SKILLED-UNSKILLED MATHEMATICAL PROBLEM SOLVERS: JORDANIAN-STUDENTS' DIFFERENCES IN SOLVING GEOMETRICAL PROBLEMS

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Abstract

A written geometry test in the subject of triangles that required students to produce the answer was administered to 240 ninth-grade students (70 skilled-mathematical problem solvers and 170 unskilled-mathematical problem solvers)**.** A quantitative-content analysis methodology was used to analyze students' responses to the test. Overall findings suggest that most skilled-mathematical problem solvers seemed to have adequate mathematical knowledge, skills and reasoning that have helped them understand geometrical problems in a way that helped them in devising a suitable plan then carrying out this plan and making sure that their solutions are reasonable and make sense to them. On the contrary, most unskilled-mathematical problem solvers seemed not to have such adequate knowledge, skills and reasoning. As a result, skilled mathematical problem solvers' behaviors of solving geometrical problems differ from that of unskilled mathematical problem solvers' behaviors. Classroom implications and suggestions for further research are included.

Keywords: Geometrical problems, skilled students, unskilled students, *Van Hiele's Model of geometrical thinking.*

Introduction

A problem arises when a person has a goal but does not know how to reach this goal (Polya, 1957; Jonassen, 2000; Florida Department of Education, 2010). The National Council of Teachers of Mathematics, in its document <u>Principles and Standards for School Mathematics</u> (NCTM,2000), has placed problem solving first in the list of the processes' standards at every grade level. Also, based on other documents issued by NCTM in the field of mathematics education such as <u>Professional Standards for teaching</u>

Mathematics (NCTM, 1991), Assessment Standards for School Mathematics (NCTM, 1995), Curriculum Focal points for Prekindergarten though Grade 8 Mathematics: A Quest for Coherence (NCTM, 2006), and Problem Solving Research Brief: Why is Teaching with Problem Solving Important to Student Learning (NCTM, 2010) one might conclude that problem solving is the primary goal of mathematics curriculum at every grade level. The main reason behind this conclusion is that the situations which the students face in their daily lives require the ability to solve problems, and mathematics teachers and mathematics curriculum have an important role to play in order to help students develop the art of problem solving.

In 1957 Polya wrote a book entitled How to Solve it: A New Aspect of Mathematical Method. In this book, Polya mentioned four steps that help students in solving mathematical problems. These steps are understanding the problem, devising a plan, carrying out the plan, and looking back and evaluating the obtained solution. According to this book, understanding the problem needs a careful analysis of the problem to decide what is given and what is the unknown and to determine whether it possible to find the unknown or not. Then, devising or creating a suitable plan needs a careful connection between the unknown and the given data. Moreover, looking for possible connections between the given problem and other related problems that the student had solved before helps in creating a suitable plan for the new problem. After analyzing a problem and deciding on what is to be done, there remains the process of manipulating the data involved in the problem to reach the correct answer. This step is usually easier than the second step especially if the student has the necessary mathematical skills needed to carry out the devised plan. Finally, the student can benefit from the looking back step in evaluating the obtained solution and predicting suitable strategies for other related problems (Polya, 1957).

A careful look at the research literature shows that problem solving studies can be grouped according to two major themes. Some researchers such as Kirkley, (2003), Amen (2006), Hoaglund (2008), Jacobbe (2008), Maria & Carlos (2008) Comesana (2009), Yeo (2009), Tello (2010), Hensberry (2012), Hickendorff (2013), Hickendorff (2013)and Sajadi, amiripour, & Malkaliteh, (2013) emphasized the importance of mathematical language of the problem, understanding the problem, planning for the solution and using the required mathematical skills. In another group of esearch studies, particular attention has been given to studying the individual differences between skilled and unskilled students in their ability to solve mathematical problems (Lazakidou, Paeaskeva,& Retalis, 2007; Maria & Carlos, 2008; Muir, 2008; Comesana, 2009; Ellison, 2009; Yeo 2009; Vista, 2012).

The first step in solving a mathematical problem is reading the problem and interpreting the facts and relationships among its conditions (Lazakidou et al., (2007; Maria & Carlos, 2008; Muir, 2008; Comesana, 2009; Yeo, 2009). Also, some researchers have found that the mathematical language of the problem is a major source of difficulty in problem solving but students' understanding of mathematics vocabulary enhances their ability to solve mathematics problems (Kirkley, 2003; Amen, 2006; Hoaglund, 2008; Lesh & Fennewald, 2008; Maria & Carlos, 2008; Comesana, 2009; Yeo, 2009; Tello, 2010; Hickendorff, 2013). Moreover, there are other factors or skills that seem to help students become successful problem solvers. Some of these skills are understanding the problem, devising a plan for the solution, carrying out the plan, and looking back and evaluating the obtained solution (Herbst, 2002; Stylianou, 2002; Lazakidou et al., 2007; Ellison, 2009; Dockter & Heller, 2009; Sutherland, 2009; Yeo, 2009; Tello, 2012; Hensberry, 2012; Maetin, 2012; Hickendorff, 2013).

Regarding the second group of research studies, the mathematics-education researchers focused attention on studying differences between skilled and unskilled students in their behaviors to solve mathematical problems(Kirkley, 2003; Stylianon & Silver, 2004; Stylianon, Chane & Blanton, 2006; Lazakidou et al., 2007;Stylianou, Chae, & Blanton, 2006; Osta & Labban, 2007; Muir, 2008; Comesana, 2009; Dockter et al., Heller, 2009; Yeo, 2009;; Ellison, 2009; Sutherland, 2009; Yeo, 2009; Tello, 2010; Maetin, 2012; Ozerem, 2012; Hickendorff, 2013). Overall findings of these research studies indicated that both skilled and unskilled students have their own behaviors on solving mathematical problems. Also, researchers such as Herbst (2002), Mansi (2003), Christou, Mousoulides, Pittalist, & Pitta-Pantazi (2004), Elis (2007), Weber (2008), Johnson, Noblet, & Rozner (2010), Brandt & Rimmasch (2012), Jones, Fujita, & Kunimune(2012), Inglis & Alcock (2012), highlighted the importance of students' ability to write mathematical proofs or validate written proofs. Overall findings of these research studies indicated that writing proofs or validating written proofs are difficult especially for students who lacked mathematical content knowledge. Moreover, some mathematics educators such as, Dimakos, Nikoloudakis, Ferentlinos, & Choustoulakis (2007), Torregrosa & Quesada (2008), VanSpronsen (2008), Panaoura & Gagatsis, (2009), Weber & Mejia-Ramous (2011), Inglis & Alcock (2012)indicated that skilled students focus on deep features of arguments, whereas unskilled students spend more time focusing on trivial features of arguments.

Based on different NCTM documents and research studies mentioned so far, one may raise the following question. "Does having sub-skills such as reading and understanding the problem, planning the solution, manipulating the data in the problem guarantee successful problem solving?" It is not necessarily true that the one who understands the problem and performs its basic skills can also solve it. When a student starts solving a problem, something more than reading it and performing its basic skills is needed. A good approach to understanding the problem solving behavior of a student is the skilled-unskilled approach. Here the focus is on the differences between skilled and unskilled problem solvers in terms of the processes used by each group of them.

The present study takes the position that it is obvious to find differences between skilled and unskilled students in their ability to solve geometrical problems, but it is relevant to study the nature of these differences between them in their behaviors of solving geometrical problems. Instructors of mathematics at all grades and levels, mathematics education researchers, and publishers of mathematics textbooks could benefit from this study.

Methodology

Purpose of the study

The present study investigated major differences that can be observed between skilled and unskilled mathematical problem solvers in their behaviors of solving geometrical problems. In particular, the study addressed the following question: What are the differences that can be observed between skilled and unskilled mathematical problem solvers in terms of their behaviors of solving geometrical problems?

Data Sources and Analysis

A type of purposeful sampling called "Maximum Variation Sampling" in three different districts of Jordan was used. Five teachers from five different schools who teach ninth-grade students were kindly requested to participate along with their students in this study. The teachers' information about their classes was considered when choosing two groups of students. The first group consisted of students who seemed to be skilled mathematical problem solvers, and the second group consisted of students who seemed to be unskilled mathematical problem solvers. The sample size was 240 students (70 skilled mathematical problem solvers and 170 unskilled mathematical problem solvers). Those 240 students agreed to participate in the study and permissions from the teachers and students were obtained prior to the study. The mean age of those students was 14.8 years.

A written geometry test in the subject of triangles that required students to produce the solution was used. The test consisted of three essay-geometrical problems. For each problem, students were asked to answer the following sub-questions according to the following categories:

1) Understanding the Problem:

- What are you given in the problem?
- What are you asked to find out?
- Graph a geometrical shape that may represent the problem.

2) Devising a Plan:

- State the connection between the given data and the unknown.
- State one related problem you had solved before.
- State all key words stated in the problem and translate these words into mathematical language.
- State any concept and theorem that could be useful to solve the problem.

3) Carrying Out the Plan:

- Give reasons for each step of the solution.
- Check each step.
- Can you see that the answer is correct?

4) Looking Back:

- Does your solution make sense to you? verify your response.
- Do you have another method of solving the problem? mention this method.
- State another related geometrical problem for which the method will work.

A quantitative-content analysis methodology was used to analyze students' response to the test(University of Texas at Austin, 2011). This methodology allows the researcher to draw conclusions about major differences between skilled and unskilled-mathematical problem solvers in terms of their behaviors of solving geometrical problems. In this study, an analytical instrument that based on Polya's strategy of problem solving was developed and used to answer the research question. Eight expert panelists in the field of mathematics and mathematics education were kindly requested to examine the validity of this instrument. Therefore, this instrument was considered valid to be used for the purpose of collecting data in the study. Also, the researcher and two raters independently worked to analyze students answers for each problem of the test and the interrater agreement was calculated based on Cooper's coefficient. The value of Cooper's coefficient was found to be 0.89. This value is considered to be quite high for this type of instrument.

Definitions of Terms Used in the Study

1)Geometrical Problems: Mathematical problems that relate to the content of geometry. Those problems are more difficult to solve than routine geometrical exercises and their solutions are not known in advance by students. For the purpose of this study, three essay-nonroutine geometrical problems were used.

2)Skilled Mathematical Problem Solvers: Students who seemed to have adequate necessary skills needed to solve mathematical problems starting from understanding the problem, devising and carrying out a suitable plan, till reaching the solution and judging whether it is reasonable or not. For the Purpose of this study, the teachers' information about their classes was considered when choosing the skilled mathematical problem solvers to participate in the study.

3)Unskilled Mathematical Problem Solvers: Students who seemed not to have adequate necessary skills needed to solve mathematical problems starting from understanding the problem, devising and carrying out an adequate plan, till reaching and judging whether it is reasonable or not. For the Purpose of this study, the teachers' information about their classes was considered when choosing the unskilled mathematical problem solvers to participate in the study.

4) Van Hiele's Model of Geometrical Thinking: This model consists of five sequential levels of geometrical- thinking processes. These levels are: Level 1 (The Visual level). In this level, children are able to recognize a geometrical figure based on its physical appearance. Level 2(The Analytical Level). In this level, children are able to describe a geometrical figure based on its characteristics and components. Also, students at this level can build simple logical arguments using concrete reasoning. Level 3 (The Informal-Deduction Level). In this level, students can build simple logical arguments or complete a part of geometrical proofs using abstract reasoning. Level 4 (The Formal-Deduction Level), In this level, students can construct or create geometrical proofs using theorems, axioms and postulates. Level 5 (The Rigor Level). In this level, students can understand different types of geometrical system such as Euclidean and Non-Euclidean geometry using different systems of theorems and axioms (Van de Wale, 2001).

Results of Data Analysis

To answer the main research question "What are the differences that can be observed between ninth-grade skilled and unskilled mathematical problem solvers in terms of their behaviors of solving geometrical problems?", the following four analytical questions were posed:

1) What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to understand the geometrical problem?

2) What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to device a plan that help to solve the geometrical problem?

3) What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to carry out the devised plan to solve the geometrical problem?

4) What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to evaluate their solution of the geometrical problem?

Table (1) shows major differences that were found between skilled and unskilled mathematical problem solvers in terms of their behaviors of solving geometrical problems.

Polya's Step		Skilled Students (70)		Unskilled Students (170)	
	Students' Behavior	N*	%**	N*	%**
Understanding The Problem	Deciding What is Given	64	91	44	26
	Deciding What is Required	64	91	44	26
	Drawing a Suitable Geometrical Figure	60	86	37	22
Devising a Plan	Searching for Connections Between Given Data and the Unknown	58	83	34	20
	Searching for Related Problems	58	83	28	16
	Focusing on Key Words on the Problem	58	83	28	16
	Stating Suitable Formulas or Theorems	58	83	28	16
	Translating Key Words into Mathematical Language	58	83	28	16
Carrying out the Plan	Using Logical steps	58	83	22	13
	Using Formulas Correctly	58	83	22	13
	Getting the Right answers	58	83	22	13
	Verification with Examples and Induction Proofs	10	14	148	87
	Verification with logical Reasoning and Mathematical Proofs	60	86	22	13
Looking Back	Making Sure if the Answer is Reasonable	38	54	12	7
	Determining Whether there is Another Method of Finding the solution	24	34	0	0
	Stating Similar Problems for Which the Method will Work	24	34	0	0

Table(1): Frequency and Percentage of Types of Skilled and Unskilled Mathematical problem Solvers' Behaviors in Solving Geometrical Problems

*The number in each cell represents the average number of students who responded correctly to each part of the problems. For example, numbers of students who decided what is given from skilled students are 63, 65 and 64 to problems 1, 2, and 3 respectively. Therefore, the average number was 64.

** The percentage is calculated by dividing the average number by number of students for each case.

Regarding the first analytical research question "What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to understand the geometrical problem?", table (1) shows that most skilled mathematical problem solvers as compared to unskilled mathematical problem solvers started by breaking dawn each problem into its main parts. It is shown from table (1) that most of the skilled mathematical problem solvers (91%) stated correctly what is given and what is the unknown from the problem, whereas only 26% of the unskilled mathematical problem solvers did so. Moreover, 86% of the skilled mathematical problem solvers graphed suitable geometrical shapes needed to solve the given problems, whereas only 22% of the unskilled mathematical problem solvers did so.

Regarding the second analytical research question "What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to device a plan that help to solve the geometrical problem?", table (1) shows that a large number of skilled mathematical problem solvers (83%) started their efforts of devising suitable plans needed to solve geometrical problems by searching for major connections between given data and the unknown then searching for related problems they solved before. Moreover, 83% of the skilled mathematical problem solvers used accurate geometrical concepts and theorems needed to solve the given problems; Those students stated and used correctly the following relevant geometrical concepts, theorems and formulas: The sum of the three interior angles of the triangle, The Pythagorean Theorem, The Concept of Similarity and Congruence of Triangles, The Midpoint of a Segment Line, The Area of a Triangle. The Concept of Perpendicular. On the other hand, only a few number of the ubskilled mathematical problem solvers (20%) were able to search for possible connection between given data and the unknown, and only (13%) stated related problem and mentioned suitable geometrical concepts, theorems and formulas needed to solve the given geometrical problems.

Regarding the third analytical research question "What are the observed differences between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to carry out the devised plan to solve the geometrical problem?”, table (1) shows that a large number of the skilled mathematical problem solvers (83%) were able to follow up logical steps in solving geometrical problems, whereas only a few number of unskilled mathematical problem solvers(13%)did so. Also, a large number of the skilled mathematical problem solvers(83%) were able to apply the suitable concepts, theorems and formulas to get the right answers, whereas only a few number of the unskilled mathematical problem solvers (13%) did so.

Moreover, table (1) shows other major differences between skilled and unskilled students in their efforts of solving geometrical problems related to their way of handling geometrical arguments and proofs; for example, a large number of the skilled mathematical problem solvers(86%) handed their arguments based on logical steps and formal reasoning, whereas a large number of the unskilled mathematical problem solvers (87%) handled their geometrical arguments and proofs based on examples and informal reasoning and only 13% were able to handle their arguments based on formal reasoning.

Finally, regarding the fourth analytical research question "What are the observed differences observed between ninth-grade skilled and unskilled mathematical problem solvers in terms of their efforts to evaluate their solution of the geometrical problem?", table(1) shows that around half of the skilled mathematical problem solvers(54%) were able to make sure if their solutions are reasonable and make sense to them, whereas a very few number of the unskilled mathematical problem solvers (7%) did so. But when it came to stating other methods of solving the problem or stating similar problems for which the method will work only around one-third of the skilled mathematical problem solvers(34%) stated other methods of finding the solution and stated similar problems for which the methods will work, whereas no one of the unskilled mathematical problem solvers did so.

Discussion of Findings and Conclusion

This study was conducted to search for major differences that might be found between skilled and unskilled mathematics problem solvers in solving geometrical problems. Overall findings indicated that skilled and unskilled mathematical problem solvers are different in how they think and the way they solve geometrical problems. There are three possible reasons that could be given as an evidence to support the conclusion made. First, most skilled mathematical problem solvers seemed to engage in a careful planning of the geometrical problem through focusing on relevant information on the problem in their effort of devising an adequate strategy of finding solution using a suitable geometrical graphs, concepts and formulas, then carrying out the plan to get the correct solution and evaluating the result. On the contrary, most unskilled mathematics problem solving seemed to analyze the problem based on irrelevant information on the problem to get quickly a wrong or incomplete solution. This result is consistent with overall findings of other related studies such as Kirkley (2003), Lazakidon et al. (2007), Osta & Labban (2007), Comesana (2009), Dockter & Heller (2009). These research studies indicated that unskilled students often don't have adequate mathematical and language skills needed to solve mathematical problems and always look at the problem as a whole focusing on the superficial and irrelevant information without displaying any concern about major connections between given data and the unknown, whereas skilled students always analyze the problem based on its mathematical structure and search for major connections between given data and the unknown. Moreover, even if both groups might sometimes develop a plan for solutions, they differ in carrying out the plan and evaluating the results.

Second, most skilled mathematical problem solvers seemed to have strong and adequate mathematical knowledge relating to geometrical concepts, formulas and procedures that are necessary to solve geometrical problems, whereas most unskilled mathematical problem solvers seemed not to have such mathematical knowledge and do not have a clear link between the geometrical concepts, formulas and their applications to a specific problem or situation. This result is consistent with overall findings of other research studies such as Kirkley (2003), Lazakidon et al. (2007), Lesh et al. (2008), Dockter & Heller (2009). These research studies indicated that one major factor that impede unskilled students ability to solve mathematical problems is their misunderstanding of mathematical concepts and theorems and their use in specific problems, whereas skilled students don't have such obstacles. As a result, skilled students are more successful than unskilled students in solving mathematical problems.

Third, most skilled mathematical problem solvers seemed to build their arguments and validate their geometrical proofs based on general cases using logical steps and formal reasoning, whereas most unskilled mathematical problem solvers seemed to build their arguments and validate their geometrical proofs based on examples and special cases using informal reasoning. This case may lead to the conclusion that most skilled students may reach the formal-deduction level (level 4 according to Van Hiele’s model) of geometrical thinking in which students can produce proofs and draw conclusions based on abstract thinking, but in contrast most unskilled students are still at the informal-deduction level or less (level 3 or 2 according to Van Hiele’s model) in which students can produce proofs and draw conclusions based on special cases and concrete thinking (Van de Wale, 2001). This result is consistent with overall findings of other research studies such as Stylianon et al. (2006), Osta & Labban (2007),Dockter & Heller (2009),Panaoura & Gagatsis (2009), Jones et al.(2012), Inglis & Alcock (2012). These research studies indicated that unskilled students build their reasoning based on surface features of arguments using concrete approaches and informal thinking, but skilled students build their reasoning based on relevant features of arguments using abstract approaches and formal thinking.

Classroom Implications and Suggestions for Further Research

1)Findings of this study support the assumption that the behaviours that distinguish skilled from unskilled mathematics problem solvers would suggest that developers of mathematics curriculum and textbook writers and mathematics teachers should consider instructions of problem solving that is based on a variety of approaches to meet the needs of both skilled and unskilled students.

2) The overall results indicated that students from both groups don't pay attention to the looking-back step of Polya’s strategy of problem solving, especially thinking of solving problems in different methods or posing related problems. Therefore mathematics textbooks publishers, textbooks writers and teachers of mathematics should pay attention to this important step in solving geometrical problems because this step may help in creating what is called reflective problem solving students.

3) The essay test that was used in the current study was administered to both skilled and unskilled groups as part of a research effort, and some students may not have taken it seriously. Therefore, it might be appropriate for ninth-grade mathematics teachers to administer this test as a part of students' evaluations in order to check whether the results obtained from the present study hold true in a classroom evaluation setting or not.

4)This study focuses on major differences between skilled and unskilled mathematical problem solvers on their behaviour of approaching geometrical problems based on cognitive factors, but there are other factors that could be found between skilled and unskilled students and may effect their ability in solving geometrical problems such as their motivations toward solving geometrical problems and it could be appropriate for further research.

5) Overall findings of data analysis revealed that most ninth-grade skilled students may reach the formal deduction level according to Van Hiele’s model of geometrical thinking, whereas most ninth-grade unskilled students may only reach the informal level or less. But classifying students according to Van Hiele’s model was beyond the scope of the present study and could be appropriate for further research.

6) This study used quantitative-content analysis methodology to analyze students’ responses to a geometrical essay test. This methodology may not give a complete picture regarding major differences between skilled and unskilled students in solving geometrical problems. This study recommends research studies that used a qualitative methodology. In this methodology, a think-aloud technique protocol may be employed and students from both groups will be asked to explain their reasoning aloud while solving geometrical problems similar to those given in the current study.

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