THE EVALUATION OF STOCHASTIC MODELING IN THE CONTEXT OF INSURANCE COMPANIES THROUGH THE ILLUSTRATION OF BRANCHING PROCESS

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Abstract

Choice of stochastic modeling attempts to model the decision process of an individual or segment in a particular context. Choice models are able to predict with great accuracy how individuals would react in a particular situation. Discrete choice models determine the probability that a decision maker will take a certain action based upon several attributes and choices. This study provides evidence on the evaluation of an individual's choice of alternative Insurance Companies. We have used some discrete choice stochastic models to assess the factors which influence on the evaluation of stochastic modeling in the context of Insurance Companies. Branching process model is used to construct a complete picture of Insurance companies' activities in terms of their earning pattern and subscriber's enrollment in their business.

One of the most important factors in modeling of Insurance companies' activities is to forecast the future profit and number of client's involvement to know the long run performance of the company. This information can be extract by stochastic modeling. Especially for Branching process we consider Rupali Life Insurance Company. The branching process is introduced and evaluated to get idea of the distribution of client's enrolment as well as it is also a stochastic population model based on explicit descriptions of individual life and reproduction .This model is also used in characterizing the distribution of the size of the population for different generations and the probability of extinction of the population.

Keywords: Introduction, Literature review, Illustration of Branching process (Data structure, Tree diagram for different generation), Probability of ultimate extinction.

1. Introduction:

A Branching process is a mathematical description of the growth of a population for which the individual produces off-springs according to stochastic lows. A typical process is the following form.

The Branching process was proposed by Galton and the probability extinction was first obtained by Watson by considering the probability generating function for the number of children in the n^{th} generation. This mathematical model was known as the Galton-Watson Branching Process and had been studied thoroughly in literature. (See, Jagers, 1975)

In probability theory, a branching process is a Markov process that models a population in which each individual in generation n produces some random number of individuals in generation n + 1, according to a fixed probability distribution that does not vary from individual to individual. Branching processes are used to model reproduction; for example, the individuals might correspond to bacteria, each of which generates 0, 1, or 2 offspring with some probability in a single time unit. Branching processes can also be used to model other systems with similar dynamics, e.g., the spread of surnames in genealogy or the propagation of neutrons in a nuclear reactor.

A central question in the theory of Branching processes is the probability of ultimate extinction, where no individuals exist after some finite number of generations. It is not hard to show that, starting with one individual in generation zero; the expected size of generation equals μ^n where μ is the expected number of children of each individual. If μ is less than 1, then the expected number of individuals goes rapidly to zero, which implies ultimate extinction with probability 1 by Markov's inequality. Alternatively, if μ is greater than 1, then the probability of ultimate extinction is less than 1 (but not necessarily zero; consider a process where each individual either dies without issue or has 100 children with equal probability). If μ is equal to 1, then ultimate extinction occurs with probability 1 unless each individual always has exactly one child. In theoretical ecology, the parameter μ of a branching process is called the basic reproductive rate.

2. Literature Review:

2.1 Branching model

Consider an organism, cell, particle or an individual, whose life time has length one unit. At the end of its life time, it produces a random number ξ of off-spring where

$$P[\xi = k] = a_k, (k = 0,1, \dots), (\sum a_k = 1). \quad (2.1)$$

Each one of the off-spring at the end of its life time produces off-springs the number of which has the same distribution.

Let X_n be the number of particles at the end of the n^{th}generation. We shall take $X_0 = 1,$ since there is only one individual initially. The number of particles in the $(n + 1)^{th}$ generation are the off-spring of those of the n^{th} generation. If $X_n = i$ and ξ_i denote the off-spring of the i individuals.

$$P[X_{n+1} = j|\, X_{n1} = i_1, X_{n2} = \ i_2, \dots X_n = i], \qquad n_1 < n_2 < \cdots < n,$$
$$= P[X_{n+1} = j|\, X_n = i],$$
$$=P[\xi_1+\xi_2 + \cdots \xi_i = j] \quad (2.2)$$

This equation depends on i and the off-spring distribution $\{a_k\}$. $\{X_n\}$ is Markov chain with state space (0,1,2,…).

The model as given above is called Galton-Watson branching process. (See, Baht, 2000)

2.2 Assumptions of Branching process

Even though some of the assumptions such as independent identical off-spring distributions, on-overlapping of generations, are stringent, because of the simplicity of the model. Branching process has found applications in many areas. (See, Baht, 2000)

I. The number of individuals initially present, denoted by $X_0 = 1$ is called the size of the 0^{th}generation.
II. All offspring of the 0^{th} generation constitute the first generation and their number is denoted by X_1.
III. In general $\{X_n\}$ denote the size of the n^{th} generation and follows that $\{X_n, n = 0, 1, \dots\}$ is a Markov chain having as its state space that the set of non negative integers.
IV. To find the characteristics of the distribution of the population for different generation.
V. To find the probability of extinction of the population.

2.3 Mean and Variance of the branching process

For $X_0 = 1$, m = E (X_1) = E (k) = A' is the off-spring mean. Let $\sigma^2 = \mathrm{Var}(X_1) = \mathrm{Var}(k)$ be its variance. Then

$$E\,(X_{n+1}) = \emptyset'_{n+1}(1) = \emptyset'_n(A(s))A'|_{s=1} = m^{n+1}\ ;n=1,2,\dots \quad (2.3)$$

If X_0 = i, then E (X_{n+1}) = i m^{n+1}.

Variance of X_n is

$V(X_n) = \emptyset''_n(1) + \emptyset'_n(1) - [\emptyset'_n(1)]^2$, thus

$V(X_n) = \sigma^2 m^{n-1}\left(\frac{m^n-1}{m-1}\right)$, if $m \neq 1$

$= n\sigma^2$, if m=1 (2.4)

If X_0 = i, then E (X_n) = i m^n.

$V(X_n) = i\sigma^2 m^{n-1}\left(\frac{m^n-1}{m-1}\right)$, if $m \neq 1$

$= in\sigma^2$ If m=1. (2.5)

2.4 Determining the Probability of Extinction of the Population

Let π_0 denote the probability of die out (under the assumption that $X_0 = 1$). More formally,

$$\pi_0 = \lim_{n\to\infty} P\{X_n = 0|X_0 = 1\}$$

The problem of determining the value of π_0 was first raised in connection with the extinction of family surnames by Galton in 1889.

We first note that $\pi_0 = 1$ if $\mu < 1$.this follows since

$\mu^n = E(X_n) = \emptyset'_{n+1}(1) = \emptyset'_n(A(s))A'|_{s=1}$, n = 1, 2,…

Since $\mu^n \to 0$ when $\mu < 1$,

Then the Probability of Extinction of the Population is given by

$$P\{X_n = 0\} \to 1$$

In fact it can be shown that $\pi_0 = 1$, when $\mu = 1$. When $\mu > 1$, it turns out that $\pi_0 < 1$, and an equation determining π_0 may be derived by conditioning on the number of offspring of the initial individual as follows

$$\pi_0 = \sum_{k=0}^{\infty} P\{\text{Populationdisout} \mid X_1 = k\}P_k$$

No, given that $X_1 = k$, the population will eventually die out if and only if each of the k families started by the members of the first generation eventually dies out.

Since each family is assumed to act independently and since the probability that any particular family dies out is just π_0, this yields

$$P\{\text{Populationdisout} \mid X_1 = k\} = \pi_0^{\ k}$$

And thus π_0 satisfies

$\pi_0 = \sum_{k=0}^{\infty} \pi_0^{\ k} P_k$ (2.6)

In fact when $\mu > 1$, it can be shown that π_0 is smallest positive number.

3. Result and Analysis

3.1 Data structure

This data structure is taken from Rupali Life Insurance Company (RLIC).Which shows that how many individuals are involved in this company under one individual to another individual as a branching way. In the following figure empty circle indicates that individuals are further included and on the other hand cross circle indicates that individuals are not further included.

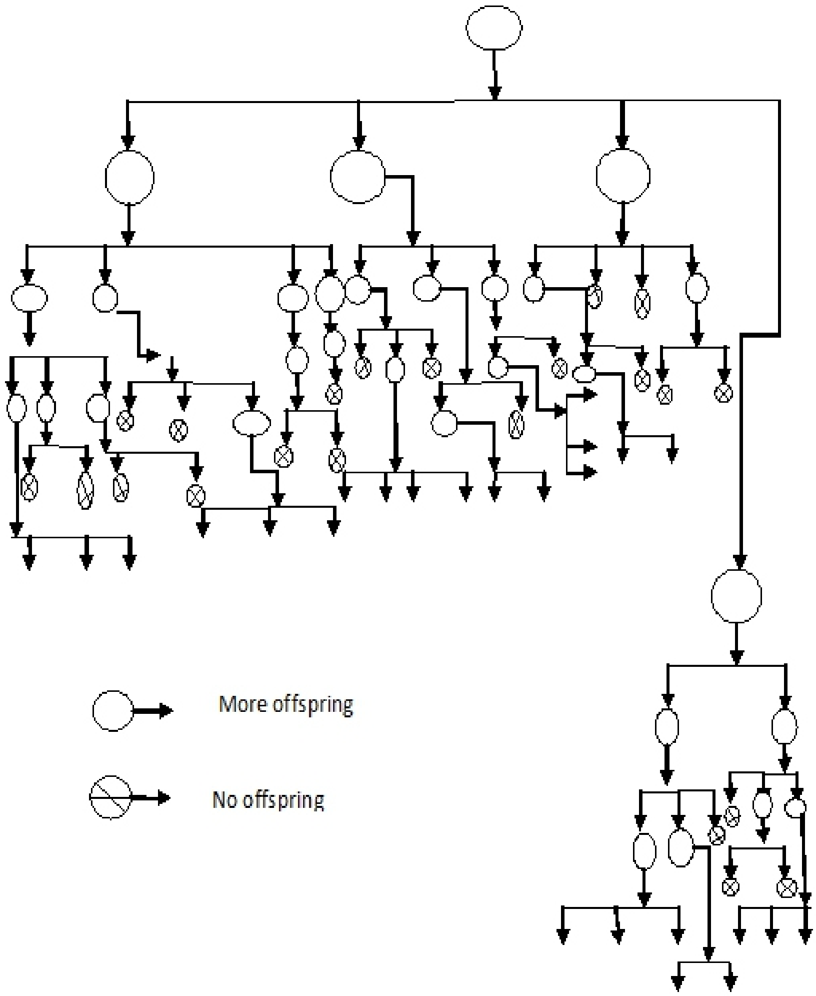

***Figure 3.1**: Data structure of involved individuals in Rupali Life Insurance Company.*

Table 3.1: Total number of clients involved under each immediate senior client

S.N. of clients	Subscribed no. of clients	S.N. of clients	Subscribed no. of clients	S.N. of clients	Subscribed no. of clients	S.N. of clients	Subscribed no. of clients	S.N. of clients	Subscribed no. of clients
1	4	11	0	21	0	31	2	41	0
2	4	12	0	22	2	32	0	42	0
3	3	13	0	23	3	33	0	43	0
4	4	14	1	24	3	34	4	44	3
5	2	15	1	25	3	35	0	45	2
6	3	16	3	26	2	36	2	46	0
7	0	17	2	27	2	37	0	47	0
8	3	18	2	28	0	38	3	48	2
9	0	19	2	29	0	39	0	49	3
10	0	20	0	30	2	40	2	50	2

Table 3.2: Distribution of offspring of the surveyed individual involved in Rupali Life Insurance Company.

No. of offspring (X)	Frequency	Probability (P_i)
0	20	0.4
1	2	0.04
2	14	0.28
3	10	0.2
4	4	0.08
Total	**50**	**1**

Mean of the down line clients (offspring) is calculated below:

$$\mu = E(X_1) \quad (3.1)$$

$$= \sum_{k=0}^{4} kp_k$$

$$= 0 \times p_0 + 1 \times p_1 + 2 \times p_3 + 3 \times p_3 + 4 \times p_4$$

$$= 0 \times 0.4 + 1 \times .04 + 2 \times .28 + 3 \times .20 + 4 \times .08 = 1.52$$

Hence, the variance is;

$$\sigma^2 = var(X_1) = E\left(X_1{}^2\right) - \{E(X_1)\}^2 \quad (3.2)$$

$$= (0^2 \times p_0 + 1^2 \times p_1 + 2^2 \times p_2 + 3^2 \times p_3 + 4^2 \times p_4) - (\mu)^2$$

$$= (0^2 \times .4 + 1^2 \times .04 + 2^2 \times .28 + 3^2 \times .20 + 4^2 \times .08) - (1.52)^2$$

$$= 1.93$$

Since our calculated mean is μ=1.52 which is greater than 1, hence we can say that our obtaining Branching process is supercritical.

3.3 Different Generations Structure of down line Clients

At the first step, we can observe the 1st generation client's structure by tree diagram in branching process.

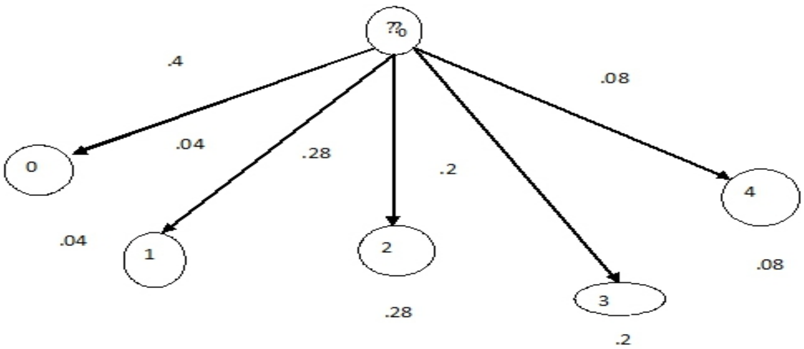

***Figure 3.2**: Tree diagram of the first generation offspring distribution with their corresponding probabilities*

Figure 3.2 represents that there is maximum chance (40%) to enroll zero clients followed by 28% with two clients, 20% with three clients in the first generation and only 4% associate with one client.

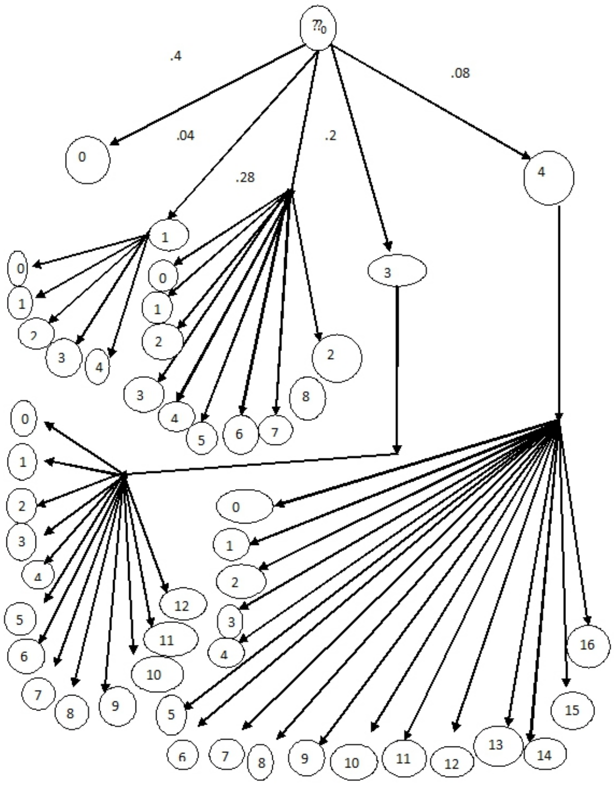

***Figure 3.3**: Tree diagram for the second generation of the offspring distribution with their corresponding probabilities.*

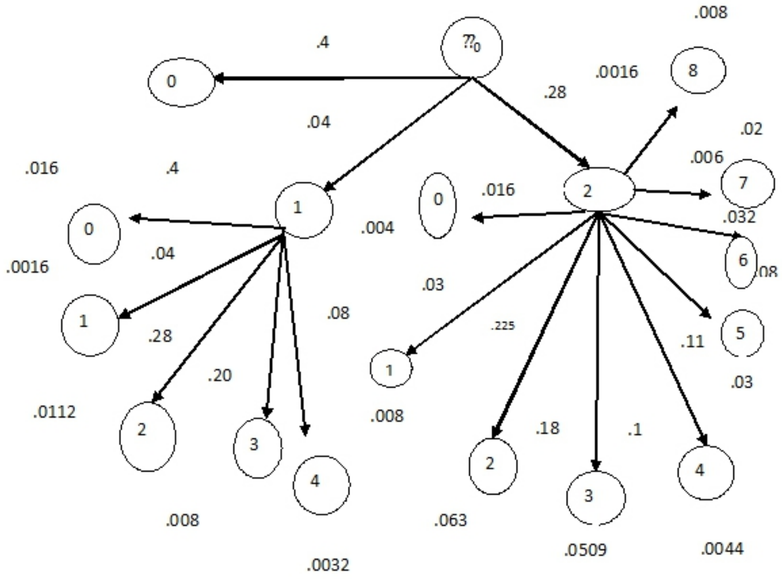

***Figure 3.4**: Tree diagram for the second generation of the offspring distribution for one and two individuals with their corresponding probabilities.*

Table 3.3: Probability of offspring distribution for the second generation for one individual:

Probability(p_i)	Probability values of number of i^{th}offspring
p_0	.016
p_1	0.0016
p_2	0.112
p_3	0.008
p_4	0.0032

Table 3.4: Probability distribution of offspring for the second generation of two individuals:

Probability(p_i)	Probability values of number of i^{th}offspring
p_0	0.0044
p_1	0.0089
p_2	0.063
p_3	.0509
p_4	0.0044
p_5	0.0330
p_6	0.0224
p_7	0.0089
p_8	0.0016

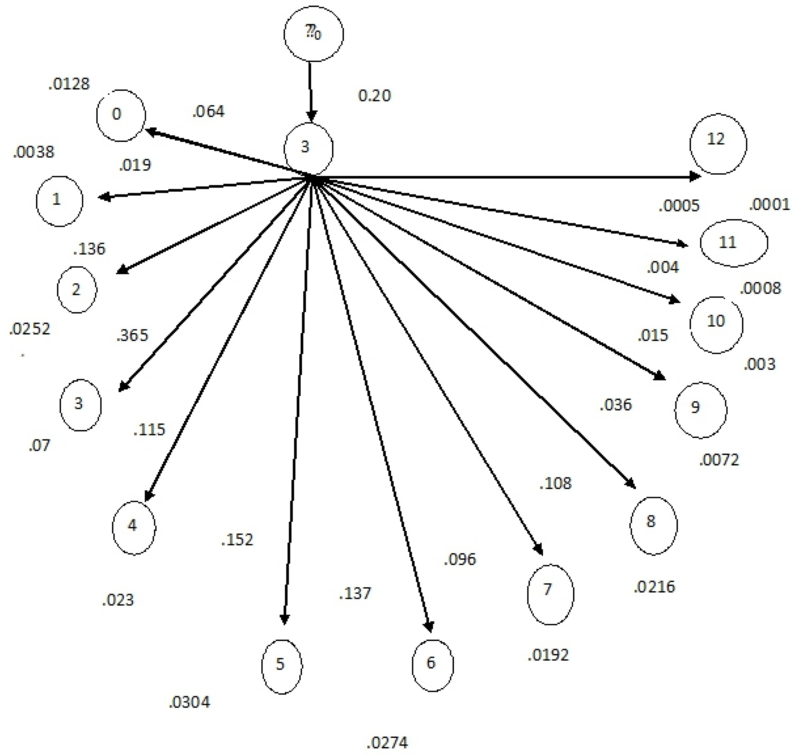

***Figure 3.5**: Tree diagram for the second generation of the offspring distribution for three individuals with their corresponding probabilities.*

Table 3.5: Probability of offspring distribution for the second generation of three individuals

Probability(p_i)	Probability values of number of i^{th}offspring
p_0	0.0128
p_1	0.0038
p_2	0.0252
p_3	0.073
p_4	0.023
p_5	0.0304
p_6	0.0274
p_7	0.0192
p_8	0.0216
p_9	.0072
p_{10}	.003
p_{11}	.0008
p_{12}	.0001

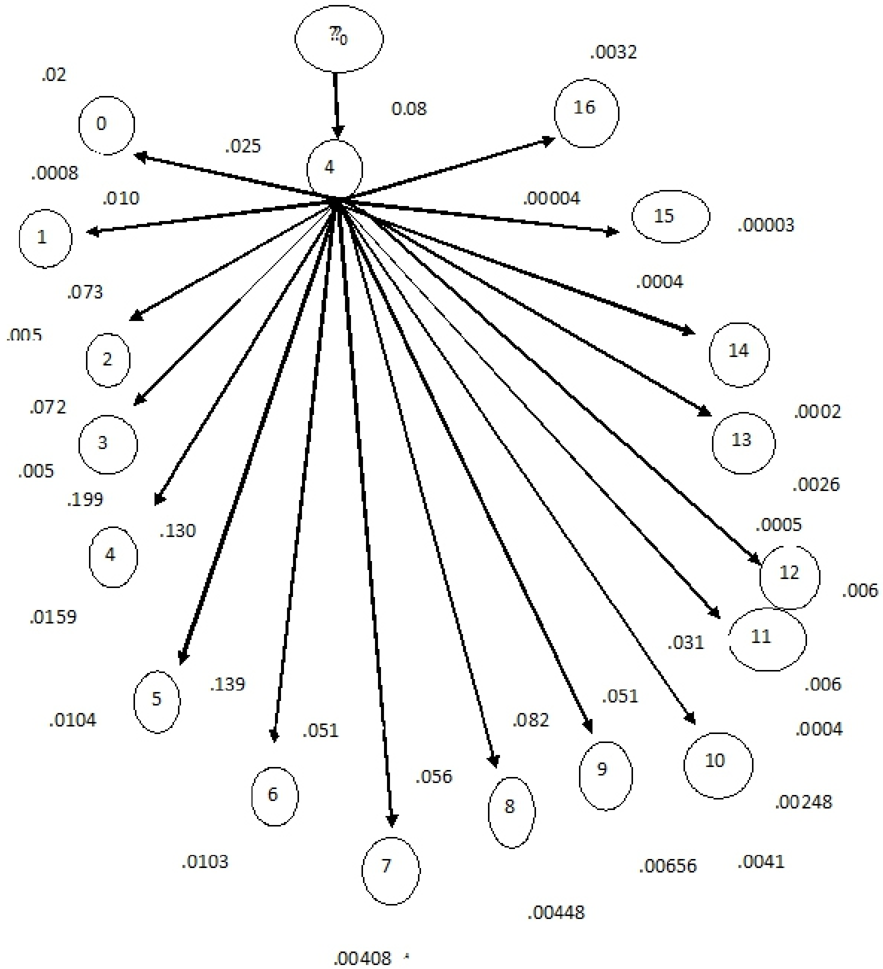

Figure 3.6: *Tree diagram for the second generation of the offspring distribution for four individuals with their corresponding probabilities.*

Table 3.6: Probability of offspring distribution for the second generation of four individuals

Probability(p_i)	Probability values of number of i^{th}offspring	Probability(p_i)	Probability values of number of i^{th}offspring
p_0	.02	p_8	.00448
p_1	.0008	p_9	.00656
p_2	.00589	p_{10}	.0041
p_3	.00576	p_{11}	.00248
p_4	.01592	p_{12}	.00048
p_5	.0104	p_{13}	.0005
p_6	.0103	p_{14}	.0002
p_7	.00408	p_{15}	.000032
		p_{16}	.0032

We found that the probability of large offspring size is near to 0 for future generations. To establish the similar findings we can calculate the mean and variance of different generations.

We know that,

$$E(X_n) = \mu^n \quad (3.3)$$

$$\text{Variance}(X_n) = \frac{\mu^{n-1}(\mu^n - 1)}{\mu - 1}\sigma^2 \text{ if } \mu \neq 1 \quad (3.4)$$

And probability of ultimate extinction

$$u_n = \sum_{k=0}^{\infty} p_k u_{n-1}^k$$

$$u_n = p_0 u_{n-1}^0 + p_1 u_{n-1}^1 + p_2 u_{n-1}^2 + p_3 u_{n-1}^3 + p_4 u_{n-1}^4$$

$$u_n = .40 u_{n-1}^0 + .04 u_{n-1}^1 + .28 u_{n-1}^2 + .20 u_{n-1}^3 + .08 u_{n-1}^4$$

$$u_n = .40 + .04 u_{n-1}^1 + .28 u_{n-1}^2 + .20 u_{n-1}^3 + .08 u_{n-1}^4 \quad (3.5)$$

Now if n=1 then $u_0 = 0$ so substituting the different values of n=1, 2…we get,

The probability of ultimate extinction for 20th generation using equation (3.5) as follows:

Table 3.7: Mean variance and probability of ultimate extinction of Down Line Clients for Different Generations until 20th generation using equation (3.3), (3.4) and (3.5) respectively:

Generation Number	Mean	Variance	Probability of ultimate extinction	Generation Number	Mean	Variance	Probability of ultimate extinction
1	1.52	1.93	0.40	11	100.064	24205.124	0.5433
2	2.31	7.392	0.48	12	152.097	56116.644	0.5434
3	3.51	21.539	0.51	13	231.188	129945.445	0.54348
4	5.34	56.541	0.525	14	351.406	300672.150	0.5435
5	8.11	140.936	0.535	15	534.138	695351.152	0.54353
6	12.33	341.278	0.541	16	811.890	1607570.19	0.54355
7	18.75	812.292	0.542	17	1234.07	3715697.11	0.54356
8	28.49	1912.899	0.5426	18	1875.79	8587128.37	0.54357
9	43.31	4474.555	0.5430	19	2851.20	19843321.6	0.54357
10	65.83	10421.60	0.5432	20	4333.82	45851513.2	0.54358

Thus the probability of ultimate extinction at or prior to 20th generation is $u_{20} = .543$, Mean $\mu^{20} = 4333.828$ and $var(x_{20}) = 45851513.24$

3.4 Probability of Extinction:

According to the theory of probability of ultimate extinction is 0.54.Now we want to estimate the probability of extinction using probability generating function .Since, $\mu > 1$ so we have to solve the following quadratic equation as

$$s = p(s) \quad (3.3.6)$$

$$s = \sum_{k=0}^{4} p_k s^k \quad (3.3.7)$$

$$s = p_0 s^0 + p_1 s^1 + p_2 s^2 + p_3 s^3 + p_4 s^4$$

$$s = .40 + .04s + 0.28\, s^2 + 0.20 s^3 + 0.08 s^4$$

$$2s^4 + 5s^3 + 7s^2 - 24s + 10 = 0$$

$$(s-1)\,(2s^3 + 7s^2 + 14s - 10) = 0$$

$$s = 1, .54, -2.02, -2.02$$

So the probability of extinction is .54.Since the probability of extinction is not very high so we think that the insurance company has not a great chance to abolish soon. So it will be better for customers to make any further policy with this company.

4. Conclusion:

Under this study we have observed from the tree diagrams or down line structures for different generations using the branching process that the number of client's enrolment will be increased gradually in Rupali Life Insurance Company from one generation to the further more generation for a long period until n^{th} generation.

Again we have examined from the mean variance and probability of ultimate extinction table that the down line client's mean is 1.52 and variance is 1.93 and probability of ultimate extinction is 0.40 of Rupali Life Insurance Company for the first generation. Whereas the down line client's mean is 2.3104 and variance is 7.392 and probability of ultimate extinction is 0.48 for the second generation and so on that means mean, variance and as well as the probability of ultimate extinction is increasing from the first generation to the further more generations until 20^{th} generation and thus the probability of ultimate extinction at or prior to 20^{th} generation is near about $u_{20} = .543$, Mean $\mu^{20} = 4333.828$ and $var(x_{20}) = 45851513.24$ and it will be gradually increased for a long period until n^{th} generation. Again according to Galton Watson process (Branching process) we know that if $\mu > 1$, branching process will be super critical. Since the resulting mean of the branching process used in Rupali Life Insurance Company is $\mu = 1.52 > 1$ for the 1^{st} generation and it will be same for further more generations hence we can say that the branching process is super critical. Further we have also found from the solution of quadratic equation that the probability of extinction is 0.54. Since the probability of extinction is not very high so we can draw an inference that the insurance company has not a greater chance to abolish soon. So it will be better for clients to make any further policy with this company.

References:

Athanasios Papoulis, S. UnnikrishnaPillai (2002), "*Probability, Random Variables and StochasticProcesses*", 4th ed., Tata McGraw-Hill Publishing company Ltd, New Delhi.

Barbu, V.S, Limnios, N. (2008) *Semi-Markov Chains and Hidden Semi-Markov Models toward Applications: Their Use in Reliability and DNA Analysis*. ISBN 978-0387731711

Bhat, B.R. (2000), "*Stochastic Models*", New Age International (p) Ltd. Publishers, New Delhi.

Bhat, U.N.(1984), "*Elements of Applied Stochastic Processes*", 2nd edition, John Wiley and Sons, New York.

Bruss, F. Thomas (1984): "A Note on Extinction Criteria for Bisexual Galton– Watson Processes". *Journal of Applied Probability*21: 915–919.

C. Kulp and J. Hall, (1968)*Casuality Insurance,* Fourth Edition, , page 35

Chakravart, Laha, and Roy 1967, A note on Kolmogorov-Smirnov Goodness-of-Fit Test. Journal of Statistics.

Daneil, Wayne W. (1978): *Applied Non-parametric Statistics,* Houghton Mifflin Company, Boston Dallas, Geneva.

Dodge, Y. The Oxford Dictionary of Statistical Terms, OUP.ISBN 0-19-920613-9.

Everitt, B.S.(2002): The Cambridge Dictionary of Statistics. CUP. ISBN 0-521-8109 -x.

F. Galton, *Problem 4001, Educational Times*, 1 April,1873 P.17.

Gujarati, D.N. (2003), "*Basic Econometrics*", 4th ed., McGraw Hill, New York.

Hamdy A. Taha(1999), "*Operations Research An Introduction*", 6th ed. Prentice Hall of India (p) Ltd, New Delhi.

Hines, William W.; Douglas C. Montgomery; David M. Goldsman and ConnineM.Borror (2003): *Probability and Statistics in Engineering*, John Wiley and Sons Inc.

Hollis and Strauss, 2007,*privacy, Driving Data and Automobile Insurance: An Economic Analysis (HTML).*

http:// www.ifsl.org.uk/uploads/CBS_Insurance_2007.PDF

Insurance Information Institute. Business insurance information. What does a business owners policy cover? Retireved on 2008-11

Jagers, P. (1975), "*Branching Process with Biological Applications*" , Wiley.

Latouche, G. and V. Ramaswami (1999): *Introduction to Matrix Analytic Methods inModelling,* 1st edition.

M.K.Roy (2004), "Fundamentals of Probability & Probability Distributions", 3rded.

Margaret E.Lynch, Editor, Health Insurance Temiology,*”* Health Insurance Assopciated with a defined attribute”Gregory D. Squires(2003) Racial Profiling, *Insurance Style: Insurance Redlining and the Uneven Development of Metropolitan Areas Journal of* Urban Affairs Volume 25 Issue 4, November 2003

Markov process, (mathematics)-Britannica online Encyclopedia.

Mason, R.D & Lind, D.A, “*Statistical Techniques in Business & Economics*”, 12th ed., McGraw Hill, New York.

Medhi, j.(1994), “ *Stochastic Processes*”,2nd edition, New Age International (P) Ltd. Publisher, New Delhi.

Peskun , P. (1973) Optimum Monte Carlo sampling using Markov Chains, Biometrica 60.

Ross, S. (2003), “ *Probability Models*”, 8th edition, Academic Press, USA.

Ross, S.M. (1995) *Stochastic Processes*. Wiley.ISBN 978-0471120629.

S.C. Gupta, V.K. Kapoor (2008), “*Fundamentals of Mathematical Statistics*”, 12th ed., Sultan Chand & Sons, New Delhi.

Schlechter, Kira (Monday March 02, 2009). "Hershey Medical Center to open redesigned . *The Patriot-News*.

Strauss and Hollis, 2007, *Insurance Markets When Firms are Asymmetrically Informed:* A Note (HTML)

SuddhenduBiswas (2004), “*Applied Stochastic Process*”, 2nded. New central book agency (p) Ltd, New Delhi.

U.S Patent Application 20060287896 “*Method for providing crop[insurance for a crop associated with a definite attribute*

Virtamo, J.,*"Birth-death processes "[1]*, 38.3143 Queueing Theory.

William W.Hines, Douglas C. Montgomery, David M. Goldsman, Connie M.Borror (2004), “*Probability and Statistics in Engineering*”, 4th ed., John Wiley & Sons, Inc.