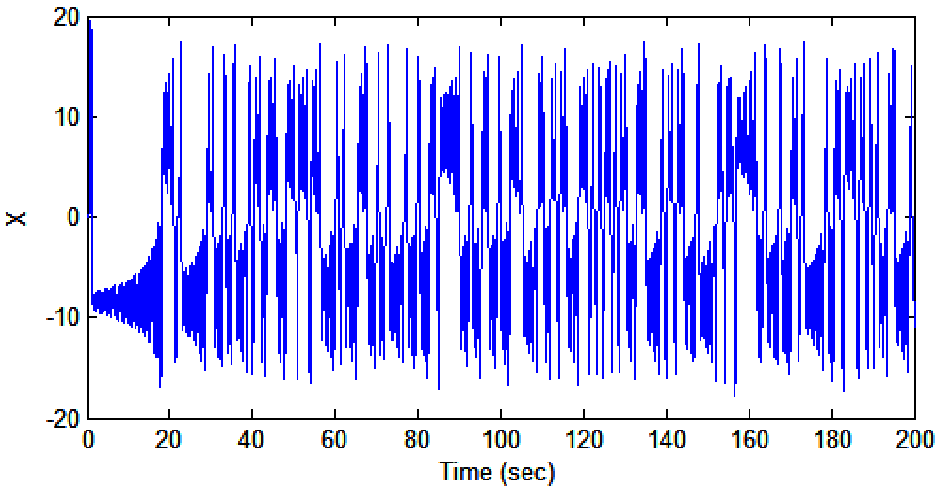


Figure 2: The time series of the Lorenz chaotic system for the x signal

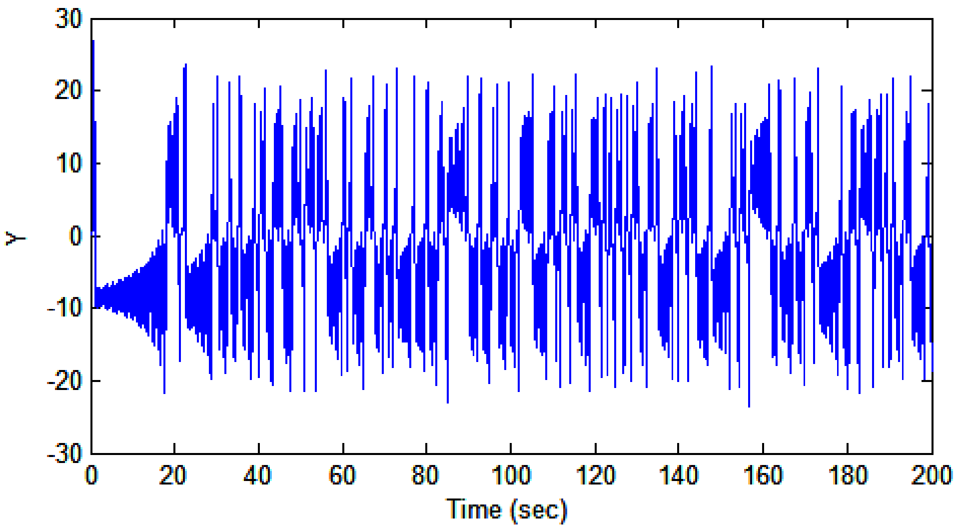


Figure 3: The time series of the Lorenz chaotic system for the y signal

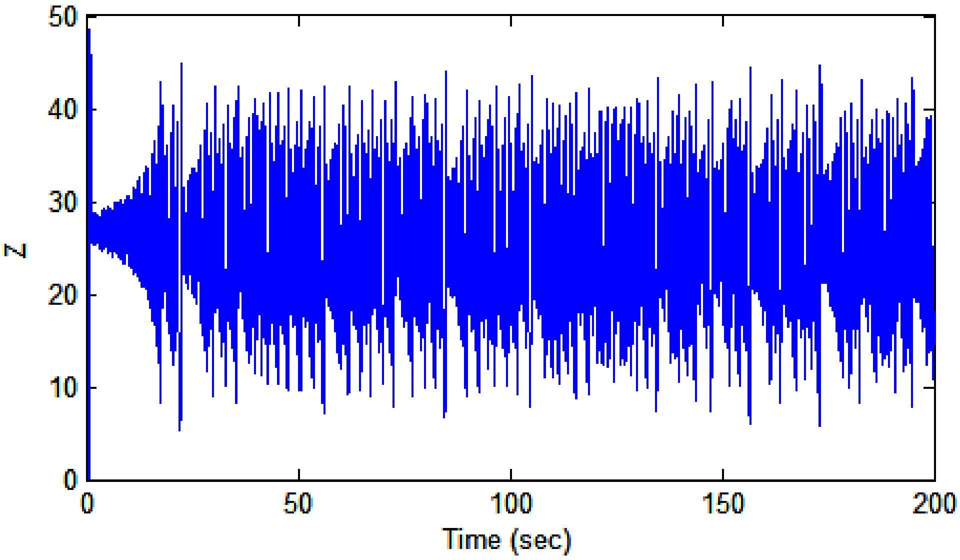


Figure 4: The time series of the Lorenz chaotic system for the z signal

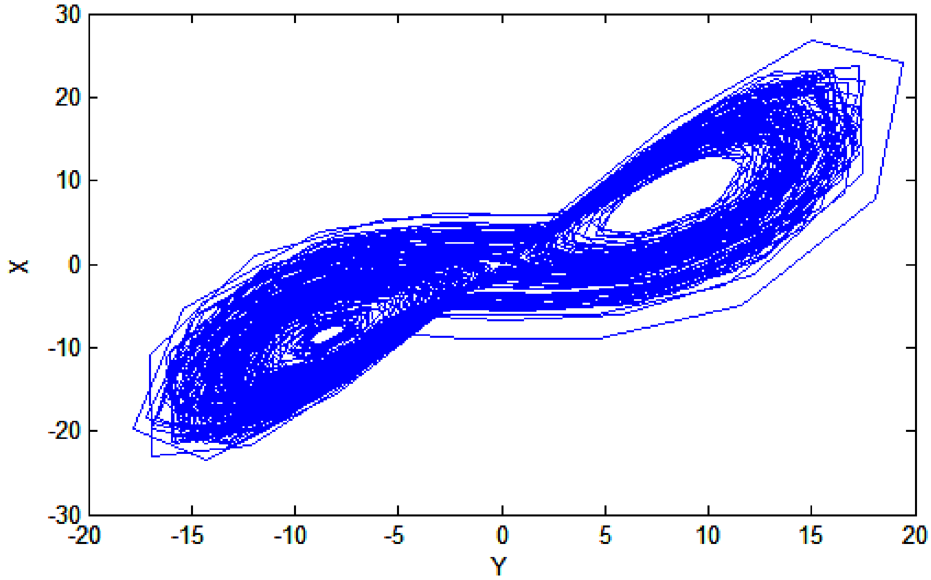


Figure 5: xy phase portait of the Lorenz Systems

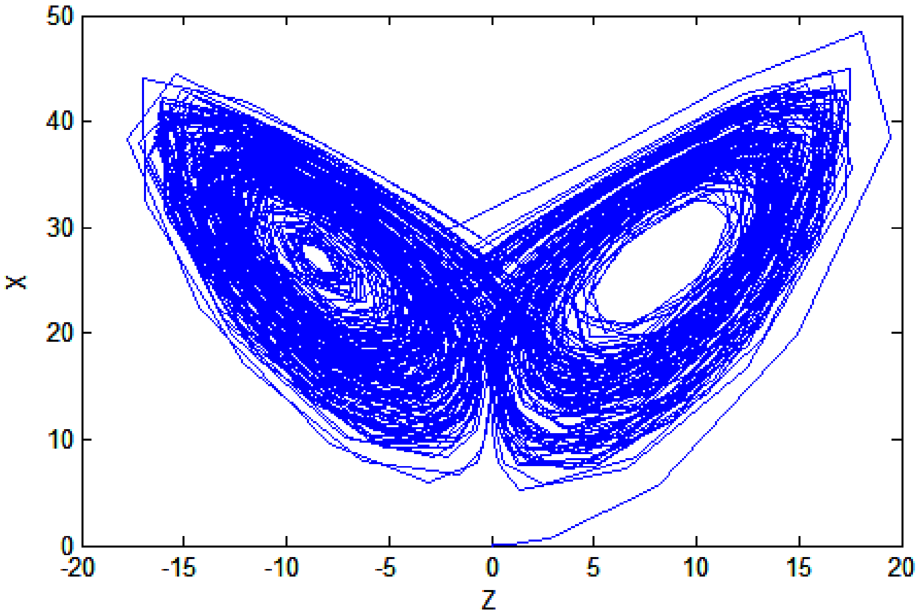


Figure 6: xz phase portait of the Lorenz Systems

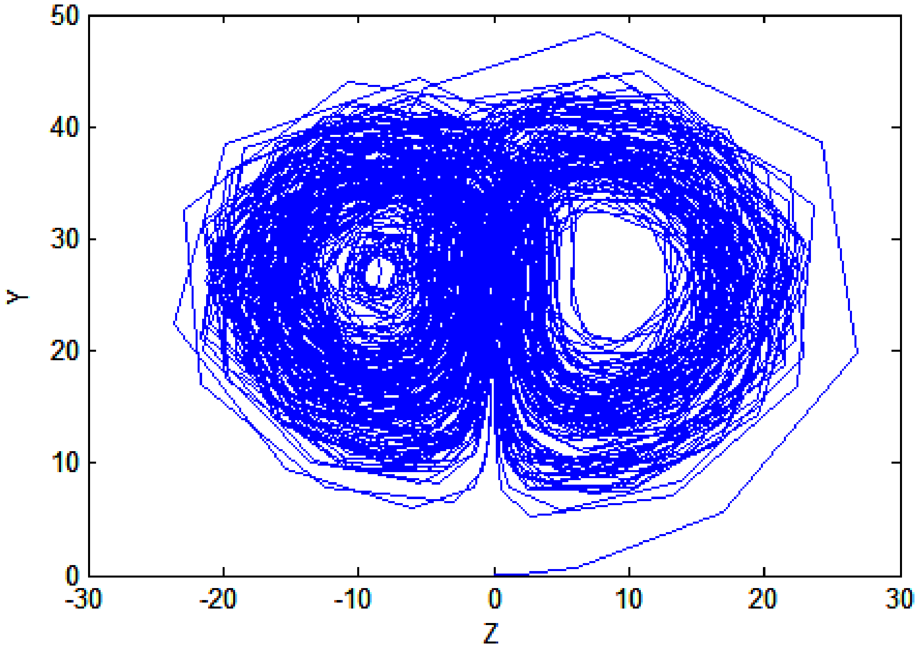


Figure 7: yz phase portait of the Lorenz Systems

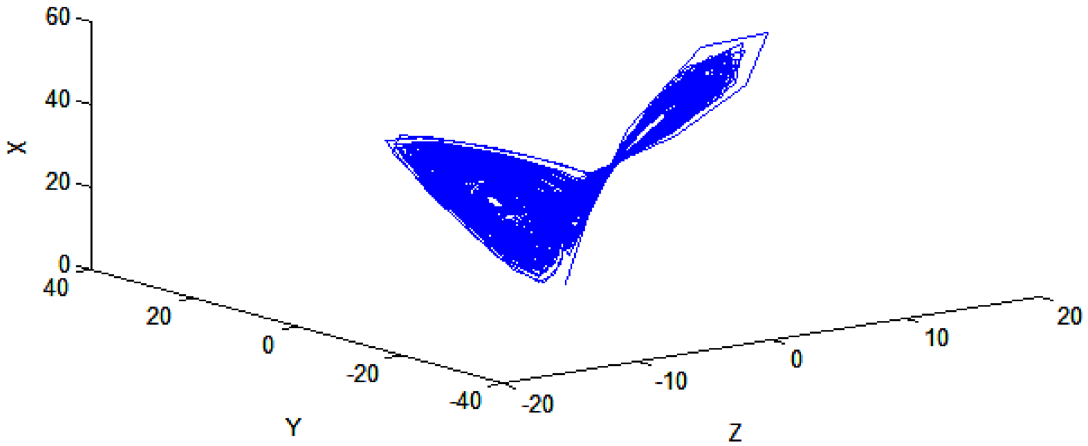


Figure 8: xyz phase portait of the Lorenz Systems

Phase portraits are showing in Figure 5, Figure 6, Figure 7 and Figure 8. The butterfly effect was seen in the expected output graphs.

The stable and unstable points of the sytem variables was examined by obtaining the eigenvalues. The system is linearization to accomplish eigenvalues and then the Jacobian Matrix is obtaing by using the first terms of linear elements.

$$\begin{aligned} f_1 &= \dot{x} = a \cdot (y - x) \\ f_2 &= \dot{y} = x \cdot (c - z) - y \\ f_3 &= \dot{z} = x \cdot y - b \cdot z \end{aligned} \tag{3}$$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{pmatrix} = \begin{pmatrix} -a & a & 0 \\ c - z & -1 & -x \\ y & x & -b \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 \\ 28 - z & -1 & -x \\ y & x & -8/3 \end{pmatrix} \tag{4}$$

The eigenvalues are found by solving the characteristic equation,

$$A = |\lambda I - J| = \begin{vmatrix} \lambda + 10 & -10 & 0 \\ -28 + z & \lambda + 1 & x \\ -y & -x & \lambda + (8/3) \end{vmatrix} = 0 \tag{5}$$

Using eig (A) MATLAB code for eigenvalues of Lorenz chaos system.

Inital eigenvalues of $\lambda_1 = 22.8277$, $\lambda_2 = -11.8277$, $\lambda_3 = 2.6667$ for $x_0=0.001$, $y_0=0.001$ and $z_0=0$. Inital eigenvalues are shown in Figure 9 (a, b, c,). All of eigenvalues are shown in Figure 10.

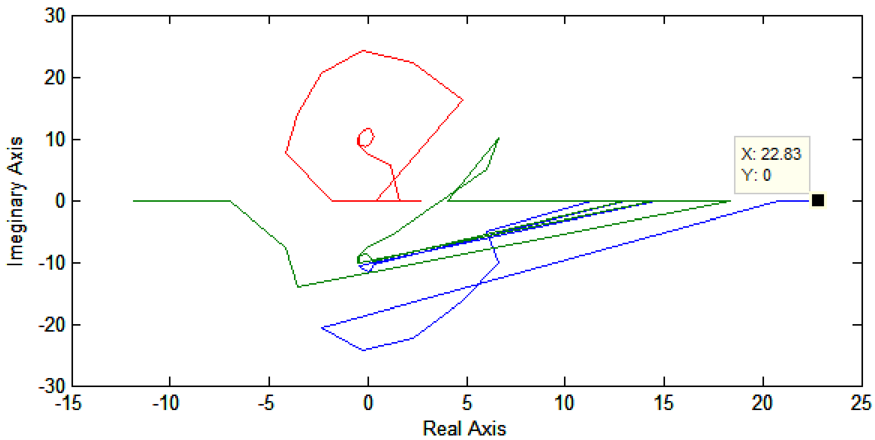


Figure 9(a): Eigenvalues coefficients in complex plane

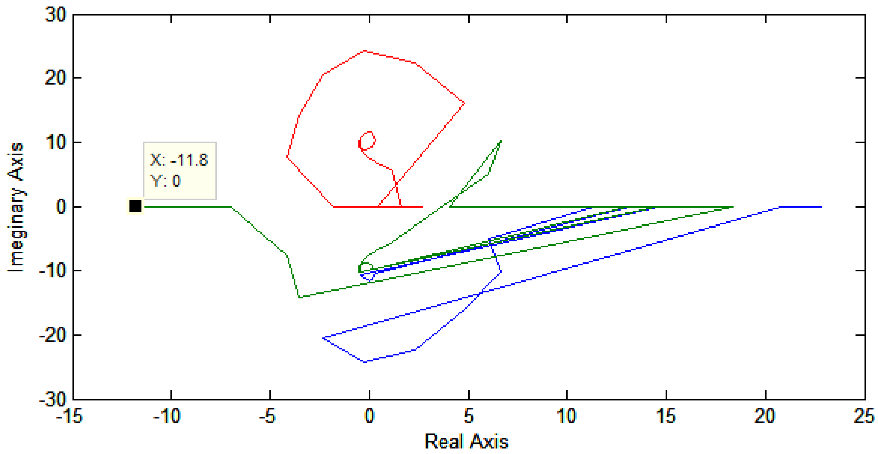


Figure 9(b): Eigenvalues coefficients in complex plane

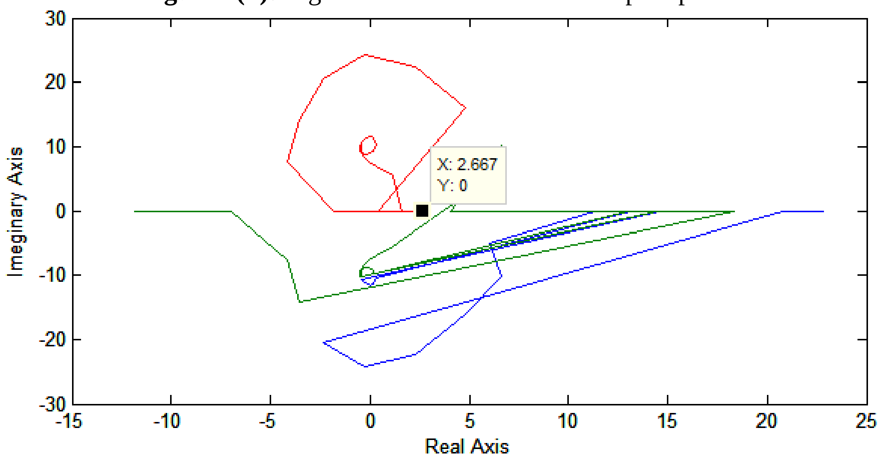


Figure 9(c): Eigenvalues coefficients in complex plane

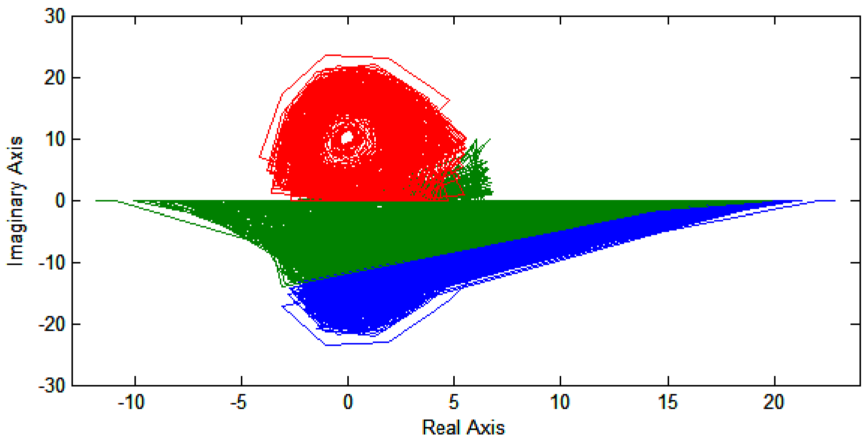


Figure 10: Eigenvalues coefficients in complex plane

Conclusion

The aim of this paper is to investigate the stable of the Lorenz chaotic systems. It is seen in Figure 10 that the eigenvalues coefficients in complex plane are located on the right side of the complex plane axis. As a result of unstable of the system is causes the nonlinear behaviors. The eigenvalues of the system is show in Figure 9(a,b,c). The eigenvalues given referance 5 were achived same values that in Figure 9(a,b,c). The various of eigenvalues coefficients in compex plane are revealed a chaotic behavior. In addition the butterfly effect was seen in the output graphs.

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