FINANCIAL EVALUATION OF LONG TERM INVESTMENTS: THE ROLE OF EXPLICIT PRODUCTION FUNCTIONS

Juan Miguel Massot, PhD

Escuela Superior de Guerra Naval, Universidad del Salvador, Argentina

Abstract

In the practice of evaluation of investment projects, the technique of discounted cash flows applies on income with functions such as logistic or neoclassical functions, which tend to be adverse on remote-time flows. However, these functions are not necessarily representative of the long term pathway of projects which are part of portfolios closely related to innovation or events with long-run externalities, such as education or environment. This paper concludes that production functions with human capital and externalities may cause distant flows to take values other than zero and produce relevant alterations of investment decisions.

Keywords: Financial evaluation of investment projects, innovation, externalities, environment, growth models

Introduction

This paper offers to reconsider an aspect of the methodology of investment projects and project portfolios financial evaluation, by using explicit production functions adequate to phenomena dominated, for instance, by innovation.

Investment project financial evaluation has been criticized for many reasons. Firstly, projects providing out-of-market goods (public or quasi-public goods) lead to certain problems such as the absence of selling prices which hinders the estimation of monetary benefit and, therefore, a return pathway which can be discounted. This promoted the evolution of non-corporate project evaluation methodologies, such as the cost-benefit method, cost-effectiveness method and multicriteria methodologies (Aliberti, 2012; Grassetti and García Fronti, 2012; Pacheco and Contreras, 2008).

Secondly, project financial evaluation has been criticized for being biased towards flows originated in the short run and, therefore, for affecting both projects organization and the composition of investment projects portfolios (Dumrauf, 2010; Aliberti, 2012; Grassetti and García Fronti, 2012). If investment projects or project portfolios have long-run and very long-run returns (for example those in the field of science and technology, health, education, institutional quality, environment), then they are excessively adverse on discounted returns of these projects, which may bring about suboptimal levels of investment in these sectors and affect long term growth and economic and social development (Smith and Parr, 2005 : 297; Hernández, 2005 : 159 and ss.; Romer, 2006 : 102 and ss.; Keifman, 2012).

Thirdly, financial evaluation models are rigid, in the sense that they assume the investment decision is made only once on the whole project at the time of evaluation, giving the decision maker a passive nature when facing the changes in context or during the progress of the project implementation. It is more reasonable to assume that the agent is able to make decisions about delaying the beginning of an investment, modifying the rhythm, increasing or reducing the amount of investment during the implementation period. This lack of flexibility

reduces its effectiveness as a methodology of evaluation, especially, in most complex and long term projects (Grassetti and García Fronti, 2012).

Finally, it is difficult to accept certain usual theoretical assumptions which underlie the financial evaluation models, such as the functions of logistic production or the neoclassical functions which usually support the flow of income in investment projects or projects portfolios related to innovation or to certain public goods such as public health, basic education and pollution, where internal and external economies of scale, externalities, or agglomeration economies may be found (Hernández, 2005 : 289 and ss.; Smith and Parr, 2005: 229 and 234).

In this context, this paper states that a simplified modelling yet more adequate of the production function supporting the income flow, allows for the application of the standard financial approach minimizing or even totally compensating for the bias in the short run, thus modifying the standard conclusions about expected results of the assessments based on financial models.

Reconsideration of discounted cash flow methodology in investment projects:
Basic evaluation model

The approach begins with the standard criterion of investment project financial evaluation. In accordance with Branson's presentation of the neoclassical investment model (Branson; 1989: 297 and ss.) capital accumulation may follow the maximization of a function of benefits (RN) as follows:

$$max_{Nt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} [P_t \, y \, (N_t, K_t) - W_t N_t - P_t^I i_t] \quad (1)$$

Here N_t is the amount of labour in the period t, K_t is capital in period t, i_t is investment in this period, r is interest rate, P_t is price, y is product, W_t is wage and P_t^I is the price of capital goods.

In other words, the criterion is reduced to find the combination of production factors (labour, capital) and maximize the current value of said function of benefit over time, subject to certain restriction $K_{t+1}= (1-\delta) K_t + i_t$, where δ represents the expected capital depreciation rate.

Using Lagrange multipliers, the problem to be solved is the following:

$$max \, L_{Nt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} [P_t \, y \, (N_t, K_t) - W_t N_t - P_t^I i_t]$$

$$+ \sum_{0}^{\alpha} \lambda \, [i_t + (1-\delta)K_t - K_{t+1}] \quad (2)$$

The lagrangian derivative respect to N results in the fact that the business owner hires labour until the marginal product of labour is equal to real wage:

$$y_N(N_t, K_t) = \frac{W_t}{P_t} \quad (3)$$

This means that the business owner hires capital until his marginal product is equal to the cost of opportunity:

$$y_K = \frac{\delta \, P_t^I + r P_{t-1}^I - (P_t^I - P_{t-1}^I)}{P_t} \quad (4)$$

The numerator on the right side of the equation is the cost of capital, C_t, equivalent to the implicit price of capital lease herein. The first component is depreciation, the second one is the interest paid for stock at the beginning of the period, and the last one is any capital gain between the beginning and the end of the period.

In the end, the marginal product of capital turns out to be:

$$y_K(N_t, K_t) = \frac{C_t}{P_t} \equiv c_t \quad (5)$$

Since it is an investment project, it is important to estimate the equilibrium capital stock, which is a function of the production volume Y, the cost of capital c and the price of product P:

$$K^E = K^E(Y, C, P) \quad (6)$$

Thus $\partial K^E / \partial Y$, $\partial K^E / \partial P > 0$, and $\partial K^E / \partial C < 0$

Within a neoclassical model such as the one presented, the Cobb-Douglas function may be adopted for the equilibrium capital stock function, which has the characteristic of generating constant returns to scale and diminishing returns to a factor. The resulting function is the following:

$$y = \alpha K^\alpha N^{1-\alpha} \quad (7)$$

In practice, it turns out:

$$K^E = \frac{\alpha P y}{C} = \frac{\alpha y}{\frac{c}{P}} \quad (8)$$

Where, K^E is the stock of equilibrium capital of the project which increases as the value of production increases and it is reduced when the cost of capital use increases.

Besides the importance of a "selling price" or the discount rate –aspects which are considered by other reviews to the financial approaches on project evaluation–, from equations (1), (7) and (8) we can infer the importance of the explicit production function whose accurate formulation determines the temporary pathway of returns. Equations (7) and (8) present the result of a Cobb-Douglas function which has particular conditions which adapt to the case under study.

In order to see these differences, this paper resorts to four function models: logistic model and Bass model, both used in the innovation theory, and neoclassical model and human capital model, used in the economic growth theory.

Logistic model

This type fulfils the conditions of logistic deterministic growth models based on technological diffusion models that take the time variable as the main determinant, such as Foster and Wild, 1999 (see Hernández, 2004: 272 – 274). In this model, a process of production growth is generated; it depends on historic time (t), a production maximum (y_{max}) and a diffusion or growth parameter (β).

$$y_{t-1} = \beta \, y_t(y_{max} - y_t) \quad (9)$$

If a period is accumulated, the difference between both periods will be:

$$y_t - y_{t-1} = y_{t-1} \, \beta \, \left[1 - \left(\frac{y_{t-1}}{y_{max}}\right)\right] \quad (10)$$

The time necessary to reach y_{max} depends on parameter β. When parameter β has a higher value, the curve grows, increasing the current value of income for equal values of interest rate r and variable y.

Focus is placed on the time variable (t) and the growth parameter β. The model does not depend on economic variables such as capital stock or human capital, as explained in the equation (1). In the model, y_{max} is exogenously determined, as well as the value of β.

The incorporation of production functions according to a deterministic process based on variable (t), on an exogenously adopted parameter β and on variable y_{max} arising from market research (determination by demand) or from engineering studies (determination by technological restriction or rigidity, or determination by offer) are common both in literature and in the practice of investment project evaluation. Besides the simplicity of the calculation,

the pathways are not objected by the management of the investment project evaluation, which tends to accept the existence of product cycles as a central component of the theoretical and empirical corpus.

Finally, the temporal pathway of the production of a logistic model corresponds quite well with that of the neoclassical production functions previously discussed, even when, as previously mentioned, one of them has its centerpiece in economic variables such as capital and labour, and the other one, only in the time variable. Therefore, according to the context and objectives of the investment project analysis (amounts involved, available information, uncertainty about certain variables, technical ability of the team of evaluators to perform a prospective study, among other factors), the selection of different functions may lead to equivalent results and, depending on the value of the parameters, to similar decisions based on more complex quantitative methods.

Bass Model

The Bass model belongs to the family of S-Curve or sigmoidal models widely used in the analysis of the economic evaluation of innovation projects. It is basically supported by the product cycle theory with four stages (introduction, growth, maturity and decline) which is reduced to three stages (invention, innovation, and standardization; decline is not included), once adapted to technology. Even when its mathematical presentation varies if compared to the logistic model previously stated, in a broad sense there is no difference. The model, which combines an innovation model and an imitation model, counts on the existence of a maximum quantity of sales and two parameters, coefficient of innovation (or market penetration) and coefficient of imitation. The quantity increases inasmuch as the former is higher and diminishes inasmuch as the latter is higher.

According to Smith and Parr (2005: 214) the production equation of the period is the following:

$$y_t = y_{t-1} + \left(g + \left(q * \left(\frac{y_{t-1}}{y_{max}} \right) * (y_{max} - y_{t-1}) \right) \right) (11)$$

Where g is the coefficient of innovation and q is the coefficient of imitation, having a range of variation between 0 and 1 for both parameters.

As well as in the logistic model, parameters are exogenously determined (by market researches, engineering studies, etc.) therefore the function is not associated to the production functions depending on economic variables, as it commonly happens in microeconomics. Again, depending on the context, it is a very useful function and, in the practice, it is widely used in empirical works on introduction of new goods in the market (Smith and Parr, 2005: 234).

A feature that should be highlighted again is that, neither the Bass model nor the logistic model, assume a decline in production as from a certain date, which does occur in the model of the product cycle. Likewise, as it will observed later on, said maximum level of production may grow at a positive rate if the parameter Y_{max} is transformed into a function that depends on time or on any other variable, for example, economic or demographic variables.

The effects of a maximum level of production differ substantively in the case under study, if such level is stable or if it grows at a certain rate over the time span relevant to the analysis. In the first case, whenever the production stabilizes, sooner or later its present value will be close to zero; in the second case, this is not necessarily true, since it depends on the difference between the growth rate in the steady state and the discount rate; besides, for the same discount rate, the value of the product which turns to be zero at the time of evaluation is located ahead in time for any growth rate higher than zero.

As it can be clearly seen, these observations are actually relevant for the purpose of this paper, since they imply a change of a key variable such as the maximum production in the long run.

In short, both logistic and Bass models may be explicit models of the income function, which may be the relevant economical-technological context; this may strongly affect the evaluation of a project. This will ultimately depend upon the specific parameters adopted for modelling and more specifically if Y_{max} is a parameter or a function growing over time or over other (economic, demographic, etc.) variables.

Neoclassical model with exogenous technological progress

The work by Robert Solow (1956, 1957) on the theory of growth, commonly called neoclassical growth model or model with exogenous technological progress, sets forth a growth of product per capita equation compatible with the function expressed in equation (1).

The basic equation for the total product is as follows:

$$Q \equiv Y = F(K, L) \ (12)$$

Where total production is function of the capital stock and the applied labour. The production function is regarded as an homogeneous function of degree 1, with constant returns to scale, and decreasing returns to a factor, which makes it compatible with the Cobb-Douglas function, as shown in equations (7) and (8). Hence, implicit function (12) of total production turns out to be:

$$Y = A K^\alpha L^{1-\alpha} \ (13)$$

Where A is total productivity of the factors, and α is both the partial elasticity of production to capital, as well as the proportion of the use of this factor in the production.

In per capita terms, it translates as:

$$y = a k^\alpha \ (14)$$

Where a is the total productivity of the per capita factors, and k is the per capita capital.

Deriving the per capita production from the per capita capital, it results in:

$$y_k = a \, \alpha k^{1-\alpha} \ (15)$$
$$y_{kk} = a \, \alpha \, (\alpha - 1) k^{\alpha-2} \ (16)$$

As the first derivative (15) is positive, and the second (16) is negative, the function grows at a decreasing rate; that is to say, capital productivity grows yet it does so at a decreasing rate.

When comparing the results obtained in the neoclassical model and in the logistic model, both functions show growing trends but at a decreasing growth rate. The neoclassical model also allows one to know the growth rate of total production as well as the per capital production.

The labour accumulation function is:

$$L_t = L_0 e^{nt} \ (17)$$

Where n is the labour accumulation rate over time. Conversely, capital accumulation function is as follows:

$$\frac{dK}{dt} \equiv I_t = S_t = sY_t \ (18)$$

Where I_t is investment, and S_t, saving. Writing the equation (18), it results in:

$$\dot{k} = \frac{sy}{k} - n = \frac{s f(k)}{k} - n \ (19)$$

From this equation it turns out that per capita capital accumulation over time is the function of saving rate s, and labour growth rate n.

Using this model it can be obtained the per capita capital level in the steady-state, $k*$, by equalising equation (18) to zero, which results in the equality below:

$$\frac{s\,f(k^*)}{k^*} = n \quad (20)$$

Such equality can be rewritten as a function, and thus obtain the per capita production function in the steady-state y^*:

$$y^* = f\,(k^*) = \frac{n}{s}k^* \quad (21)$$

In the model with no technological progress, total production, capital, and labour increase in the long run at the same n rate, which is the labour growth rate. Hence, in the steady-state, per capita production growth comes to a halt when the situation shown in (21) occurs.

The introduction of the exogenous technological progress, which modifies labour efficiency and, consequently augments its productivity in the long run, changes the steady state condition previously mentioned. Exogenous technological progress is carried out by introducing the variable effective labour per capita, that is to say, labour which is modified by technological progress and which augments its efficiency:

$$E_t = L_t e^{\lambda t} = L_0 e^{nt} e^{\lambda t} = L_0 e^{(\lambda+n)t} \quad (22)$$

Here λ is the growth rate of the effective labour per capita.

Once technological progress is introduced in this manner, the result is that growth of the total variables in the long run will be:

$$\dot{K} = \dot{E} = n + \lambda \quad (23)$$

And, consequently, growth of per capita production in the long run is equal to λ.

The model can be widened if capital depreciation is taken into account, in a magnitude equal to δK, which reduces the growth rate shown in (22) to $n+\lambda-\delta$.

Assuming the production function associated to an investment project has a formula as the one expressed between equations (12) to (23), and that n, λ and δ are constant, then if $r > n+\lambda-\delta$, it is possible to find a current flow value compatible with what is pointed out in the literature of financial evaluation of projects. If this is the case, then, there would be an economic model for the flow function compatible with most literature and praxis in the financial project evaluation field.

Unlike the logistic model (2.2.1.) and Bass model (2.2.2.), in the neoclassical model, function depends on economic variables. The major relative difficulty of the latter is that it can only be used in cases where information allows for correct mathematical modelling of the flow. For this reason, in the other cases, the former models are chosen, since they are enriched by data derived from market research (demand) and from experts in technology (offer conditions).

A variant of the presented case is that of a growth model with exogenous technological progress but with growing returns within a range of capital accumulation. Provided such modifications were made, production could become even more similar to the functions which are more usual in the product cycle theory, since it would be made up of three phases. The first one, in which it would grow at increasing rates; the second, in which it would grow but at decreasing rates; and the final phase, in which the product would finally decline.

In this case, production function *f(k)* has two intersections with function *[(n+δ)/s] k*, where the former is unstable *(k**)* and the latter is stable *(k*)*. In the former, capital productivity per each additional factor unit is higher than *[(n+δ)/s] k*, which is the reason why it is decided to keep increasing production. In the latter, the opposite happens, thus reaching the steady-state.

The variant introduced in the production function helps understand the possible implications of increasing returns to scale. Nonetheless, the restriction imposed by the very product cycle theory (y_{kk} is in the second negative phase), necessarily leads to the steady-state

k^*, which does not alter substantially the before mentioned conclusions, unless the production range relevant to the project being analysed falls below k^*.

In this final case, there is an efficiency problem which, although it exceeds the aim of this paper, must be considered as a marginal factor in cases of private project evaluation (unless there are marginal benefits of increasing production in the future, provided the other conditions remain invariable). In non-private cases, such as the ones shown, they are included in some of the observations to be made as a consequence of the following model.

Human capital model

The above explained concept regarding the production function in Solow's model but with increasing returns for a given function range –which matches the product cycle theory– allows for speculation on what might happen if phase I of the cycle (increasing returns) were always true, i.e. that neither phase II or III existed where the law of decreasing returns were present.

If as it does happen in production dominated by innovation, production functions introduced to the fund flow of the projects included in an investment project portfolio were characterized, for example, by externalities, human capital as a production factor, clustering effects, and *learning by doing*, then the production function should adapt to possess such characteristics.

Based on an *ad hoc* adaptation of the contributions by Jones and Manuelli (1992) and by Lucas (1988), the production function proposed is the one below:

$$Y = A\bar{H} + B\,K^{1-\alpha}L^{\alpha} \quad (24)$$

Where Y is the product, $\bar{H}$ is the applied human capital, K is the physical capital, and L is the workforce. Accumulation functions of production factors L and K match equations (17) and (19), and applied human capital increases according to the following function:

$$\dot{h} = \gamma\,(\emptyset\,u\,h\,) - \delta_h\,h \quad (25)$$

Where $\emptyset$ is the efficiency of human capital, u is the fraction of time of the people in charge of accumulating human capital (the rest is meant for working), δ_h is the depreciation of human capital, and γ is the degree of the portfolio internalization of the trickle down effects of each project. Human capital is accumulated based on the amount of time devoted to that task, but subject to correction due to its efficiency and to the depreciation it undergoes.

This model is especially interesting to project portfolios which encompass externalities, clustering effects, and *learning by doing* generated in the system as a consequence of its organization as such. These effects of the portfolio organisation are shown by parameter γ, which shows the degree of appropriation that the projects have regarding the effects generated by the whole group of the portfolio projects. The trickle down effects generated towards the inside of the very system and which are derived from the main production factor applied to innovation, which is the human capital, will depend on the quality of the coordination of activities over time and space.

A production function like (24) may grow indefinitely over time, depending on the values taken by the parameters. In order for all the relevant range to be $h > 0$, in equation (25) the first term must be higher than the second one, which forces all γ, $\emptyset$, u to be positive and that $(\gamma\,\emptyset\,u\,) > \delta_h$. Provided this condition is met, human capital accumulation per capita as defined herein will not come to a halt for all the range of relevant time.

In the case being studied, for equal values of parameters $\emptyset$, u, and δ_h, higher levels of γ appropriation involve higher growth rates. At the limit, when the model tends to 0, it resembles the neoclassical model; when the model tends to 1, the role of non-decreasing returns from human capital is at its highest expression. Ultimately, the values the latter rate might hold are key in the premise of this approach, and as it can be observed in graph 3, they are not trivial in the growth pathway.

Finally, the second part of the right term of the equation (24) is to be analysed. Even though applied human capital ($\bar{\bar{H}}$) is not subject to the decreasing returns, to a factor as in the neoclassical function, term $B\ K^{(1-\alpha)}\ L^{\alpha}$ is indeed subject to it, as it was shown in the neoclassical model (2.2.3.)

From previous paragraphs it can be deducted that the production function with human capital (24) can display various pathways depending on the parameters adopted by the accumulation of $\bar{H}$ and its participation in the total product, given that the rest of the function grows at a decreasing rate, thus affecting maximization of benefit associated to the financial evaluation criterion of a project.

Comparison of the four models

In this title the maximization problem associated with the financial evaluation of projects but applied to the four types of production functions given above is taken up; i.e. logistic models, Bass, neoclassical and human capital.

Since going back to equation (1):

$$max_{Nt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} [P_t\ y\ (N_t, K_t) - W_t N_t - P_t^I i_t] \quad (1)$$

Production function may adopt various functional forms, which translates into four maximisation problems:

a. Logistic model:

$$max_{Nt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} \left\{ P_t y_{t-1} \left[1 + \beta \left(1 - \frac{y_{t-1}}{y_{max}} \right) \right] - W_t N_t - P_t^I i_t \right\} \quad (26)$$

b. Bass model:

$$max_{Nt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} \left\{ P_t y_{t-1} \left(g + \left(q * \left(\frac{y_{t-1}}{y_{max}} \right) * (Y_{max} - Y_{t-1}) \right) \right) - W_t N_t \right.$$
$$\left. - P_t^I i_t \right\} \quad (27)$$

c. Neoclassical model:

$$max_{Lt,Kt,it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} \{ P_t [A\ K^{\alpha}\ L^{1-\alpha}] - W_t N_t - P_t^I i_t \} \quad (28)$$

d. Human capital model:

$$max_{Lt,Kt,\overline{Ht},it} \sum_{0}^{\alpha} \frac{1}{(1+r)^t} \{ P_t [A\bar{H} + B\ K^{1-\alpha} L^{\alpha}] - W_t N_t - P_t^I i_t \} \quad (29)$$

Provided all projects have the same factor ($W_t N_t - P_t^I i_t$) regarding costs, and P_t remains invariable over time, then the relevance of the flows which are more distant from the evaluation moment will depend on the ratio between the interest rate r of the discount factor, and the evolution over time of total productivity of each function.

In other words, if production growth depends on accumulation of production factors (capital, labour and applied human capital, as the case may be), it will be critical for the flow evaluation the ratio between total productivity of factors over time of each specific production function and the interest rate. While in equations (26), (27) and (28) productivity grows at a decreasing rate, in equation (29), the growth rate depends on the values taken by

the parameters, which can show non-decreasing growth rates over time, or for a relevant period of time. This means that, depending on the organisation of productive units, the increase in human capital stock, and that of the productive fabric which uses it, there could exist a growing production pathway over time, without it necessarily being subject to the decreasing returns to all factors.

Likewise, if in equations (26) and (27) y_{max} is a growing function over time – depending either on the time variable, or on any other variables, such as economic or demographic ones–, both the logistic and Bass models tend to resemble the human capital model regarding the evolution over time the net flow may have.

From what has been pointed out, it can then be deducted that in equation (29) as well as in both equations (26) and (27), provided the maximum production level increases for all the relevant range, if the growth rate of the function in the long run were to be higher than the applied discount rate, *ceteris paribus*, the current value of the future flows will never be zero, and, thus, even under the implementation of the discounted flow method, flows from each period will contribute –positively or negatively– to establishing the current value of the project at issue.

However, as it has been previously mentioned, if in the long run $(\partial P_t)/(\partial t) < 0$, then there is a more complex ratio which involves the interest rate, and the growth of both the product and the price in the long run. Assuming that the rest of the elements in the function remain constant, the fundamental comparison is between the net flow growth over time $(\partial RN_t / \partial t)$ and r. When $(\partial RN_t / \partial t) < r$, there will come a moment in time as from which the current value of the flows to come will tend to zero. Otherwise, subsequent flows will still keep economic relevance at the time of decision making.

Numeric simulation of the presented production function models

In this section, comparative results of numeric simulations of the four models previously presented are shown. Results refer to the growth rate between periods for a total of 30 periods of total income. Results of three logistic model cases, and results of three Bass model cases corresponding to various values of their parameters are simulated, as well as results of a neoclassical model case, and two human capital model cases with different parameter values. For the simulation purposes, the price is supposed to be constant and equal to 1.

The variation rate between periods in percentage of the total income provides a first look to the question treated in this paper, that is to say, whether the time pathway of the discounted flow may not be zero after a certain number of periods. If the function grows at a decreasing rate, or even if it does not grow at all, or falls as from a given period, the desired scenario cannot be met. Consequently, the fact that the function grows at a non-decreasing rate, or if it does, that it is at a slightly decreasing rate, must then be a necessary condition, though still not sufficient enough, to solve the problem. In other words, the growth pathway of total income is a necessary condition but still not sufficient enough for the current value of the distant flows to be different from zero, given that in order for the necessary and sufficient condition to be met, it is necessary to know exactly the time pathway of the total cost and the discount factor to be applied. In this case, as it was initially pointed out, the aim of this paper is focused on the condition of the total income pathway as being *conditio sine qua non* (or necessary condition).

The (arbitrary) parameters of each of the functions are the following:
Logistic function L 1: β =0.78
Logistic function L 2: β = 0.88
Logistic function L 3: β = 0.93
Bass function B 1: g=0.005 and p = 0.8

.   Bass function B 1: g= 0.05 and p = 0.5
Bass function B 3: g= 0.5 and p = 0.3
Neoclassical function NC: Lo =120; K0= 120; n= 0.03; s=0.20; α=0.5
Human capital function HK 1: H_0 = 120; other neoclassical parameters when appropriate, γ= 0.9; Ø = 0.5 ; u =0.3 ; δ=0.01; and A=0.5 and B= (1-A)
Human capital function HK 1: H_0 = 120; other neoclassical parameters when appropriate, γ= 0.5; Ø = 0.5 ; u =0.3   ; δ=0.01; and A=0.5 and B= (1-A)

Even though they are arbitrary, the selected parameters provide the growth pathways of the total income corresponding to each production function, and thus the growth rates between periods for each model. Growth rates can be seen in chart 1.

As it can be observed, total income in logistic functions and in Bass model increase at high rates in the first sections, yet once maturity of the productive cycle is reached, growth rates tend to zero. By contrast, in the neoclassical model, total income grows at a decreasing rate all along the selected range (30 periods). In human capital models, which differ between themselves depending on their level of appropriation (high and low), even though growth rates differ at certain levels, in both cases, rates do not decrease significantly all over the range.

Chart 1Growth rate between periods of the total income per function type

	L1	L2	L3	B1	B2	B3	NC	HK1	HK2
1									
2	78%	88%	93%	90%	55%	80%	15%	12%	9%
3	78%	88%	93%	90%	53%	58%	13%	12%	8%
4	78%	88%	93%	90%	52%	48%	12%	11%	8%
5	78%	87%	92%	89%	51%	42%	11%	11%	8%
6	77%	87%	92%	89%	51%	38%	10%	11%	8%
7	77%	86%	91%	88%	50%	36%	10%	11%	8%
8	76%	84%	88%	86%	50%	34%	9%	11%	8%
9	74%	81%	84%	82%	49%	33%	9%	11%	8%
10	71%	75%	77%	76%	49%	32%	8%	11%	7%
11	65%	66%	64%	65%	48%	31%	8%	11%	7%
12	57%	51%	46%	49%	47%	30%	8%	11%	7%
13	45%	32%	24%	29%	46%	30%	7%	11%	7%
14	30%	14%	8%	11%	44%	29%	7%	10%	7%
15	16%	4%	1%	2%	41%	28%	7%	10%	7%
16	6%	1%	0%	0%	37%	28%	7%	10%	7%
17	2%	0%	0%	0%	32%	27%	6%	10%	7%
18	0%	0%	0%	0%	26%	26%	6%	10%	7%
19	0%	0%	0%	0%	20%	24%	6%	10%	7%
20	0%	0%	0%	0%	14%	23%	6%	10%	7%
21	0%	0%	0%	0%	9%	21%	6%	10%	7%
22	0%	0%	0%	0%	5%	19%	6%	10%	7%
23	0%	0%	0%	0%	3%	17%	5%	11%	7%
24	0%	0%	0%	0%	2%	15%	5%	11%	7%
25	0%	0%	0%	0%	1%	13%	5%	11%	7%
26	0%	0%	0%	0%	0%	10%	5%	11%	7%
27	0%	0%	0%	0%	0%	8%	5%	11%	7%
28	0%	0%	0%	0%	0%	7%	5%	11%	7%
29	0%	0%	0%	0%	0%	5%	5%	11%	7%
30	0%	0%	0%	0%	0%	4%	5%	11%	7%

Source: Own estimates

It can then be concluded that, even if the cost function and the discount factor are not considered, in logistic models, Bass model, and the neoclassical model, the current value of the more distant flows over time tend to zero. By contrast, in the human capital functions herein presented, since income grows constantly depending on the cost functions and the

discount factor, the current value of the net flows distant in time will not be necessarily irrelevant.

Just as a mere pedagogical or illustrative example of what has been mentioned, assuming the case of a constant price equal to 1, with cost functions in which costs are a fixed proportion of the total income (not a usual case in economic theory or praxis, yet useful for pedagogical purposes), a comparison between growth rate of total income (10-11% and 7% in the chart cases) and the interest rate of the discount factor is required. If the interest rate were of 6%, the net flows are relevant for all the selected range. In other words, unlike the other models in which –depending on the case– after periods 10 and 15, the current flow values tend to zero, human capital models still have non-trivial values and different from zero.

Consequently, probability that the flows distant in time are different from zero is higher in the case of production functions with human capital than in the functions traditionally used in the economic and financial evaluation of investment projects. Therefore, it is relevant to appropriately set out such function at the time of exploring, both theoretically and empirically, the importance of distant flows in time, which are as usual in investments in scientific and technology facilities, as in other phenomena closely related to human capital (basic education, primary healthcare, etc.) or as in environment-related matters.

Conclusion

Explicit production functions that cover the phenomenon of growth derived from accumulation of human capital, externalities, the effects of learning, among other questions treated within the modern theory of economic growth, not only represent more appropriately the function applicable to intensive investments in technological innovation, but also avoid the short-term bias affecting all types of flows generated in the very long run and that exceeds the above mentioned cases.

Therefore, what has been concluded regarding the growth pathways resulting from appropriate explicit functions, would allow for:

a) Making the returns distant in time become relevant, for example, those occurring after twelve or fifteen years;

b) Appropriately dealing with the growth of net flows of the investment projects belonging to project portfolios that generate externalities among projects and market imperfections, such as the ones dominated by innovation;

c) Taking into account the impact on future generations of certain phenomena, such as the formation of scientific and technological systems, human capital (education, health, etc.), environment damage/recovery, among other issues, which would provide for a more accurate interpretation of results in the long run.

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