SIMPLE FORMULA TO REPRESENT FLEXURAL CAPACITY OF PRESTRESSED CONCRETE

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Abstract

It is imperative for the structural designer to possess simple and reliable formulas to predict different aspects in structural behavior of reinforced concrete. Nowadays, mathematical formulas, as simple as possible, are required for computer aided design. This is a significant point to the present study. A theoretical review for the ACI Code analysis approach is presented. Theoretical formulae, based on the ACI Code ordinary approach, are derived to predict maximum and optimum prestressing steel. Theoretical simple formula is suggested to predict maximum flexural capacity for the prestressed concrete section. Theoretical factor R, that depends on f_c' and f_{ps}, is presented to relate maximum to optimum prestressing steel. Another formula is derived based on the ACI Code ordinary approach to predict maximum flexural capacity for comparison with the suggested formula.

Keywords: Beam, concrete, maximum, optimum, prestress

Introduction

The problem of construction of the structures that are strong as well as durable was faced from the early times during the progress of human civilization. The Assyrians and Babylonians used bitumen to bind stones and bricks together, the Egyptians began to use mud mixed with straw to bind dried bricks, and the Romans used brick dust and volcanic ash with lime to produce hydraulic mortar. In 1926, high tensile steel wires (yield stress over 1240 MPa) have been used to prestress concrete sections.

In reinforced concrete, the analysis means finding the resisting moment of the reinforced concrete section $M_R = \varphi M_n$. This is necessary when checking an existing structure, or element, to determine if the strength provided (supply) by the section is sufficient to satisfy M_u that is calculated (demand) from the loads (Youkhanna 2014). This also makes it possible to calculate the maximum live load that may be permitted on the structure or element. Analysis of beams can be performed when all parameters that influence the ultimate strength are known. These are: dimensions of the section (*b*, *d*), materials used (f'_c and f_y), and tensile reinforcement area (*A*s).

Compressive stresses, in prestressed concrete members, are introduced to reduce the tensile stresses that result from applied loads including self-weight of the member (Notes on ACI 2008).

Prestressed Concrete

The following three fundamental concepts of action of reinforced concrete were clearly initiated in literature:

1. Concrete is weak in tension so that all the tension can be assumed to be taken by steel.
2. The transfer of stress between concrete and steel takes place through the bond strength developed between steel and concrete on setting of concrete.
3. The volume changes in concrete and in steel due to the atmospheric change of temperature are more or less equal.

Pre-stressing induces compression on the tension side of concrete members (such as beam) and when the design load is applied, tension is produced which neutralizes the compression already set up by pre-stressing. The aim of pre-stressing thus is to completely neutralize the stresses due to the design load as shown in Fig. 1 (Youkhanna 2009). The pre-stress is set up in a concrete beam by stretching several wires of high tensile strength in the concrete.

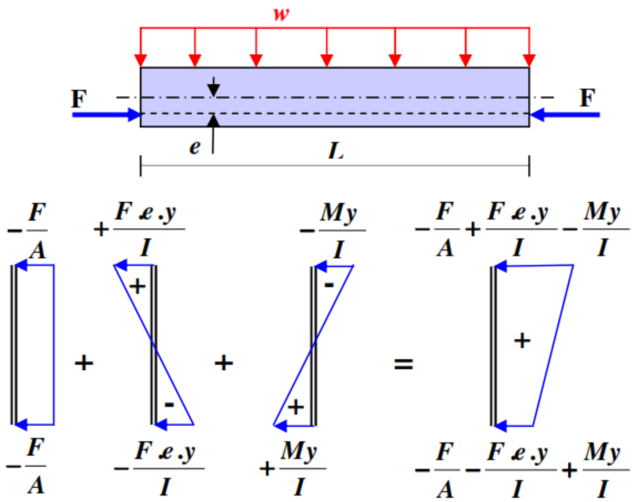

Fig. 1: Prestressed concrete beam.

Strength and behavior at service conditions (at all stages that will be critical during the life of the structure) shall be the basis of design of prestressed members (ACI Code 2011). The following assumptions are applicable in design of prestressed members:

1. Direct proportionality between strain in reinforcement and concrete with the distance from the neutral axis shall be applicable, i.e. strains vary linearly with depth through the entire load range.
2. At extreme concrete compression fiber, maximum strain shall be assumed equal to 0.003.
3. Concrete strength in tension shall be neglected.

The reduction of cross-sectional dimensions and consequent weight savings is the aim of modern structural engineering. Such development is particularly important in the field of reinforced concrete, where the dead load represents a substantial part of the total load. Prestressed concrete may fulfill this task (Nilson et al 2004).

Research Significance

The significance of this research is to find the maximum effect that can be achieved using prestressing steel only. This is performed by deriving simple formula based on the ACI Code approach to predict maximum flexural capacity. The presented formula is useful in computer calculations.

Theory

Strength design method (ACI Code 2011) shall be used to compute the design moment strength of flexural members. For prestressing steel, f_{ps} shall be substituted for f_y in strength equations. Refer to Fig. 2 which is based on Whitney equivalent rectangular stress block (Whitney 1942).

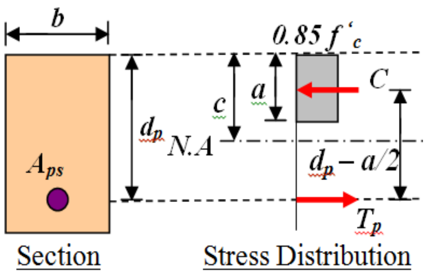

Fig. 2: Equivalent stress block.

$$T_p = C$$

$$A_{ps} \cdot f_{ps} = 0.85 f_c^{'} b.a \tag{1}$$

in which

$$a = \frac{A_{ps} \cdot f_{ps}}{0.85 f_c^{'} b.} \tag{2}$$

$$a = \beta_1 . c \qquad (3)$$

and β_1 is to be taken equal to **0.85** for f'_c up to and including 30 MPa. For f_c' above 30 MPa, β_1 is to be calculated from Eq.(4) but shall not be taken less than **0.65** (McCormac & Nelson 2006).

$$\beta_1 = 0.85 - 0.008(f_c' - 30) \geq 0.65 \qquad (4)$$

Eq. (2) may be written as:

$$a = \frac{\rho_p . f_{ps} . d_p}{0.85 f_c'} \qquad (5)$$

where

$$\rho_{ps} = \frac{A_{ps}}{b . d_p} \qquad (6)$$

The nominal moment can be represented as:

$$M_n = T_p . (d_p - \frac{a}{2}) = A_{ps} . f_{ps} (d_p - \frac{a}{2}) \qquad (7)$$

And the ultimate flexural strength is:

$$M_u = \varphi . M_n = \varphi . A_{ps} . f_{ps} (d_p - \frac{a}{2}) \qquad (8)$$

Where φ is the strength reduction factor, and its variation is shown in Fig. 3 (ACI Code 2011). For tension controlled sections, φ is equal to 0.90.

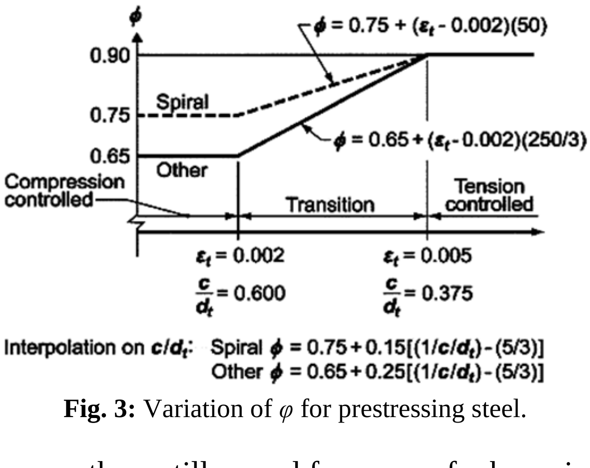

Fig. 3: Variation of φ for prestressing steel.

In some cases, there still a need for some of rebar reinforcement to be added to prestress tendons in order to provide the required bending capacity of the beam section (Nilson et al 2004). A check should be made to ensure that the beam is still under reinforced. At ultimate load stage, the prestressed section behaves the same way as ordinary reinforced concrete section, except that the stress is very much higher (Gilbert and Mickleborough 2004). Depending on the amount of the prestressing steel, a section can be under-reinforced or over-reinforced, and the transition situation is called a balanced condition, hence:

$$M_u = \varphi.M_n = \varphi.A_{ps}.f_{ps}(d_p - \frac{a}{2}) \qquad (8)$$

$$\rho_{pb} = 0.85\frac{f_c'}{f_y}\beta_1\frac{600}{600+f_{ps}}.\frac{d_t}{d} \qquad (9.a)$$

When there is only one layer of reinforcement, $d_t/d = 1.0$, hence

$$\rho_{pb} = 0.85\frac{f_c'}{f_y}\beta_1\frac{600}{600+f_{ps}} \qquad (9.b)$$

Making use of

$$\rho_{\max} = 0.75\rho_{pb} = 0.6375\beta_1\frac{f_c'}{f_y}\frac{600}{600+f_{ps}} \qquad (10)$$

Substituting Eq. (2) into Eq. (8) and rearranging:

$$M_u = \varphi.d_p.f_{ps}.A_{ps} - \frac{\varphi.f_{ps}^2}{1.7f_c'.b}A_{ps}^2 \qquad (11)$$

Derive Eq. (11) w.r.t. A_{ps} and equate to zero (Fong et al 2003), the optimum prestress steel (theoretical maximum prestress steel) can be found as

$$(A_{ps})_{opt} = 0.85\frac{f_c'}{f_y}.b.d_p \qquad (12)$$

Substitute Eq. (12) into Eq. (6):

$$(\rho_{ps})_{opt} = 0.85\frac{f_c'}{f_y} \qquad (13)$$

Let the factor R be the ratio of maximum to optimum prestressing steel:

$$R = \frac{(A_{ps})_{\max}}{(A_{ps})_{opt}} = \frac{(\rho_{ps})_{\max}}{(\rho_{ps})_{opt}} \qquad (14)$$

From which

$$(A_{ps})_{\max} = R.(A_{ps})_{opt} \qquad (15)$$

or

$$(\rho_{ps})_{\max} = R.(\rho_{ps})_{opt} \qquad (16)$$

Substitute Eq.(10) and Eq.(13) into Eq.(16) and arrange:

$$\boldsymbol{R} = 0.75 \frac{600}{600 + \boldsymbol{f_y}} . \boldsymbol{\beta}_1 \qquad (17)$$

From equation (17), it can be seen that the factor R depends on the values of $f_c^{'}$ and f_{ps} only. Table 1 gives values of the factor R predicted from Eq. (17) for some of the values of $f_c^{'}$ and f_{ps}.

It can be seen that the factor R takes same values for $f_c^{'} \leq 30$ MPa, Same thing is noticed for $f_c^{'} \geq 55$ MPa. As a result, Table 1 may be replaced by the simple one, i.e. Table 2. The relation of the factor R with the values of $f_c^{'}$ and f_{ps} is shown in Fig. 3.

Suggested Simple Formula for Maximum Flexural Capacity

Substituting Eq. (15) into Eq. (11) and making use of Eq. (12), a simple formula is suggested to find maximum flexural capacity of concrete section with prestressing steel to be:

$$(\boldsymbol{M_u})_{\max} = 0.425 \boldsymbol{\varphi} . \boldsymbol{f_c^{'}} \boldsymbol{b} . \boldsymbol{d_p^2} (2\boldsymbol{R} - \boldsymbol{R}^2) \qquad \text{(Suggested)} \qquad (18)$$

Where factor R can be determined from either Table 2 or Fig. 3.

Table 1: Values of factor R.

f_{ps} (MPa)	$f_c^{'}$ (MPa)	R		f_{ps} (MPa)	$f_c^{'}$ (MPa)	R
1500	25	0.182		1600	25	0.174
	30	0.182			30	0.174
	35	0.174			35	0.166
	40	0.165			40	0.158
	45	0.156			45	0.149
	50	0.148			50	0.141
	55	0.139			55	0.133
	60	0.139			60	0.133
1700	25	0.166		1800	25	0.159
	30	0.166			30	0.159
	35	0.158			35	0.152
	40	0.151			40	0.144
	45	0.143			45	0.137
	50	0.135			50	0.129
	55	0.127			55	0.122
	60	0.127			60	0.122

Table 2: Values of factor *R*.

f_c' (MPa)	f_{ps} (MPa)			
	1500	1600	1700	1800
	R			
≤ 30	0.182	0.174	0.166	0.159
35	0.174	0.166	0.158	0.152
40	0.165	0.158	0.151	0.144
45	0.156	0.149	0.143	0.137
50	0.148	0.141	0.135	0.129
≥ 55	0.139	0.133	0.127	0.122

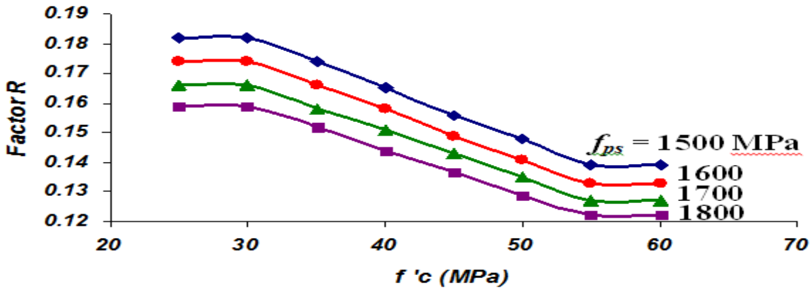

Fig. 3: Relation of *R* with values of f_c' and f_{ps}.

Maximum Flexural Capacity using Ordinary Approach of the ACI Code

Substitute Eq. (6 & 10) into Eq. (11), another formula for the maximum flexural capacity based on the ACI Code ordinary approach can be found as:

$$(M_u)_{\max} = \frac{382.5\varphi.\beta_1 b.d_p^2 f_c'}{600 + f_{ps}}\left[1 - \frac{382.5\beta_1}{1.7(600 + f_{ps})}\right] \quad (19)$$

Application of Eq. (18) is more convenient and simple than application of Eq. (19). This is proved by solving numerical examples.

Conclusion

Reviewing the previous paragraphs, the following may be concluded.

1. A theoretical review for the ACI Code analysis approach is presented.
2. Theoretical formulae, based on the ACI Code ordinary approach, are derived to predict maximum and optimum prestressing steel.
3. Theoretical simple formula is suggested to predict maximum flexural capacity for the prestressed concrete section.
4. Theoretical factor *R*, that depends on f_c' and f_{ps}, is presented to relate maximum to optimum prestressing steel amount.
5. Another formula is derived based on the ordinary approach of the ACI Code to predict maximum flexural capacity for comparison with the suggested formula.

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Notation

a : depth of equivalent compression rectangular stress block (mm).

A_{ps} : area of pre-stressed tensile reinforcing steel (mm^2).

$(A_{ps})_{opt}$: optimum area of pre-stressed tensile reinforcing steel (mm^2).

b : width of the compression face of a flexural member (mm).

c : distance from extreme compression fiber to neutral axis (mm).

d_p : effective depth of a section measured from extreme compression fiber to centroid of prestressing steel (mm).

R : factor relates maximum tensile reinforcement to optimum reinforcement.

f'_c : specified compressive strength of concrete (MPa).

f_{ps} : specified stress in the prestressed reinforcing steel (MPa).

M_u : ultimate flexural capacity (kN.m).

β_1 : a factor to be multiplied by the distance c to obtain the depth of the equivalent rectangular stress block.

ρ_p : ratio of prestressed reinforcement in a concrete section.

ρ_{pb} : ratio of tensile prestressing steel producing balanced strain condition.

ρ_{pmax} : maximum ratio of prestressed reinforcement.

$(\rho_p)_{opt}$: optimum ratio of non prestressed reinforcement.

φ : capacity reduction factor.