

EFFECTS OF VISCOUS DISSIPATION ON MHD NATURAL CONVECTION FLOW OVER A SPHERE WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY IN PRESENCE OF HEAT GENERATION

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Abstract

The objective of this research is to investigate the combined effects of heat generation and viscous dissipation on MHD natural convection flow of an electrically conducting fluid over an isothermal sphere with variable thermal conductivity. Thermal conductivity is considered as a linear function of temperature. The governing equations are solved numerically by numerical solution strategy as per requirement and suitability. Solution method such as finite difference method with killer box scheme has been employed. The computational findings for the dimensionless velocity, temperature profiles as well as for the skin-friction coefficient and surface heat transfer rate are displayed graphically.

Keywords: Magnetohydrodynamic (MHD), Variable thermal conductivity, Heat generation, viscous dissipation

Nomenclature:

a	Radius of the sphere	T_w	Temperature at the surface
C_f	Skin friction coefficient	u	Dimensionless velocity component along x direction
C_p	Specific heat at constant pressure	v	Dimensionless velocity component along y direction
f	Dimensionless stream function	U	Velocity component along the surface
Gr	Grashof number	V	Velocity component normal to the surface
g	Acceleration due to gravity	X	Axis in the direction along the surface
k	Thermal conductivity	Y	Axis in the direction normal to the surface
k_f	Thermal conductivity of the fluid	ξ	Dimensionless coordinate along to the surface
k_∞	Thermal conductivity of the ambient fluid	η	Dimensionless coordinate normal to the surface
M	Magnetic parameter	ψ	Stream function
N	Viscous dissipation parameter	τ_w	Shearing stress
Nu	Local Nusselt number	ρ	Density of the fluid
Pr	Prandtl number	μ	Viscosity of the fluid
q_w	Heat flux at the surface	ν	Kinematics viscosity of the fluid
Q	Heat generation parameter	θ	Dimensionless temperature function
Q_0	Constant	β	coefficient of thermal expansion
r	Radial distance from the symmetric axis to the surface	β_0	Strength of magnetic field.
T	Temperature of the fluid in the boundary layer	γ	Thermal conductivity variation parameter
T_∞	Temperature of the ambient fluid	σ_0	Electric conductivity

Introduction

Natural convection heat transfer has gained considerable attention because of its numerous applications in the areas of energy conservation cooling of electrical and electronics components, design of solar collectors, heat exchangers etc.. Many practical heat transfer applications involve with the conversion of some forms of mechanical, electrical, nuclear or chemical energy to thermal energy. A vast number of research papers have been published considering different fluid flow systems. Alam et al. [1] analyzed the viscous dissipation effects on MHD natural convection flow along a sphere. Miraj et al. [2] studied the effects of pressure work and radiation on natural convection flow around a sphere with heat generation. Mixed convection boundary layer flow about a solid sphere with Newtonian heating is investigated by Salleh et al. [3]. Huang and Chen [4] considered laminar free convection from a sphere with blowing and suction. The effect of

viscous dissipation on external natural convection flow over a surface was examined by Gebhart and Mollendorf [5].

None of the above mentioned paper considers the thermal conductivity with variable behavior. However physical property may change with the change of temperature. Sarma and Singh [6] have shown that the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Mollah et al. [7] analyzed the natural convection flow from an isothermal sphere with temperature dependent thermal conductivity. Boundary layer flow in a porous medium past a moving vertical plate with variable thermal conductivity and permeability is carried out by Singh [8].

It is observed that the effect of variable temperature thermal conductivity in presence of heat generation on natural convection flow near the lower stagnation point over a sphere has received a little attention. So, in the present study, it is proposed to investigate the conjugate effects of heat generation and viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity.

Formulation of the problem:

A steady two-dimensional natural convection boundary layer flow of an incompressible viscous and electrically conducting fluid over a sphere of radius a has been considered. In this analysis T_w is assumed as the constant temperature at the surface of the sphere, and T_∞ being the ambient temperature of the fluid, and T is the temperature of the fluid within the boundary layer. The conservation equations for the flow characterized with the continuity, momentum and energy equations which are written as follows:

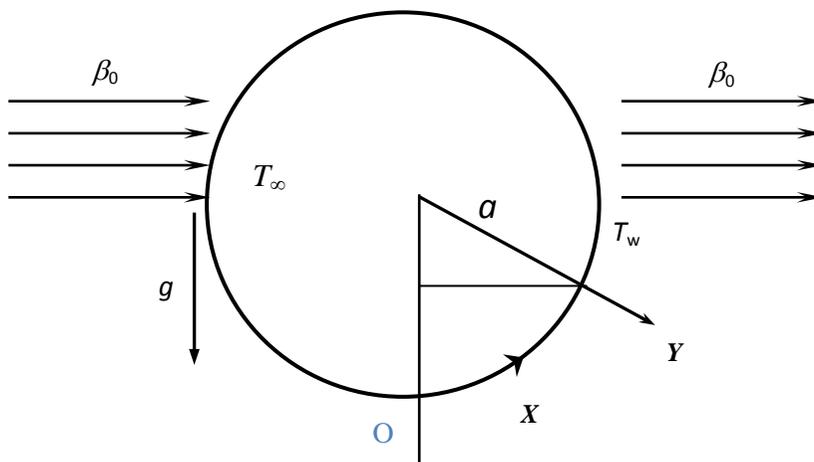


Fig. 1: Physical model and coordinate system

$$\frac{\partial}{\partial X}(rU) + \frac{\partial}{\partial Y}(rV) = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \nu \frac{\partial^2 U}{\partial Y^2} + g \beta (T - T_\infty) \sin\left(\frac{X}{a}\right) - \frac{\sigma_0 \beta_0^2}{\rho} U \tag{2}$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{\rho C_p} \frac{\partial}{\partial Y} \left(k_f \frac{\partial T}{\partial Y} \right) + \frac{\nu}{C_p} \left(\frac{\partial U}{\partial Y} \right)^2 + \frac{Q_0}{\rho C_p} (T - T_\infty) \tag{3}$$

The boundary conditions for the governing equations are

$$\left. \begin{aligned} U = V = 0, \quad T = T_w \quad \text{on} \quad Y = 0 \\ U \rightarrow 0, T \rightarrow T_\infty \quad \text{at} \quad Y \rightarrow \infty \end{aligned} \right\} \tag{4}$$

$$r(X) = a \sin\left(\frac{X}{a}\right) \tag{5}$$

where $r = r(X)$

Where a is the radius of sphere, r is the radial distance from the symmetrical axis to the surface of the sphere, $k(T)$ is the thermal conductivity of the fluid depending on the fluid temperature T . The amount of heat generation per unit volume is $Q_0 (T - T_\infty)$, Q_0 being a constant which may either positive or negative. The source term represent the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$. Here we will consider the form of the temperature dependent thermal conductivity which

was proposed by Charraudeau [9] as $k_f = k_\infty \left(1 + \gamma^* (T - T_\infty) \right)$

where k_∞ is the thermal conductivity of the ambient fluid and γ^* is defined

as $\gamma^* = \frac{1}{k_f} \left(\frac{\partial k}{\partial T} \right)_f$. Equation (3) can be reduced into the following form

$$\begin{aligned} U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} &= \frac{1}{\rho C_p} \left(\frac{\partial k}{\partial Y} \frac{\partial T}{\partial Y} + k \frac{\partial^2 T}{\partial Y^2} \right) + \frac{\nu}{C_p} \left(\frac{\partial U}{\partial Y} \right)^2 \\ &= \frac{1}{\rho C_p} \left(\frac{\partial k}{\partial Y} \frac{\partial T}{\partial Y} \right) + \frac{1}{\rho C_p} \left(k \frac{\partial^2 T}{\partial Y^2} \right) + \frac{\nu}{C_p} \left(\frac{\partial U}{\partial Y} \right)^2 + Q\theta \end{aligned} \tag{6}$$

The above equations are non-dimensionalized using the following substitutions:

$$\xi = \frac{X}{a}, \eta = \text{Gr}^{\frac{1}{4}} \frac{Y}{a}, u = \frac{a}{\nu} \text{Gr}^{\frac{-1}{2}} U, v = \frac{a}{\nu} \text{Gr}^{\frac{1}{4}} V, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \theta_w = \frac{T_w}{T_\infty} \tag{7}$$

Thus (5) becomes $r(\xi) = a \sin \xi$

The equations (1) to (3) can be converted into dimensionless forms using equation (7) as follows:

$$\frac{\partial}{\partial \xi}(ru) + \frac{\partial}{\partial \eta}(rv) = 0 \tag{8}$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \theta \sin \xi - Mu \tag{9}$$

$$u \frac{\partial \theta}{\partial \xi} + v \frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}(1 + \gamma\theta) \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{Pr} \gamma \left(\frac{\partial \theta}{\partial \eta} \right)^2 + N \left(\frac{\partial u}{\partial \eta} \right)^2 + Q\theta \tag{10}$$

since $\frac{\partial T_\infty}{\partial \xi} = \frac{\partial T_\infty}{\partial \eta} = 0$ and $\nu\rho = \mu$

and the boundary conditions (4) becomes

$$\left. \begin{aligned} u = v = 0, \quad \theta = 1 \quad \text{at } \xi = 0, \text{ for any } \eta \\ u = v = 0, \quad \theta = 1 \quad \text{at } \eta = 0, \xi > 0 \\ u \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \xi > 0 \end{aligned} \right\} \tag{11}$$

Here, $Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2}$ is the Grashof number and θ is the non

dimensional temperature function, $N = \frac{\nu^2 Gr}{\rho a^2 C_p (T_w - T_\infty)}$ is the viscous

dissipation parameter, $Q = \frac{a^2 Q_0}{C_p \mu Gr^{1/2}}$ is the heat generation

parameter, $M = \frac{\sigma_0 \beta_0^2 a^2}{\mu Gr^{1/2}}$ is the magnetic parameter, $\gamma = \gamma^*(T_w - T_\infty)$ is the

non-dimensional thermal conductivity variation parameter and $Pr = \frac{\mu C_p}{k_\infty}$ is

the Prandtl number. To solve equations (9) and (10) subject to the boundary conditions (11), we assume the following variables u and v as follows and $\theta = \theta(\eta, \xi)$

$$u = \frac{1}{r} \frac{\partial \psi}{\partial \eta} \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial \xi} \tag{12}$$

where $\psi = \xi r(\xi) f(\xi, \eta)$ is a non-dimensional stream function .

Putting the above value in equation (9) and (10), we have

$$\frac{\partial^3 f}{\partial \eta^3} + \left(1 + \frac{\xi}{\sin \xi} \cos \xi\right) f \frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta}\right)^2 + \theta \frac{\sin \xi}{\xi} - M \frac{\partial f}{\partial \eta} \tag{13}$$

$$= \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial \xi} - \frac{\partial f}{\partial \xi} \frac{\partial^2 f}{\partial \eta^2} \right)$$

$$\frac{1}{Pr} (1 + \gamma \theta) \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{Pr} \gamma \left(\frac{\partial \theta}{\partial \eta}\right)^2 + \left(1 + \frac{\xi}{\sin \xi} \cos \xi\right) f \frac{\partial \theta}{\partial \eta} \tag{14}$$

$$+ N \xi^2 \left(\frac{\partial^2 f}{\partial \eta^2}\right)^2 + Q \theta = \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right)$$

The corresponding boundary conditions are :

$$\left. \begin{aligned} f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1 \text{ at } \eta = 0 \quad \text{for any } \eta \\ f = \frac{\partial f}{\partial \eta} = 0, \quad \theta = 1 \text{ at } \eta = 0, \quad \xi > 0 \\ \frac{\partial f}{\partial \eta} \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad \xi > 0 \end{aligned} \right\} \tag{15}$$

At the lower stagnation point of the sphere i.e. $\xi \approx 0$ and the equations (13) and (14) reduced to the following ordinary differential equations:

$$f''' + 2f f'' - (f')^2 + \theta - Mf' = 0 \tag{16}$$

$$\frac{1}{Pr} (1 + \gamma \theta) \theta'' + \frac{1}{Pr} \gamma (\theta')^2 + 2f \theta' + Q \theta = 0 \tag{17}$$

The boundary conditions as mentioned in Equation (15) then take the following form

$$\left. \begin{aligned} f(0) = f'(0) = 0, \theta(0) = 1 \\ f' \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \tag{18}$$

Where primes denote the differentiation with respect to η . The physical quantities of the principle interest are shearing stress in terms of the skin-friction coefficient and the rate of heat transfer in terms of the Nusselt number, which can be written, in non-dimensional form as

$$Nu = \frac{aGr^{-1/4}}{k_f (T_w - T_\infty)} q_w \text{ and } C_f = \frac{Gr^{-3/4} a^2}{\mu \nu} \tau_w \tag{19}$$

Where $\tau_w = \mu \left(\frac{\partial U}{\partial Y} \right)_{Y=0}$ and $q_w = -k_f \left(\frac{\partial T}{\partial Y} \right)_{Y=0}$ are the shearing stress and heat flux, respectively. Using the new variables (7), we have the simplified form of the heat transfer and the skin- friction coefficient as

$$Nu = - \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} \text{ and } C_f = \xi \left(\frac{\partial^2 f}{\partial \eta^2} \right)_{\eta=0} \tag{20}$$

Discussion of Results:

Numerical computations have been carried out for different values of the parameters entering into the problem. The velocity profiles, temperature profiles, average skin friction and Nusselt numbers are found for different physical parameters such as Prandlt number Pr magnetic parameter M , thermal conductivity variation parameter γ , heat generation parameter Q and viscous dissipation parameter N , which are presented in Figures 2 to 9. In order to verify accuracy of the present work, the values of heat transfer rate are compared with those reported by by Molla et al. [7] and Nazar et al.[10]that is shown in theTable-1. The results are found to be in good agreement.

Table-1: Compares the present numerical value of Nu for the values of $Pr = 0.70$ without the effect of M , γ , Q and N with those obtain by Molla et al. [7] and Nazar et al.[10]

$Pr = 0.70$			
ξ in degree	Nazar <i>et al.</i> [10]	Molla [7]	present
0	0.4576	0.4576	0.4576
10	0.4565	0.4564	0.4565
20	0.4533	0.4532	0.4533
30	0.4480	0.4479	0.4480
40	0.4405	0.4404	0.4406
50	0.4308	0.4307	0.4310
60	0.4181	0.4188	0.4192
70	0.4046	0.4045	0.4049
80	0.3879	0.3877	0.3882
90	0.3684	0.3683	0.3689

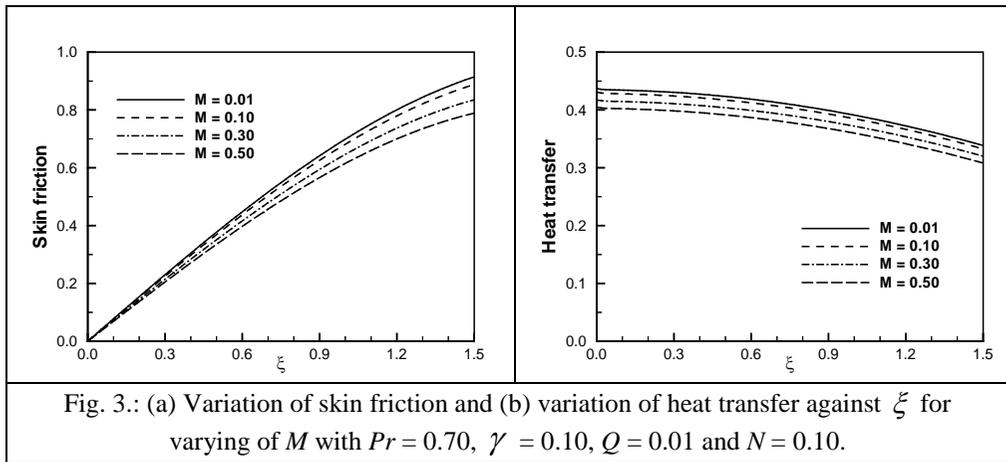
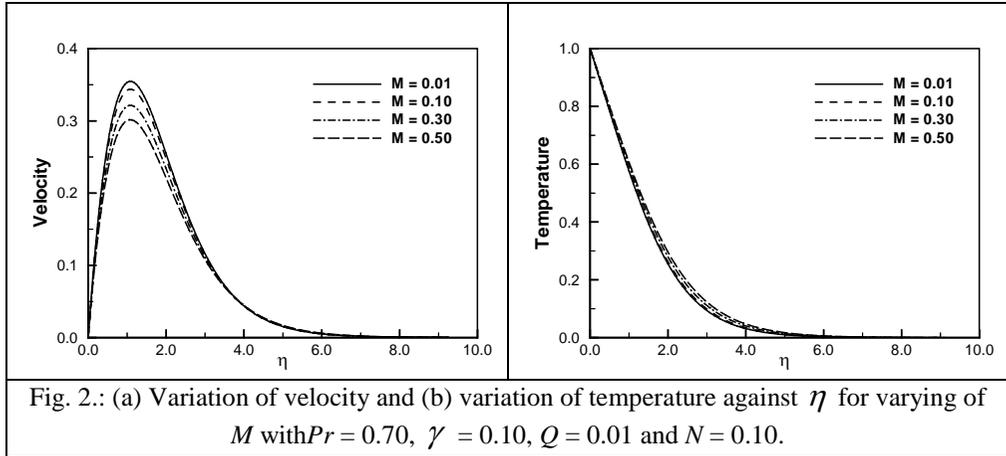
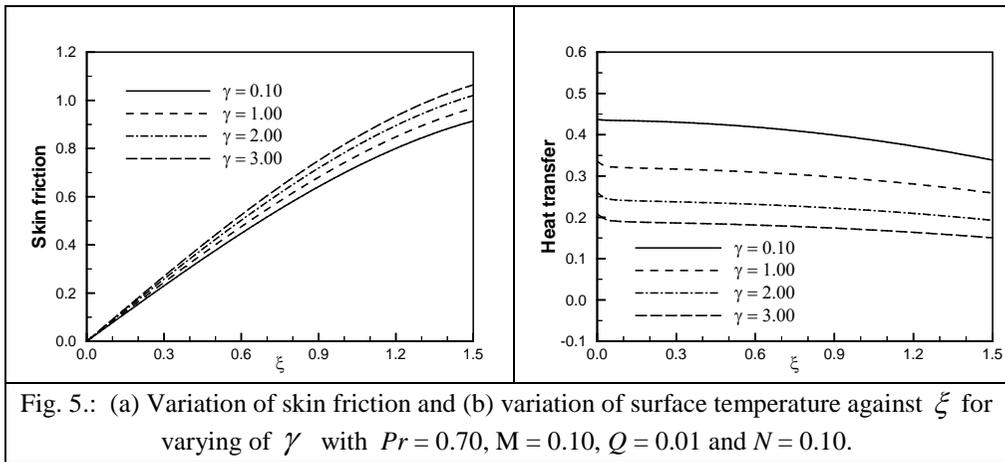
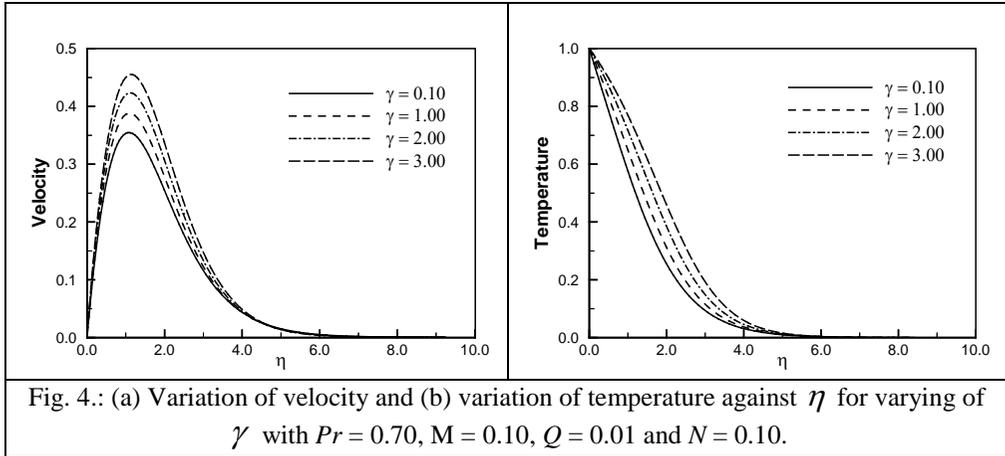


Figure 2(a) and Figure 2(b) deal with the effect of magnetic parameter M on the velocity and temperature distributions against η with Prandtl number Pr , thermal conductivity variation parameter γ , heat generation parameter Q and viscous dissipation parameter N . Here it is found from Figure 2(a) that the velocity distributions decreases slightly as the magnetic parameter increases but near the surface of the sphere velocity increases up to the peak and then decreases and finally approaches to zero. The temperature increases owing to the increasing values of magnetic parameter M that is presented in Figure 2(b). The variation of the local skin friction coefficient C_f and the local rate of heat transfer Nu for the selected values of magnetic parameter M are shown in Figures 3 (a) and 3(b), respectively. It is clear from both Figures that the skin-friction and heat transfers coefficient are decreased with the increasing values of magnetic parameter due to the increased M decreases the fluid velocity as well as the heat flow from the solid to fluid.



The effect of thermal conductivity variation parameter γ on the velocity and temperature profiles with the fixed value of the controlling parameters are shown in Figures 4(a) and 4(b), respectively. In Figures 4(a)-4(b), it is found that both the velocity and temperature increases for the change of thermal conductivity parameter γ . From Figure 4(a), the highest values of the velocity are 0.35459, 0.36354, 0.36817 and 0.37287 for $\gamma = 0.10, 1.0, 2.0, 3.0$, respectively which take place at $\eta = 1.05539$. We come to unanimous decision that the velocity increases by 3.83 % for distinction of γ from 0.10 to 3.0. In addition, in Figure 4(b) the temperature increase with increasing γ along η direction up to the extreme value and progressively decreases to zero. Figures 5(a) and 5(b) demonstrate the effect of thermal conductivity variation parameter γ on the skin friction and heat transfer coefficient against ξ with $Pr = 0.70, M = 0.10, Q = 0.01$ and $N = 0.10$. Figures reflect that the escalating value of the thermal conductivity variation

parameter γ increases the skin-friction coefficient C_f and decreases the local heat transfer rate. The values of skin-friction coefficient increases by 16.30% and the Nusselt number Nu decreases by 66.43 % as well for particular values of γ from 0.10 to 3.0.

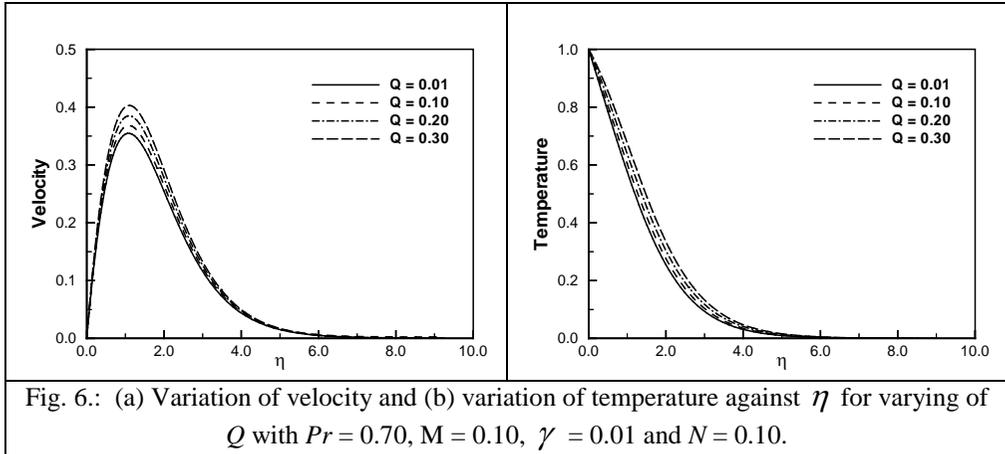


Fig. 6.: (a) Variation of velocity and (b) variation of temperature against η for varying of Q with $Pr = 0.70$, $M = 0.10$, $\gamma = 0.01$ and $N = 0.10$.

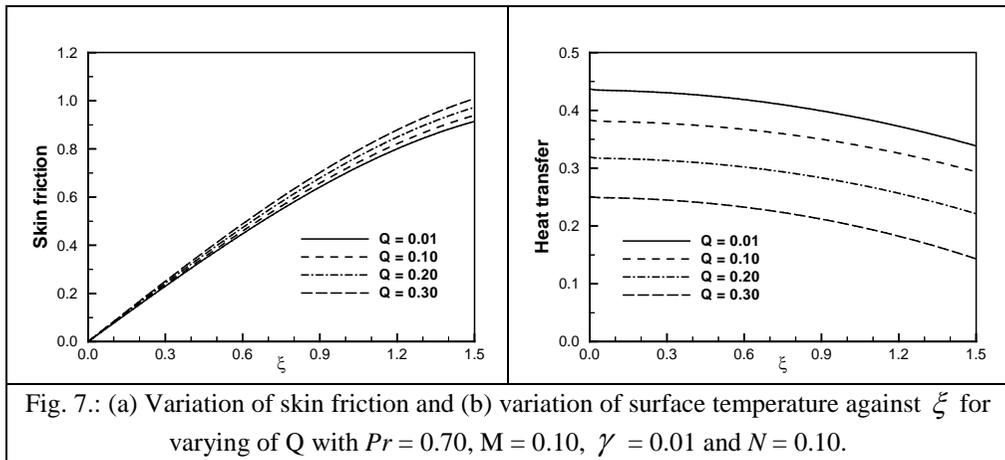
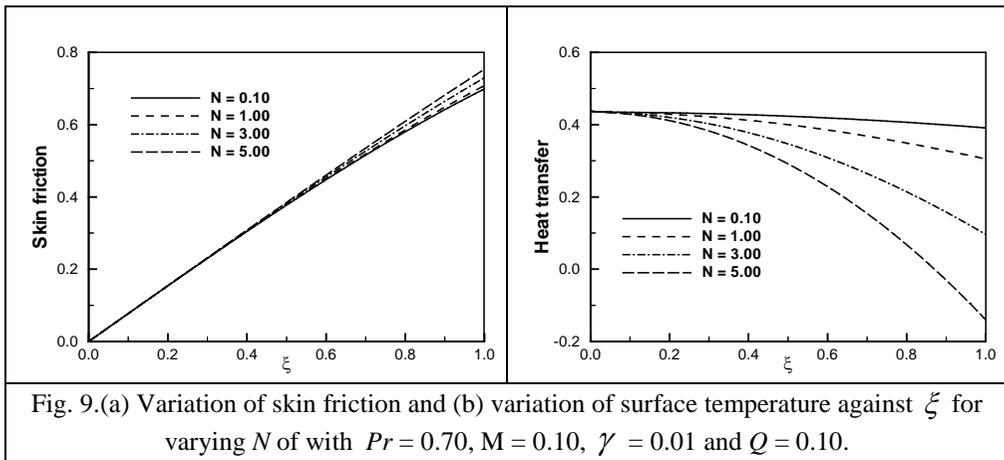
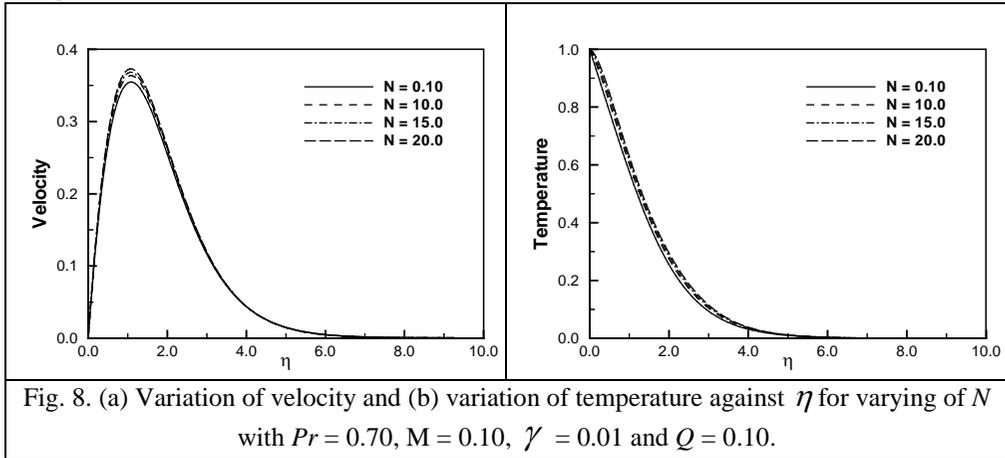


Fig. 7.: (a) Variation of skin friction and (b) variation of surface temperature against ξ for varying of Q with $Pr = 0.70$, $M = 0.10$, $\gamma = 0.01$ and $N = 0.10$.

Figures 6(a) and 6(b) display the numerical results of velocity and temperature distributions for the effect of heat generation parameter Q with Prandlt number $Pr = 0.70$, magnetic parameter $M = 0.01$ thermal conductivity variation parameter $\gamma = 0.10$ and viscous dissipation parameter $N = 0.10$. Tasting for different values of Q , it is observed that the velocity and temperature profile increase with the increase of heat generation parameter. The upper limit of the velocity are 0.35459, 0.36820, 0.38495 and 0.40297 for $Q = 0.01$, 0.10, 0.20 and 0.30, respectively, which come about at $\eta = 1.05539$ for the first maximum value and the rest of all is at $\eta = 1.11440$. It is deliberate that the velocity increases by 13.64 % as Q count in from .01 to

0.30. Again the effect of heat generation parameter Q on skin friction and the rate of heat transfer are shown in Figures 7(a) and 7(b) with $Pr = 0.70$, $M = 0.10$, $\gamma = 0.01$ and $N = 0.10$. From Figures 7(a) and 7(b) we observed that the skin friction coefficient increase sharply, on the contrary the heat transfer rate decrease monotonically for the selected value of Q along ξ direction. It is observed that the skin friction coefficient and local Nusselt number increase by 10.49% and decrease by 41.48%, respectively for distinct value of Q .



The variation of viscous dissipation parameter, N on the velocity and temperature profiles while $Pr = 0.70$, $M = 0.01$, $\gamma = 0.10$, and $Q = 0.01$ are exposed in Figures 8(a)-(b). We observed in Figure 8(a) that the velocity is zero at the boundary wall then the velocity increases up to the peak value as η increases and finally approach to zero (the asymptotic value). The maximum values of the velocity are 0.35459, 0.36354, 0.36817 and 0.37287

for $N = 0.1, 10.0, 15.0$ and 20.0 , respectively at position $\eta = 1.05539$. Counting these peak values of the velocity, we have calculated that the velocity rises by 5.15 % as N increases from 0.10 to 20.0. On the other hand, Figure 8(b) exhibits the temperature profile increases for the extend value of N . Figures 9(a)-(b) demonstrate the effect of viscous dissipation parameter N on the local skin friction coefficient and heat transfer rate against ξ with $Pr = 0.70$, $M = 0.01$, $\gamma = 0.10$, and $Q = 0.01$. It is observed from Figure 9(a) that the increasing value of the viscous dissipation parameter N , causes the greater skin friction on the surface of the sphere. On contemporary of Figure 9(b) depicts that the rate of heat transfer which is gradually decreased from positive to negative value for larger values of N .

Conclusion

In this analysis, the solution of two dimensional steady free convectional flow of viscous incompressible fluid over a sphere with the effects of viscous dissipation and heat generation in presence of variable thermal conductivity have been examined separately. The following observations and conclusions can be drawn:

- ✚ Velocity decreases with the increasing magnetic parameter, while temperature increases.
- ✚ The velocity and the temperature of the fluid within the boundary layer increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter.
- ✚ The skin friction along the surface of the sphere increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter but decreases for the increasing M .
- ✚ The heat transfer rate from the surface of the sphere to the fluid decreases with the increasing value of the magnetic parameter, thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter.

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