A MODEL TO ANALYZE WEATHER IMPACT ON APHID POPULATION DYNAMICS: AN APPLICATION ON SWALLOWTAIL CATASTROPHE MODEL

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Abstract

Aphids are important pests of most of the crops which are greatly affected by weather parameters such as temperature, relative humidity, rainfall, wind and sunshine hours. When applying eco-friendly control measures such as biological control, a good understanding of their population dynamics which is been impacted by above weather parameters is critically important. A number of experiments have been conducted to develop forecasting models or expert systems based on weather parameters as one of their driving variable for identifying the population dynamics of aphids. Considering more weather variables as a function of change of population is a difficult task and mathematical models have not been developed to overcome this for catastrophe theory applications. This research is aimed to develop a method to analyze the overall effect of more weather variables as a weather factor which could specifically be useful for catastrophe theory applications on population dynamics. In this paper a mathematical model is developed using factor analysis to do this task and implemented as a computer program. The model is applied with a swallowtail catastrophe theory model which has been developed to analyze the aphid population dynamics. The mathematical expressions of development of the model, its implementation and the results of the model verification are presented. Simulated results of the program suggest that some catastrophic movements like sudden jumps happening in growth of wheat aphid populations can clearly be explained by considering overall weather effect as the weather factor for population analysis models.

Keywords: Aphids; Catastrophe theory; Factor analysis; Weather factor, Ecological simulation

Introduction

Aphids, small sap sucking insects, are important pests of most of the crops which are greatly affected by weather parameters such as temperature, relative humidity, rainfall, wind speed and sunshine hours. Most of them are adapted specifically to the set of environmental and physiographic conditions especially for weather where crops are grown. When applying eco-friendly control measures like biological control, a good understanding of their population dynamics which is been impacted by above weather parameters is critically important. A number of experiments have been conducted (Ma & Bechinski, 2009; Wei et al., 2009; Zhao, 1989, 1991; Zhao et al., 2005; Zhao & Wang, 1993) to develop forecasting models or expert systems using weather parameter as one of their driving variables, with the aim of identifying the impact of weather in addition to other factors. Considering more weather parameters as a function of change of population is really a difficult task and mathematical models have not been developed to overcome this for catastrophe theory application. The cusp catastrophe model built by Ma and Bechinski (2009) explores the relationship between intrinsic rate of increase of aphids *Diuraphis noxia* and environmental and crop factors. They have used temperature (weather factor) and plant growth stage as controlling variables. Rashid et al. (2009) reported that the population dynamics of mustard aphid showed a positive effect with temperature whereas significant negative effect on mean relative humidity. Tomar (2010) has conducted a research on direct and indirect effects of weather parameters on population dynamics of cotton aphids. He also reported that temperature and relative humidity displayed positive association with aphid population, but the rainfall showed considerable negative direct effect. Sometimes, some parameters have indirect effects on other parameters, but sometimes not the same (Tomar, 2010). Another research has reported that effects of density-independent weather factors were relatively minor compared to density-dependent regulation of growth of buckthorn, green peach and potato aphids (Alyokhin et al., 2005). Heavy rainfall, sometimes, decrease the aphid population with other indirect consequences like virus spreading (Wallin &

Loonan, 1971). Some winged aphid forms are affected by wind, so that do not fly when wind speed is high and it is indirectly affected on migration. Some discussions of weather effects in relation to aphids or other insects can be found in Wool (2002), Day et al. (2010), Kidd (1990), Finlay and Luck (2011) and Belovsky and Slade (1995).

Most catastrophe models are descriptive in worldwide in terms of getting some understanding of relations between control factors and state variables and explaining of some biological phenomena. There are some difficulties in using catastrophe model in agriculture (Loehle, 1989); complexity of selecting control variables, difficulty of understanding catastrophe regions for higher dimensional models, lack of theoretical base of parameter estimation, and unclearness of explaining biological meanings. These difficulties seriously restrict the applications of catastrophe theory. Considering some influential weather parameters as a single weather factor in developing mathematical models especially for catastrophe theory application could be a simplification for this complex scenario. Here we proposed a mathematical model to analyze the overall effect of more weather variables as a weather factor which could particularly be useful for catastrophe theory application on population dynamics.

Factor analysis (Lattin et al., 2003) can be used to find unobserved variables or factors which can account for the correlation among a set of observed variables. The overall effect (unobserved) of weather parameters (observed) on population dynamics could be estimated using factor analysis and it is subjected to find the loadings or factors that explain the highest variation in the weather parameters over the population dynamics. Using this technology in catastrophic theory applications to derive an overall effect or factor from several observed variables is less documented. In this article, the overall weather effect is calculated by developed model and used to estimate population dynamics parameters using APHIDSim program. The APHIDSim is a simulation program (Piyaratne et al., 2013) which uses Swallowtail catastrophe theory (Glimore, 1981; Zeeman, 1976) with modified logistic growth equation (Zhao, 2005) to simulate the population dynamics of wheat aphids. It explains swallowtail behavior of aphid population dynamics using equilibrium points and plotting them in a three dimensional control space. APHIDSim uses the discriminant curve (bifurcation set) of swallowtail model and considers five basic regions (Fig 1) in the three dimensional space which could be used to explain catastrophic behavior (sudden jumps) of populations dynamics of aphids. In this paper we don’t expect to explain the APHIDSim in detail because our aim is only to explain the technologies of estimating the overall weather effect as a usable factor for catastrophic theory applications, and explain the catastrophic behavior of aphid population dynamics in terms of overall weather effect. The APHIDSim has been reported in detail in Piyaratne et al. (2013).

(a) (b)

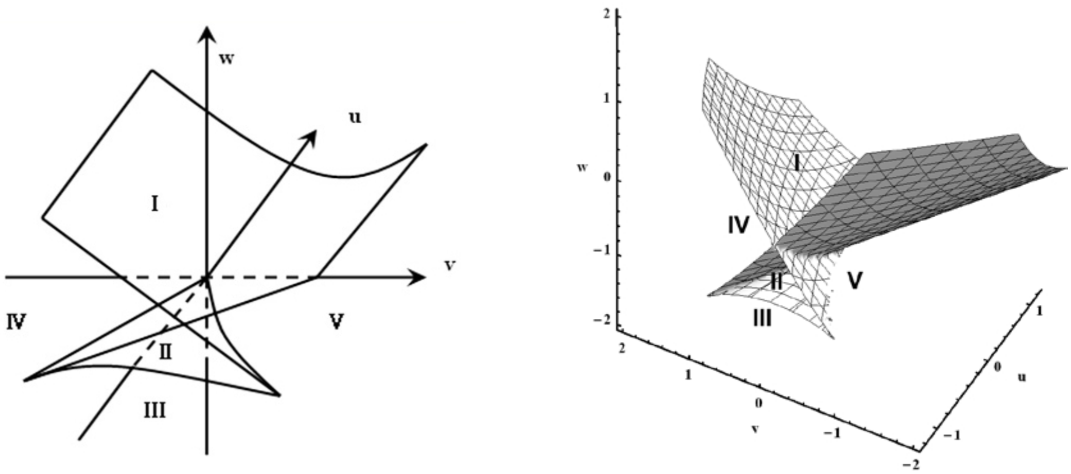

Figure 1. Basic catastrophe regions (I-V) of the swallowtail catastrophe model: (a) The schematic diagram of the bifurcation set (b) Three dimensional image of the bifurcation set (Piyaratne et al., 2013)

Materials and Methods

Three weather variables (temperature, humidity and rainfall) were considered, and the factor analysis (Lattin et al. 2003) was used to calculate the overall weather effect in the model. Here we used weather and aphids data collected by the Northwest A & F University and Meteorological Bureau in Yangling (www.ylqx.gov.cn), in 1987. Although factor analysis is a data reduction technique to simplify multivariate data by reducing it to a smaller number of factors**,** here we proposed a single factor analysis method relying on cumulative variance of all observed variables**.** This method produces one factor which is based on cumulative variance of correlation coefficients of three weather variables. This factor represents the overall effect of weather variables for change of aphid population.

Model Development and Analysis

Factor Analysis can be used to reduce the dimensionality of a data set of n number of observed correlated weather variables. Each of the n initial variables $x_1, x_2, ..., x_n$ can be given as a linear function of m uncorrelated common factors ($m < n$). Let $x_1, x_2, ..., x_n$ are the observed weather variables and $f_1, f_2 \dots, f_m$ are the common factors, then,

$x_1 = \lambda_{11}f_1 + \lambda_{12}f_2 + \ldots + \lambda_{1m}f_m$

$x_2 = \lambda_{21}f_1 + \lambda_{22}f_2 + \ldots + \lambda_{2m}f_m$

...

$x_p = \lambda_{n1} f_1 + \lambda_{n2} f_2 + \ldots + \lambda_{nm} f_m$ (1)

where λ_{jk}, $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, m$ are constants and called the factor loadings. Each factor consists of standardized (zero mean and unit variance) values called factor scores.

Then 1 can be given as a matrix form as

$x = \Lambda f$ (2)

a. Data collection: Data for three weather variables can be fed into the model manually or using Excel files as the Table 1. The model prepares a data matrix as required by the algorithms for calculation. The data include three weather variables; Temperature, rainfall and relative humidity (Table 1).

Table 1 Weather data used to calculate overall weather factor

Temperature (^{0}C)	Relative humidity (%)	Rainfall (mm)
4.70	80.00	7.60
8.30	77.60	9.60
3.50	77.40	13.60
8.06	72.50	21.00
11.92	76.60	3.60
15.14	72.60	0.00
9.68	76.20	7.80
16.40	72.60	22.00
16.66	73.20	14.40
15.76	76.80	1.20
14.16	67.00	8.20
17.54	71.60	0.00
18.44	81.80	37.40
21.24	67.60	4.40
18.61	84.33	84.33

b. Correlation matrix: Correlation matrix (Lattin et al. 2003) is estimated using observed data by calculating correlation coefficients of weather variables. Correlation coefficient *rc* (Eq. (3)) is calculated using Karl Pearson equation adapted from Nikolic et al. (2012).

$$r_c = \frac{N_w \sum x_w y_w - (\sum x_w)(\sum y_w)}{\sqrt{N(\sum x_w{}^2 - (\sum x_w)^2) N_w (\sum y_w{}^2 - (\sum y_w)^2)}} \quad (3)$$

where x_w and y_w are the weather variables and N_w is the number of observations. An algorithm developed in the model will produce the correlation matrix (Table 2) using these coefficients.

Table 2 Correlation matrix estimated by the model

	Temperature	Relative Humidity	Rainfall
Temperature	1.000	-0.225	0.249
Relative Humidity	-0.225	1.000	0.604
Rainfall	0.249	0.604	1.000

c. After calculating the correlation matrix, it is necessary to calculate Eigen values of the matrix and eigenvector associated to the Eigen values. Let *M* be a complex square matrix (here, the correlation matrix of weather variables). By definition (Strang, 2003), if ə is a complex number and *v* a nonzero complex vector satisfying Mv = əv, we call ə is an Eigen value of *M*, and *v* is called an eigenvector associated to the Eigen value ə. Then, an algorithm is developed to calculate Eigen values and eigenvectors according to *det* (*M*-ə*I*) = 0, and the algorithm was adapted from Press et al. (2007) and modified to Visual Basic .Net programming language.

d. Component matrix: Factors (λ) related to the correlation coefficients of the weather variables are extracted from the Eigen values and Eigen vectors using Principal Component Analysis (PCA) (Lattin et al. 2003). This method forms uncorrelated linear combinations of the observed weather variables. The first component has maximum variance while successive components have smaller portions of the variance and all are uncorrelated with each other. Therefore, the Eigenvectors with highest Eigen value are considered as principal components of the data set. Then the Eigenvectors are ordered by Eigen value, highest to lowest. Factors for component matrix are calculated in order to obtain the overall weather effect starting from highest Eigen value and correspondent Eigen vectors as follows:

$$\lambda = \sqrt{\text{Eigen value}} * \text{ Eigen vector}$$

The component matrix is shown in Table 3.

e. The overall weather effect (*OE*) is calculated by multiplying the cumulative major factors and original weather data and summing them up as follows:

$$OE_i = \lambda_1 x_{w1} + \lambda_2 x_{w2} \ldots + \lambda_{Nw} x_{wNw}$$

where λ_i is the cumulative value of major factor, x_w is the weather variable and N_w is the number of weather variables.

Table 3 Component matrix estimated by the model

	Factor 1	Factor 2	Factor 3
Temperature	-0.043	0.978	0.202
Relative Humidity	-0.887	-0.334	0.319
Rainfall	-0.904	0.282	-0.322

Application with Swallowtail catastrophe model

We incorporated the model into Swallowtail model of APHIDSim (Piyaratne et al., 2013) to analyze the weather parameters in order to observe the swallowtail catastrophe behavior of aphid populations. The Swallowtail behavior was estimated under both overall weather effect (overall effect of temperature, rainfall and relative humidity) and temperature (as a single variable) as well. We analyzed the behavior of population growth of wheat aphid in terms of the geometry of the swallowtail catastrophe model (Glimore, 1981; Zeeman, 1976). The bifurcation behavior of aphid population is featured on different regions of bifurcation set. Since the bifurcation set of the swallowtail model has three-dimensional control space (*u*, *v* and *w*) (Zhao, 2005) which is divided into different regions (Fig 1) (Since the behavior surface of swallowtail model has four dimensional hyper-surface, it is more difficult to visualize in a diagram, thus we use the bifurcation set, which is three dimensional, to explain swallowtail behavior of a system (Zeeman, 1976)). Piyaratne et al. (2013) has considered five basic regions as shown in Fig 1 in order to explain the swallowtail behavior of growth of wheat aphid population. Critical points which could initiate a catastrophe of the system could be on the bifurcation set itself when manipulative points cross the bifurcation set. The relevant regions were determined by plotting the equilibrium points in the three dimensional curved surface. The equilibrium points were derived with applying the both weather factor (overall effect) and a single weather variable (Temperature) separately using APHIDSim simulation model.

Results and Discussion

Analyzing the surface, the swallowtail behavior of population growth could be predicted for different cases of *u* (Piyaratne et al. 2013). The *u*, *v* and *w* values estimated by APHIDSim using the data collected by the Northwest A&F University and Meteorological Bureau in Yangling (www.ylqx.gov.cn) are furnished in Table 4 and 5 together with aphid population data. The underlying theory of estimating *u*, *v* and *w* values and other results (weather and predator data) are documented in detail in Piyaratne et al. (2013). Equilibrium points were plotted in graphs visualizing the Swallowtail behavior in three dimensional control spaces according to *u*, *v* and *w* values, and are shown in Fig 2 and Fig 3 respectively. The regions where the equilibrium points are positioned were observed and furnished in Table 3 and 4 (last column) together with equilibrium point values (*u*, *v* and *w*) estimated by Swallowtail model.

Table 4 *u*, *v* and *w* values estimated by swallowtail model with overall weather effect (Piyaratne et al., 2013)

Time instance	Aphid Population /100 stems	u	v	w	Region in the control space
1	7	1.58×10^{9}	3.44×10^{11}	7.31×10^{13}	IV
2	4	2.25×10^{9}	2.24×10^{12}	1.01×10^{14}	IV
3	13	1.71×10^{9}	4.98×10^{11}	1.44×10^{13}	IV

4	3	1.29×10^{10}	1.84×10^{11}	-7.53×10^{13}	IV/V
5	10	-1.23×10^{9}	-1.92×10^{11}	-1.83×10^{12}	II
6	39	-4.75×10^{9}	-1.92×10^{12}	-6.75×10^{13}	II
7	353	4.53×10^{9}	1.08×10^{12}	2.58×10^{13}	IV
8	1026	-1.72×10^{9}	-1.13×10^{12}	-4.69×10^{13}	II
9	3962	-2.62×10^{9}	-7.48×10^{12}	-3.68×10^{14}	II
10	3424	-1.22×10^{9}	-1.01×10^{13}	-5.14×10^{14}	II
11	28	1.82×10^{9}	1.39×10^{12}	5.95×10^{13}	IV
12	400	-1.76×10^{9}	1.33×10^{11}	1.84×10^{13}	II
13	248	1.48×10^{9}	1.44×10^{12}	6.43×10^{13}	IV
14	4572	-1.15×10^{9}	-4.01×10^{12}	-1.99×10^{14}	II
15	7736	9.40×10^{8}	1.09×10^{13}	5.58×10^{14}	IV

Catastrophe behavior can clearly be explained according to the equilibrium points in graphs in Fig 2 estimated with overall weather factor. According to Wei (2009) and Zhao (2005), whether the catastrophe occurs or not depends on the nature of equilibrium points when position crosses the curved surface of bifurcation section. If the point changes from the region IV and V to I or, from the region II to I, III, IV and V, the equilibrium may be changed. As a result the system will change from one stable equilibrium point to another or to an unstable region. In this situation the catastrophe may occur. The catastrophe may not occur if the point changes from the region I to II, IV and V or from the region III, IV and V to II (Wei, 2009; Zhao, 2005).

Table 5 *u*, *v* and *w* values estimated by swallowtail model with only the temperature as weather effect

Time instance	**Aphid Population /100 stems**	**u**	**v**	**w**	**Region in the control space**
1	7	-6.5×10^{9}	1.4×10^{12}	-6.2×10^{13}	II
2	4	-3.7×10^{9}	3.7×10^{12}	-1.8×10^{14}	II
3	13	-8.7×10^{9}	2.5×10^{12}	-1.2×10^{14}	II
4	3	-3.8×10^{9}	6.1×10^{10}	3.2×10^{12}	II
5	10	-2.5×10^{9}	4.1×10^{11}	-1.7×10^{13}	II
6	39	-2.0×10^{9}	8.2×10^{11}	-3.9×10^{13}	II
7	353	-3.1×10^{9}	7.6×10^{11}	-3.4×10^{13}	II
8	1026	-1.8×10^{9}	1.2×10^{12}	-6.0×10^{13}	II
9	3962	-1.8×10^{9}	5.2×10^{12}	-2.7×10^{14}	II
10	3424	-1.9×10^{9}	1.6×10^{13}	-8.2×10^{14}	II
11	28	-2.1×10^{9}	1.6×10^{12}	-8.1×10^{13}	II
12	400	-1.7×10^{9}	-1.3×10^{11}	9.5×10^{12}	II
13	248	-1.6×10^{9}	1.6×10^{12}	-8.0×10^{13}	II
14	4572	-1.4×10^{9}	5.0×10^{12}	-2.6×10^{14}	II
15	7736	-1.6×10^{9}	1.9×10^{13}	-9.8×10^{14}	II

(a) (b)

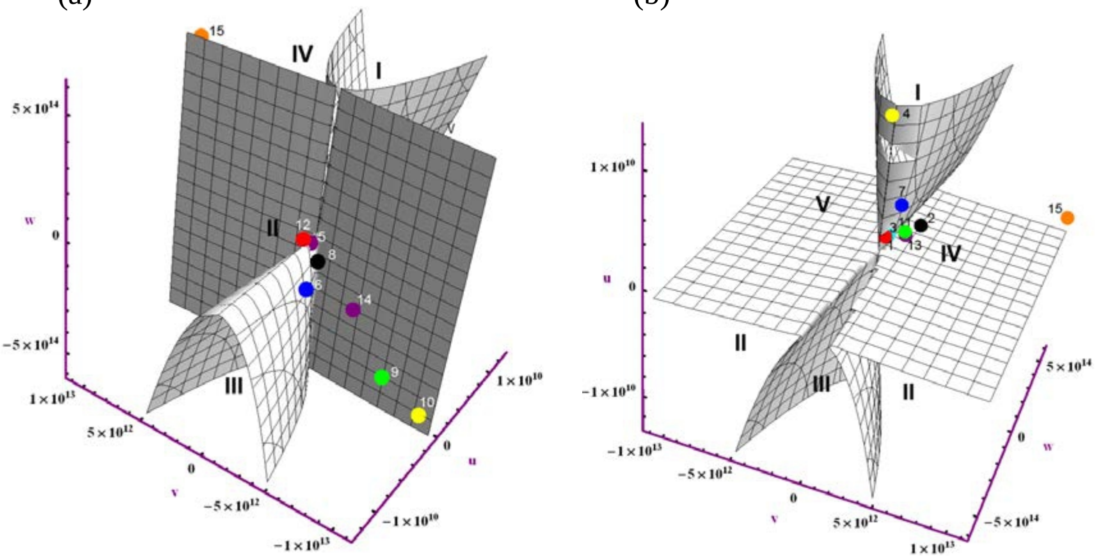

Figure 2. Three dimensional control space with plotted data points for data analysis with total weather effect. (a) points located in the region II; (b) points located in the region IV and V; points are labeled with the sample number, point colors are used to identify each point separately and therefore no any meaning related with colors used for points (Piyaratne et al., 2013)

(a) (b)

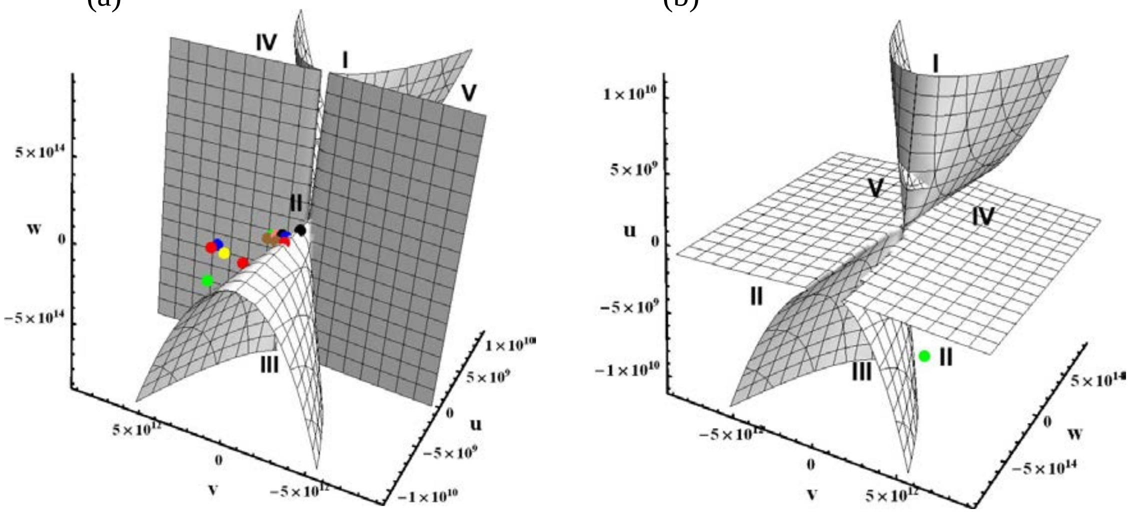

Figure 3. Three dimensional control space with plotted data points for data analysis with single weather variable. (a) all points are located in the region II; (b) None of the points can be seen in the region IV and V; points are labeled with the sample number, point colors are used to identify each point separately and therefore no any meaning related with colors used for points

The results in Table 4 show that catastrophe changes like sudden jump, occurs especially when the number of population change dramatically at 7th time instance to 8th and 10th time instance to 11th. Since a control measure was applied at 10th time instance, the aphid population change shows an irregular pattern and cannot predict with catastrophe regions where equilibrium points are positioned. In contrast, in Fig 3, there are not any observable point changes to explain the catastrophe behavior even the same population data were used to plot all graphs. It could also be explained clearly in Table 5 that all points are in the region of II, and it does not explain the catastrophe behavior. Therefore, the results from this study suggest that the overall weather factor is more closer to the real field conditions than a single weather variable when analyzing aphid population dynamics using catastrophe theory, especially for the swallowtail model.

Conclusion

A computer program based on Factor Analysis which could be used to analyze overall impact of weather parameters on aphid population dynamics has been developed and integrated into APHIDSim simulation model successfully. Simulated results of the program show that the catastrophe movements (sudden jumps) happening in growth of wheat aphid populations can clearly be explained by considering overall weather effect as the weather factor for population analysis models. It cannot be explained clearly using only the single weather variable as a factor. Thus we suggest that overall weather effect is more suitable for catastrophe theory applications in population dynamics analysis as the weather factor. Since the program has been developed as a separate object, using object oriented programming concept, it is able to integrate any other software which use the weather variables as a factor. Further the program provides positive evidence that it is possible to improve as a stand along program with including different kinds of variables to analyze the overall effects on subjected phenomena in addition to analyze weather variables on population dynamics of aphids.

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