AN APPROACH FOR MATHEMATICAL FORMULATION OF THE SUSTAINABLE ECONOMY

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Abstract

Assuming that the concepts included in the fundamental description level of the sustainable economy are the concepts of sum and direction, a model is formulated herein using a mathematical structure used by the equilibrium Macroscopic Thermodynamic Theory. For this formulation, the following items are considered: an open economy, with a public sector, which produces non-free (goods) and free (conveniences) goods and services. New concepts are introduced which, associated to macroeconomic variables, define a set of variables which are representative of each and every activity performed by the economy. Equilibrium states are conceptualized which an economy can achieve when it evolves in a sustainable manner and with certain restraints. The existence of a function, only defined for equilibrium states, is postulated which has a peak value when the economy achieves the highest welfare state. It is postulated that this function has mathematical properties that allow use the same mathematical tools of Thermodynamic Theory. Then, the model is applied to a simple economy and the equilibrium and equilibrium stability criteria are deduced. It is emphasized in the conclusions that the model provides a useful connection between the Microeconomics and Macroeconomics domains, and numerous equations that are useful for economic analysis; the obtained equilibrium criteria are compared to the traditional ones, and the economic content of the stability criteria is expressed.

Keywords: Sustainable, Economy, Mathematical, Model, Thermodynamics

Introduction

In the fundamental description level of a sustainable economy there is an arrow of time. An economy (or economic system) operates in a sustainable manner when it evolves increasing the welfare of its population. In order to achieve this objective, in addition to fulfill the necessary macroeconomic requirements, the system must ensure that the net potential availability of factors (NPAF) per inhabitant shall not diminish. The employment level and the assignment efficiency and use of these factors should not diminish either. The NPAF include all the potentially Productive Factors (the economic system´s human and capital resources) plus all the potentially imported factors (goods the system may import and could be used as factors) less the factors the economy use to produce export goods. The system must also ensure that its characterization shall not undergo any negative changes. For the purposes of this model, an economic system is sufficiently characterized by: i) a determined number of consumers; ii) a certain state of scientific and technological knowledge, for each and all of its activities; iii) a determined mean level of education, culture and public health; iv) an enforceable legal system to ensure peaceful coexistence and to regulate the interaction with other economies; v) a set of consumer preferences that determine the bundle of goods the system must produce to satisfy domestic and external demand; vi) a predisposition to produce unpriced public conveniences, and vii) the aspiration of consumers to enhance their wellbeing. On the other hand, the system must to ensure that exploitation of their natural resources is efficient; this can be achieved by maintaining renewable resources available and by making the necessary previsions for future generations to have the economic means for solving the problems that the exploitation of non-renewable natural resources might cause (WECD, 1987). In addition to the concept of direction, the other concept included in the fundamental description level of a sustainable economy is the concept of sum. These two concepts are the basis of the formulation of the equilibrium macroscopic thermodynamic theory. Consequently, by identifying the appropriate variables representing each and every activity of the economy and by conceptualizing ideal limit states, it is possible to formulate a model employing the mathematical structure used by thermodynamics. For the formulation of the model an open economy is considered having a public sector, which produces non-free (in short, goods) and free (in short, conveniences) goods and services. In order to identify the variables to be incorporated into the mathematical structure, for any instant during the evolution of the economy, the average economic availability per inhabitant is defined as $dep = (GDP+M-X)/N$. This variable, which has dimensions of [money/time.inhabitant], allows for introduction of the concept of average effort associated to achievement of a physical unit of each good produced by the economy. Thus, for example, the average effort associated to achievement of a physical unit of good j is equal to (z_j /dep), wherein z_j is the price of good j. The dimension of this new concept is [(time.inhabitant)/physical unit of good j]. Moreover, a limit state can be conceptualized as an ideal state that might be achieved by a system having a certain characterization, a certain NPAF and evolving in a sustainable manner, but restricted to having a certain factor assignment. With these concepts, the model is formulated, mathematical relations resulting from this formulation are deduced, the model is applied to a simple economic system, and the equilibrium and equilibrium stability criteria are deduced.

Model Formulation

An economy operating in a sustainable manner, having a certain characterization and a certain NPFA per capita, could achieve different limit states for different factor assignments. These states, conceptualized as ideal, will be referred to as *sustainable equilibrium states*, SES. In each of these states, the economy makes full use of its factors, operates in a sustainable manner, with a certain factor assignment, with no externalities and with an optimal tax policy. Under these conditions, the economy achieves an equilibrium state by maximizing the usefulness of all its agents and ensuring that the consumption of goods and conveniences is in equilibrium. If the restriction to operate with a particular factor assignment is eliminated, the sustainability condition will cause the economy to evolve to the state of maximum possible economic welfare. This ideal state, which belongs to the set of SES, to be called the *State of Maximum Sustainable Economic Well-being*, here abbreviated SEW. In this state, the economy adopts an optimal factor assignment and uses these factors with maximum efficiency for the available technologies.

When a system reaches a SES, its NPFA is described by a unique set of quantity and quality properties that, in a unit of time, the economy employs to satisfy its domestic demand. If the economy uses t number of distinct productive factors and it use an m variety of imported products, this set can be represented by the vectors F = ($F_1,\ldots,F_t$) y P[imp] = ($P_{1,imp},\ldots,P_{m,imp}$). F is a vector representing the productive factors allocated to satisfy domestic demand, while P[*imp*] is a vector representing the imported products allocated to satisfy the domestic demand. The coordinates that define these two vectors can be grouped in a unique variable that is expressed in *units of factor* per unit of time symbolized F. The unit of factor (*uf*) is defined as the amount of physical units of each and every factor employed to produce a "universal consumption basket" which contains different amounts of physical units of each and every good j produced in the economy. For each good j, the amount contained in this basket is the average amount consumed by a consumption unit, predetermined in each case, in the time unit selected to define the flow variables. Then, if $x_{r,j}$ and $x_{i,j,imp}$ are the amounts of physical units of the factors r and i used to produce a physical unit of j, and c_j is the amount of physical units of product *j* contained in the universal consumption basket, the factor unit is given by:

$$uf = \sum_{j=1}^{n}\sum_{r=1}^{t} c_j \cdot x_{r,j} + \sum_{j=1}^{n}\sum_{i=1}^{m} c_j \cdot x_{i,j,imp}$$

If for a SES, w_r and $z_{i,imp}$ are the prices of a physical unit of the productive factor *r* and the imported factor *i* respectively, then the price of the unit of factor is:

$$w = \sum_{j=1}^{n}\sum_{r=1}^{t} w_r \cdot c_j \cdot x_{r,j} + \sum_{j=1}^{n}\sum_{i=1}^{m} z_{i,imp} c_j \cdot x_{i,j,imp}$$

It is then possible to calculate the quantity of the units of factor per unit of time as F =A/w, where A = GDP + (M – X).

The definition of (dep) and the quantification of the concept of average effort, establishes the concept of the economic intensity of factors or products, defined as the fraction of all the consumers that are either associated with the use of a certain quantity of factors, or produce a certain quantity of products per unit of time. For example, the economic intensity of the factor units the economy uses per unit of time to satisfy its domestic demand is equal to [(w/dep).F], and the economic intensity for the production of good j is equal to $[(z_j/dep).P_j]$.

When the economy reaches a SES, there is a complete use of its NPFA. In this situation, it is possible to describe the SES by enumerating a set of variables that are associated to different activities. Given that each SES has a given characterization, a given NPFA and an a-priori efficiency in the allocation and use of factors, one of the possible descriptor set is (F, F^k, P, F^{bg}), where F^k is the quantity of units of factor used to performs those task needed to growth, P is a vector whose coordinates represents the total quantity of physical units of product per unit of time of each and all final priced goods *j* produced by the system to satisfy the population demands, and F^{bg} is the quantity of units of factor used to generate unpriced conveniences. If in this description, only the population (*N*) changes, then the variables (F, F^k, P, F^{bg}) change in the same proportion, thus these variables are extensive variables.

Postulate 1: In an economy that makes complete use of all of its NPFA the SES is fully described by the set

$(F, F^k, P_1,\ldots, P_n, F^{bg}) = (F, F^k, P, F^{bg})$

In the set (F, F^k, P, F^{bg}) there are not included those variables related to global conservation. Therefore, the economic intensity of the output of the activities of global conservation is a function of (F, F^k, P, F^{bg}). When an economy reaches its SEW, it achieves a state of maximum efficiency and minimizes the economic effort to achieve its economic objectives; therefore, the economic intensity of global conservation activities is at a minimum and the economy minimizes the average effort to produce a unit of each and all goods and conveniences.

Postulate 2: There exists a function, arbitrarily called Yala (Ya), that is a function of the extensive variables (F, F^k, P, F^{bg}), that is defined for all the SES of any economy and has the property that the values adopted by the unconstrained variables $(P_1,\ldots,P_n)$, when the economy is in the SEW, are those that minimize the Yala function for the system's characterization, the NFPA and the magnitude of F that define that SEW.

The set of the points of the space $R^{(n+3)}$ for which the function is defined, conform a continuous and differentiable manifold. Also, when F increases and (F^k,P,F^{bg}) remains constant, the Yala function necessarily increases.

Postulate 3: The Yala function (Y^a) is a homogenous first order function in each and all its extensive variables, it is also a continuous and differentiable function and is a monotonically increasing function of F.

The partial derivatives of Yala are the intensive parameters of the economy and are defined by the following equations:

$$\left.\frac{\partial Y^a}{\partial F}\right)_{F^k,\mathbf{P},F^{bg}} \hat{=} \frac{w}{dep} = \tilde{w} \qquad \left.\frac{\partial Y^a}{\partial F^k}\right)_{F,\mathbf{P},F^{bg}} = -\tilde{w}$$

$$\left.\frac{\partial Y^a}{\partial P_j}\right)_{F,F^k,P_{k(\forall k\neq j)},F^{bg}} \hat{=} -\frac{z_j^{mo}}{dep} = -\tilde{z}_j^{mo} \qquad \left.\frac{\partial Y^a}{\partial F^{bg}}\right)_{F,F^k,\mathbf{P}} \hat{=} -\tilde{w}$$

The variables that symbolize the value of one physic unit of each and all goods *j* are expressed in [monetary units/physical units of product]. For each final good *j* the symbol has a supra index (*mo*) to highlight that is determined by the sum of the prices of those factors employed in the production of that final good *j*, that depends exclusively on the production method utilized to obtain it. These variables include all direct factors, and only the indirect factors that depend on the production method employed as provision for human resources, replacement for depreciated capital goods allocated to the production of the *j*-good, producers costs to ameliorate the environmental impact of the production method used to obtain *j*, and provisions necessary to compensate the depletion of the non-renewable natural resources used by the product *j*. Vector z[*mo*] represents the price vector of those products destined to satisfy the population's demands.

The Yala function contains all the economic information to describe the system and will be called the fundamental relation of the economy. Its first differential is:

$$dY^a = \tilde{w}\cdot dF - \tilde{w}\cdot dF^k - \sum_{j=1}^{n} \tilde{z}_j^{mo}\cdot dP_j - \tilde{w}\cdot dF^{bg}$$

Postulate 3 allows derivation of an alternative form of the fundamental relation:
$F = F\,(Y^a, F^k, \mathrm{P}, F^{bg})$

This equation is the fundamental relation for an economy represented in *F* and contains all the information necessary to specify the SES.

Since the Yala function is monotonically increasing of *F*, the minimum condition specified in Postulate 2, implies a maximum for *F* when the economy reaches the SEW. This implies that the SEW is a state where Y^a has minimum when *F* is constant or *F* has maximum when Y^a is constant.

The Yala function is homogeneous of first order in all and each variable; therefore for any *u*

$$Y^a(u\cdot F, u\cdot F^k, u\cdot\mathbf{P}, u\cdot F^{bg}) = u^1\cdot Y^a(F, F^k, \mathbf{P}, F^{bg})$$

Particularly for $u = 1$

$$Y^a\left(F, F^k, \mathbf{P}, F^{bg}\right) = \tilde{w}\cdot F - \tilde{w}\cdot F^k - \tilde{\mathbf{z}}[mo]\cdot\mathbf{P} - \tilde{w}\cdot F^{bg}$$

A similar analysis for the equivalent fundamental relation on F results in:

$$F\left(Y^a, F^k, \mathbf{P}, F^{bg}\right) = \frac{1}{\tilde{w}}\cdot Y^a + F^k + \frac{\tilde{\mathbf{z}}[mo}{\tilde{w}}\cdot\mathbf{P} + F^{bg}$$

The last two equations are an application to economics of the Euler Theorem on homogenous first order forms.

To analyse the evolution of an economy through SES, it may be convenient to cast the fundamental relation as a function of other variables, for example, as a function of intensive variables. The Legendre Transformations provide the adequate formalism to transform the fundamental relation in terms of the conjugated variables and to obtain new functions, conserving all information. For example, to obtain a new function expressed as a function of the corresponding conjugated variable as the independent variable instead of *F*, then, the Legendre transformation is:

$$Y^a_{\tilde{w}}\left(\tilde{w}, F^k, \mathbf{P}, F^{bg}\right) = Y^a - \tilde{w}\cdot F$$

With the ceteris paribus condition that only F and P_j can change, the others transformed functions useful to analyse the economy evolution are:

$$Y^a_j\left(F, F^k, P_1, ..., P_{j-1}, \tilde{z}^{mo}_j, P_{j+1}, ..., P_n, F^{bg}\right) = Y^a - \left(-\tilde{z}^{mo}_j\right)\cdot P_j$$

$$Y^a_{\tilde{w},j}\left(\tilde{w}, F^k, P_1, ..., P_{j-1}, \tilde{z}^{mo}_j, P_{j+1}, ..., P_n, F^{bg}\right) = Y^a - \tilde{w}\cdot F - \left(-\tilde{z}^{mo}_j\right)\cdot P_j$$

It is important to note that when F^k, $P_1, \ldots, P_n$, and F^{bg} are transformed, the resulting Legendre´s transformation is equal to the population of the system (N):

$$Y^a_{k,1,...,n,bg}\left(\tilde{w}, \tilde{z}^{mo}_1, ..., \tilde{z}^{mo}_n\right) = Y^a + \tilde{w}\cdot\left(F^k + F^{bg}\right) + \sum_{j=1}^{n}\left(\tilde{z}^{mo}_j \cdot P_j\right) = N$$

This Legendre transformation, expresses the sum of the economic intensity of each and all the activities the system execute.

Because the fundamental relation is a function of [q = (n+3)] variables, there are [$2^{(q)}$-1] Legendre transformations possible. The Yala function and its transformed functions are called Potential Functions.

Similar analysis for the fundamental relations on F, allow one to obtain the legendre transformations of F function.

Given that potential functions have exact differentials and its partial derivatives also are continuous, it is possible to find a set of mathematical relations using the property that for this class of functions the mixed second derivatives are equal. For example, with the condition that only F and P_j can change it is possible to obtain the following reciprocity relations:

$$\left.\frac{\partial \tilde{w}}{\partial P_j}\right)_{F, F^k, P_{k(\forall k\neq j)}, F^{bg}} = -\left.\frac{\partial \tilde{z}^{mo}_j}{\partial F}\right)_{F^k, \mathbf{P}, F^{bg}} \qquad -\left.\frac{\partial \tilde{z}^{mo}_j}{\partial \tilde{w}}\right)_{F^k, \mathbf{P}, F^{bg}} = \left.\frac{\partial F}{\partial P_j}\right)_{\tilde{w}, F^k, P_{k(\forall k\neq j)}, F^{bg}}$$

$$\left.\frac{\partial P_j}{\partial F}\right)_{\tilde{z}^{mo}_j, F^k, P_{k(\forall k\neq j)}, F^{bg}} = \left.\frac{\partial \tilde{w}}{\partial \tilde{z}^{mo}_j}\right)_{F, F^k, \mathbf{P}, F^{bg}} \qquad \left.\frac{\partial P_j}{\partial \tilde{w}}\right)_{\tilde{z}^{mo}_j, F^k, P_{k(\forall k\neq j)}, F^{bg}} = -\left.\frac{\partial F}{\partial \tilde{z}^{mo}_j}\right)_{\tilde{w}, F^k, \mathbf{P}, F^{bg}}$$

Modelling a simple Sustainable Economy

Consider an economy defined, as before, by a certain characterization, PFA and operating in the SEW. This economy has N inhabitants, t productive factors, h firms producing two priced products A and B to satisfy the demand of its population, p firms producing the x products it exports, and imports a variety of m different products. Also, this economy has a public sector that ensures (i) the economy operates sustainably, (ii) supply the system's conveniences and (iii) performs those tasks needed to improve the social infrastructure.

Let the economy be displaced marginally from a SEW point, for which the only possible changes are the quantity of units of the A and B products. Because the characterization, the NPFA and F do not change, the variable F^k and F^{bg} keep constant. Them, from the first differential of Yala function and the statement of Postulate 2 can be derived the following expressions:

$$-\left.\frac{\partial P_B}{\partial P_A}\right)_{Charact., NPFA, F} = \frac{\tilde{z}^{mo}_A}{\tilde{z}^{mo}_B} = \frac{z^{mo}_A}{z^{mo}_B}(I) \qquad -\left.\frac{\partial C_B}{\partial C_A}\right)_{Charact., NPFA, F} = \frac{\tilde{z}^{mo}_A}{\tilde{z}^{mo}_B} = \frac{z^{mo}_A}{z^{mo}_B}(II)$$

Equation (I) is the micro economical relation known as the relation of products transformation, or the marginal rate of transformation, from B to A. Similarly, equation (II) is the micro economical relation known as the relation of products substitution, or the marginal rate of substitution, from B to A.

The proposed formulation allow a different path to obtain equations (*I*) and (*II*). At any point in time, the sum of the *GDP*, associated to potential productive factors availability, and the [*M* – *X*], associated to the potential factors availability that result from international transactions not related to goods exchange, is the price of the endowments available to the system. This result must be equal to the monetary sum of the total domestic consumption (that includes all end-goods, conveniences and the amount of money allocated to conserve the system as a whole) plus the system's total net savings, S_t. This balance can be write in terms of economic intensity as:

$$\frac{GDP+\left(M-X\right)}{dep}=\tilde{z}_A^{mo}\cdot P_A+\tilde{z}_B^{mo}\cdot P_B+\frac{T+O_{up}}{dep}+\frac{S_t}{dep}$$

$$\frac{GDP+\left(M-X\right)}{dep}=\tilde{z}_A^{mo}\cdot C_A+\tilde{z}_B^{mo}\cdot C_B+\frac{T+O_{up}}{dep}+\frac{S_t}{dep}$$

Where T is the amount of money allocated by government to public sector units' direct and indirect factors. Similarly O_{up} is the amount of money allocated by non-government public sector units. In the equations above it is valid to replace:

$$\frac{GDP+\left(M-X\right)}{dep}=N \qquad \frac{T+O_{up}}{dep}=Y^a+\tilde{w}\cdot F^{bg} \qquad \frac{S_t}{dep}=\tilde{w}\cdot F^k$$

Then

$$dN=d\left(\tilde{z}_A^{mo}\cdot P_A+\tilde{z}_B^{mo}\cdot P_B\right)+dY^a+d\left[\tilde{w}\cdot\left(F^k+F^{bg}\right)\right]$$

$$dN=d\left(\tilde{z}_A^{mo}\cdot C_A+\tilde{z}_B^{mo}\cdot C_B\right)+dY^a+d\left[\tilde{w}\cdot\left(F^k+F^{bg}\right)\right]$$

At the SEW the economic effort associated with one unit of each and all priced end-goods (*A* and *B*) are minimum. For the marginal displacement considered above N, F^k, F^{bg} and the average effort asociated to the unit of factor remains constant and the expressions (*I*) and (*II*) are also obtained from the last diferential equations.

As for the total system, at the SEW it is possible to write the balance in terms of economic intensity for any agent of the economy. For the *j*-consumer this balance is:

$$\tilde{w}\cdot F_j=\tilde{z}_A^{mo}\cdot C_{A,j}+\tilde{z}_B^{mo}\cdot C_{B,j}+Y_j^a+\tilde{w}\cdot\left(F_j^k+F_j^{bg}\right)$$

F_j is the amount of units of factor that have the agent *j*. The variables in brackets in the last equation symbolise the contribution in units of factor of agent *j* to improve the social infrastructure and to the production of conveniences, respectively. Then, for the marginal displacement considered above, for every $j = 1,\ldots,N$ by differentiation of the last equation:

$$-\left.\frac{\partial C_{B,j}}{\partial C_{A,j}}\right)_{Charact.,NPFA,F}=\frac{\tilde{z}_A^{mo}}{\tilde{z}_B^{mo}}=\frac{z_A^{mo}}{z_B^{mo}} \quad (III)$$

The same analysis applies to the *k*-firma, for every $l = 1,\ldots,h$. For these cases the balance in terms of economic intensity is:

$$\tilde{w}\cdot F_l=\tilde{z}_A^{mo}\cdot P_{A,l}+\tilde{z}_B^{mo}\cdot P_{B,l}+Y_j^a+\tilde{w}\cdot\left(F_l^k+F_l^{bg}\right)$$

As for consumer agent, for the marginal displacement considered above, by differentiation of the last equation:

$$-\left.\frac{\partial P_{B,l}}{\partial P_{A,l}}\right)_{Charact.,NPFA,F}=\frac{\tilde{z}_A^{mo}}{\tilde{z}_B^{mo}}=\frac{z_A^{mo}}{z_B^{mo}} \quad (IV)$$

For the case under consideration, equations (*III*) y (*IV*) allow to deduce that, at the SEW, the marginal rate of transformation for all the firms and the marginal rate of substitution for all consumers are equal.

In the space P_B versus P_A, the SEW is represented by a point. Consider a finite virtual displacement from the SEW to other points where the economy can only change the P_B and P_A quantities, thus there is a graph that represents the set of the feasible outputs of the products *A* and *B* an economy produces using efficiently the unique set of factors it allocates to this task. This output set is the production possibility frontier. The SEW is a unique point in this frontier and is the point of maximum efficiency for an economy that has achieved an optimum and sustainable level of economic wellbeing for all its inhabitants. Also the SEW is the unique point where it is possible to calculate the slope of the production possibility frontier graph. To calculate this slope let f_r be the amount of physic units of the *r*-productive factor used to produce *A* and *B*; and f_i the amount of physic units of the i-factor imported to produce *A* and *B*, then

$$f_r = x_{r,A} \cdot P_A + x_{r,B} \cdot P_B \qquad\qquad f_i = x_{i,A} \cdot P_A + x_{i,B} \cdot P_B$$

In these equations $x_{r,A}$ and $x_{r,B}$ are the quantities of physical units of r-productive factor used in the production of one physical unit of *A* and *B* respectively, and $x_{i,A}$ and $x_{i,B}$ are the quantities of physical units of *i*-imported factor used in the production of one physical unit of *A* and *B* respectively. For this simple economic system, the SEW is technically efficient if and only if $x_{r,A}$, $x_{r,B}$, $x_{i,A}$ and $x_{i,B}$ are minimum (Koopmans (1957). By differentiation of the equations

$$0 = x_{r,A} \cdot dP_A + x_{r,B} \cdot dP_B \quad (V) \qquad\qquad 0 = x_{i,A} \cdot dP_A + x_{i,B} \cdot dP_B \quad (VI)$$

Multiplying (*V*) by w_r and adding for all *j*, and multiplying (*VI*) by $z_{i,imp}$ and adding for all *i*, and adding them:

$$\left(\sum_{r=1}^{t} x_{r,A} \cdot w_r + \sum_{i=1}^{m} x_{i,A} \cdot z_{i,imp} \right) \cdot dP_A + \left(\sum_{r=1}^{t} x_{r,B} \cdot w_r + \sum_{i=1}^{m} x_{i,B} \cdot z_{i,imp} \right) \cdot dP_B = 0$$

Since the bracketed terms in last equation are equal to z_A and z_B, then at SEW the slope is:

$$\left. -\frac{\partial P_B}{\partial P_A} \right)_{SEW} = \frac{\tilde{z}_A^{mo}}{\tilde{z}_B^{mo}} = \frac{z_A^{mo}}{z_B^{mo}}$$

The last equation expresses the same as equation (*I*).

Conventionally the stability is analysed upon the bases that the set bounded above by the production possibility frontier, is a convex set. The proposed formulation allows a new path to obtain the criterion of stability for the SEW, from the second order minimum condition for the Yala function. Thus, the stability of the simple system under consideration can be analysed using the formalism of Tisza (1950). In this way it is possible to obtain the following stability conditions:

$$\left. \frac{\partial \tilde{w}}{\partial F} \right)_{P_A, P_B} > 0 \quad (VII) \qquad\qquad \left. -\frac{\partial \tilde{z}_B^{mo}}{\partial F} \right)_{\tilde{w}, P_A} > 0 \quad (VIII)$$

The stability criterion posed by equation (VII), means that: as the aim of the inhabitants is to improve their welfare, those that belong to the subsystem with lower endowment of factors (the poorest) are going to migrate to the subsystem with the highest value of the average effort associated to one unit of factor. The second stability criterion, as expressed in equation (VIII) means: if the average effort associated with the production of one physical unit of *B*-product increases then the production of *B*-product must to decrease.

Conclusion

The formulated model derives microeconomic descriptors from macroeconomic analysis and, therefore, the model provides a useful linkage between both domains. Equations (*I*), (*II*), (*III*) and (*IV*) are the microeconomic relations that provide the conditions with a Pareto´s optimum allocation (Pareto, 1909) must comply with, and they have been deduced from the proposed model. The concepts introduced yield themselves to the mathematical formalism of Legendre transformations, Euler theorem, minimum and maximum first and second order conditions, and relations of reciprocity, then making possible to derive a large number of potential functions and mathematical relations that provide a fertile ground for economic analysis. Finally, as shown in the example, this new characterization of equilibrium state expressed in the Second Postulate, allow to obtain two stability criteria whose economic content is: for a system with a given characterization, the spontaneous process induced by a deviation from the SEW is in a direction to restore the system to the SEW.

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