# MULTI-LEVEL CODING AND MULTI-STAGE DECODING FOR MODULATIONS WITH HEXAGONAL CONSTELLATION

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**Abstract**

M-ary Modulation and convolutional encoder are commonly employed in modern wireless systems; the modulation has rectangular constellation and square decision regions, and the encoder is binary based. However, we have demonstrated previously that modulations with hexagonal decision regions are more energy efficient transmission. And we have introduced a system which is compatible with binary data. It provides improved energy efficiency and spectrum utilization and minimizes probability of error, as well as the peak-to-average-power ratio for OFDM systems. Also, we introduce a ternary convolutional encoder. Associated with the mentioned modulation, it outperforms some of the existing conventional schemes, and its computational complexity is comparable to that of a binary decoder with a similar number of states. In this paper, we combine multi-level coding to the hexagonal modulation to provide a system which is capacity-approaching. This concept may be use the ternary convolutional encoder introduced previously related to the constellation used. Here, we work with three different hexagonal constellations which are the 6QAM-H, 8QAM-H, and 12QAM-H, and we design a multilevel coding based on two or three levels. This approach is shown to provide improved probability of error and may be interesting for the further investigations about the multilevel unequal error protection with multistage decoding.

**Keywords:** Capacity, Hexagonal Modulation, LDPC, Binary, and Ternary Convolutional Codes, Multi-Stage and Parallel Independent Decoder, Bit-Interleaved Coded Modulation, Trellis Coded Modulation

## Introduction

For more than 60 years, channel coding has been widely studied. The purpose was to find digital communications systems that have a capacity and a performance close to the limits found by Shannon. The researchers were mainly interested by binary codes designed for rectangular power-of-two modulation. Based on these concepts, and under iterative decoding, LDPC codes can achieve the capacity of the AWGN channel for binary modulation. This has been improved by introducing an irregularity in the parity check matrix of the LDPC code. The main drawback of LDPC codes is in the relatively high complexity of the encoder.

Previously, I proposed new modulation schemes based on hexagonal constellation and evaluate the corresponding BER performance. The new QAM-H schemes proposed was contain 3, 6, 8, and 12 constellation points to represent 1 trit, 1-bit plus 1-trit, 3-bits, and 2-bits plus 1-trit, respectively. These schemes gave a better performance that the conventional schemes. After that, we employed a ternary convolutional coding to protect the ternary digit directly using the bit/trit interleaved coded modulation technique. For QAM-H with hybrid bit and trit information, we consider a combination of binary and ternary coding. The BER performance was evaluated for different modulation schemes, including the conventional QAM, with code rates 1/2 and 3/4 to conform to the IEEE 802.11 standard. The results showed that, the new modulation and coding schemes not only can provide finer granularity adjustment for AMC, but also can replace some of the existing schemes by achieving a higher throughput with a lower BER for the given SNR region and confirmed our idea about the improving of the capacity by using the hexagonal modulation combined to the ternary convolutional coding with respect to the conventional modulation and coding.

Here, we are not interesting by the AMC scheme but our objective is the construction of capacity-approaching coding schemes for hexagonal modulations, by considering the class of coded modulations, known as Multi-Level Coded modulations (MLC), which can achieve capacity and it can be used to represent a wide range of coded modulations. This capacity-approaching using a MLC scheme will consider that it has equal/unequal capacity at each level. We will reduce largely the complexity of the system by changing the architecture of encoding and decoding. This concept may be interesting for the construction of a multilevel unequal error protection (UEP) with multistage decoding. The main results found in the literature, especially by Huber et al, regarding multilevel coded modulations are reviewed. We mainly focus on the capacity, the rate design rules and the labeling strategies of MLC. The principles of multilevel coding are applied. Two coding schemes based on bit-interleaved coded modulations and multilevel coding are shown.

The major contributions of this work are presented by the proposition of multilevel coded modulations schemes based on modulation with

hexagonal constellations with a bit/trit-interleaved coded modulation at each level.
The remainder of this paper is organized as follows. In Section II, we discuss the background and related work. Section III presents the multilevel encoding scheme, including the multistage decoding and the parallel independent decoding techniques. The system design and the system architecture are also presented in Section IV. The system performance is investigated in Section V through extensive trace-driven simulation. Finally, conclusions are given in Section VI along with the suggestions for future work.

## Related Work

*A. The basics of information theory*

The channel capacity C is defined as the maximum of the average mutual information I(X; Y) where the maximization is over all possible input probability distributions. That is,

$$C = \max_{P(x)} I\left(X;Y\right) \tag{1}$$

## Theorem 1: Channel coding theorem

*There exist channel codes that make it possible to achieve reliable communication, with as small an error probability as desired, if the transmission rate R ≤ C, where C is the channel capacity. If R > C, it is not possible to make the error probability approaches zero.*

The proof of the channel coding theorem and the calculations of the upper bound on the performance of optimal codes are based on a random coding argument and, hence, codes that achieve capacity and the performance of optimal codes may exhibit little or no structure making them not suitable for practical applications. Thus, it is of great interest to investigate the maximum reliable transmission rates and the best performance achieved by structured codes like lattice codes.

The main literature results regarding the capacity and random codes are reviewed. These results determine the best achievable performance. Digital communications systems with performance close to these limits must be found. This may be done using structured codes like lattice codes since these codes can not only achieve capacity but also perform as good as optimal spherical codes. The main problem with lattice codes is decoding. However, the results shown in the literature are not sufficient to validate lattice codes for the use in a transmission over the AWGN channel.

**Multilevel coded modulations**

The performance improvement, in the case of binary codes, is achieved by expanding the bandwidth of the transmitted signal by an amount equal to the reciprocal of the code rate. However, efficient digital communication systems have to be both power and bandwidth efficient, especially in the case of bandwidth-limited channels, where the digital communication system is designed using non-binary modulations. Coding and modulation are combined. We get the so called coded modulations. The most powerful applicable coded modulations systems are the well-known Trellis Coded Modulations (TCM) that were first described by Ungerboeck in 1982 and Multilevel Coded Modulations (MLC) introduced by Imai and Hirakawa. The common idea to these two methods is to optimize the code in Euclidean space rather than dealing with Hamming distance. The difference between TCM and MLC is in coding. In a TCM system, the LSB bits are encoded using any convolutional code and the MSBs remain uncoded. On the other hand, in an MLC system each bit of the signal point is individually protected by an individual code. Compared to TCM, the MLC approach has the advantage of providing flexible transmission rates. Furthermore, any code like block codes, convolutional codes, turbo codes, etc. … can be used as a component code.

**Multi-Level Coded Modulation**

**Multi-Level Encoding scheme**

The multilevel encoder is shown in figure 1. Each bit/trit is independently protected. Mapping is then used to select a point from the constellation. The code rate of each individual encoder is equal to $R_i$. The total rate of multilevel scheme is equal to

$$R = \frac{\sum_{i=1}^{L} k_i \cdot R_i}{\sum_{i=1}^{L} R_i} \qquad (2)$$

Where $k_i$ is the number of information bits/trits at each level $i$, and $L$ is the number of level.

Thus, a block of $k$ information bits/trits is encoded into n vectors. Each of those vectors is used to select a signal point from the constellation.

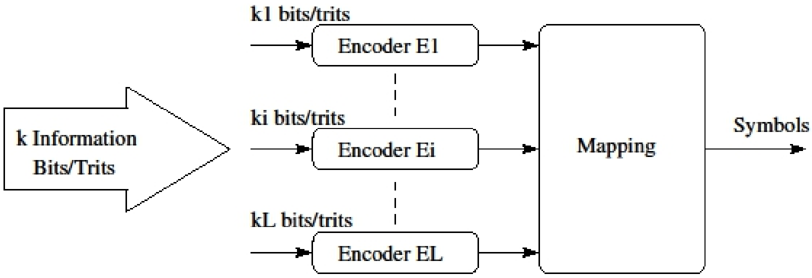



Fig. 1: Multilevel encoding scheme.

## Multi-Stage Decoding

Therefore, a sub optimal decoding technique, known as multistage decoding shown in figure 2 can be used. The constituent codes at each level i, are successively decoded. We begin by decoding the level 1. Then, at each level i, the corresponding decoder uses the decided bits/trits of lower levels in order to compute his metric and determine its own decided bits/trits.

In order to achieve better performance, interleaving between the levels, forwarding of reliability information from lower to higher levels using soft-output decoding algorithms, and especially iterative MSD have been proposed. Simulation results show that the performance of MSD is almost equal to the one we get using iterative MSD. This can be explained by the fact that MLC combined to MSD achieves the capacity of the modulation transmitted over the physical channel and that this performance is only slightly inferior from the one we have for an optimum maximum likelihood decoder. We should also note that the use of interleaving between levels increases the delay of data and the codeword length. Using such long component codes in direct way would yield a quite better performance.

Finally, it is easy to show that the capacity $C$ of a multilevel coding modulation system is equal to the sum of the capacities $C_i$ of the equivalent channels $i$, ($i$ = 1… $L$)

$$C = \sum_{i=1}^{L} C_i \tag{3}$$

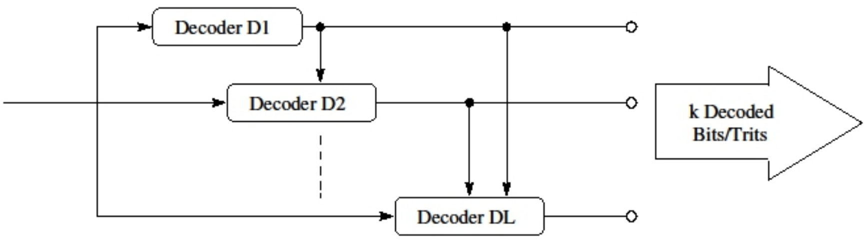



Fig. 2: Multistage Decoding of an MLC scheme.

The capacity $C$ can be approached via multilevel encoding and multistage decoding, if and only if the individual rates $R_i$ are chosen to be equal to the capacities of the equivalent channels, i.e. $R_i = C_i$.
From the simulations results we can conclude that:

1. Capacity can be approached via structured multilevel codes.
2. Sub-optimum multistage decoding can achieve the capacity if the rates $R_i$ are equal to $C_i$.

**Parallel Independent Decoding**

In MLC with parallel independent decoding (PDL) of the individual levels shown in figure 3, the decoder $D_i$ makes no use of the decisions of other levels $i \neq j$ .

Therefore, the individual rates are bounded by

$$R_i \leq C_{PDL_i},\ i = 1, 2, ..., L \quad (4)$$

Where $C_{PDLi}$ is the capacity of the equivalent channel $i$ when PDL is applied.

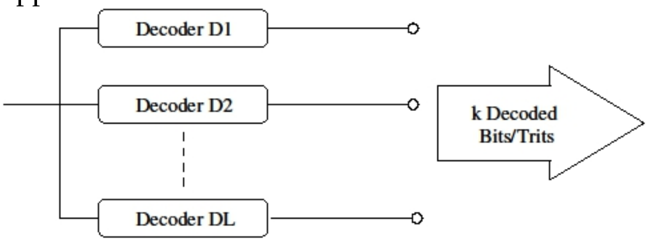


Fig. 3: Parallel independent decoding of an MLC scheme.

Thus, $C_{PDL} \leq C$, where $C$ is the total capacity of the MLC scheme when MSD is applied.

Therefore, PDL approach can be viewed as a sub-optimum decoding technique of an optimum MLC scheme. The capacity CPDL strongly depends on the labeling strategy. The gap to the total capacity C is small when Gray labeling is used, which cannot be used with hexagonal modulation. Finally, we should note that a BICM modulation, implemented using an infinite length ideal interleaver, can be viewed as an MLC scheme based on Gray labeling and decoded using a PDL. Therefore, BICM and MLC combined to Gray labeling and PDL have the same capacity. The small degradation in capacity due to PDL is similar to the one found by Caire et al regarding the small capacity loss of BICM over AWGN channel when Gray labeling is applied to an 8-PSK and a 16-QAM constellation.

**System Model**

As mentioned in our previous work, the number of constellation points in the most compact hexagonal constellations is not always an integer power-of-two. Therefore, to fully utilize these constellations for modulation, both bits and trits should be transmitted. This requires a reinvestigation of the modulation constellation geometry, the mapping of bits and trits to constellation points, the error control coding, and the multiplexing of bits and trits which were done previously.

**System Design**

Here, we propose to work independently on three different hexagonal constellations, these constellations are: 6QAM-H, 8QAM-H, and 12QAM-H schemes which are presented in figure 4. As there are more neighboring points (with the smallest distance to a constellation point) using hexagonal modulation, careful mapping is required to limit the number of bits and/or trits in error due to a symbol error. We cannot obtain a Gray type of mapping (with only one bit/trit difference between neighboring points), because the number of neighbors with hexagonal tiling often exceeds the number of bits and trits represented by each symbol.

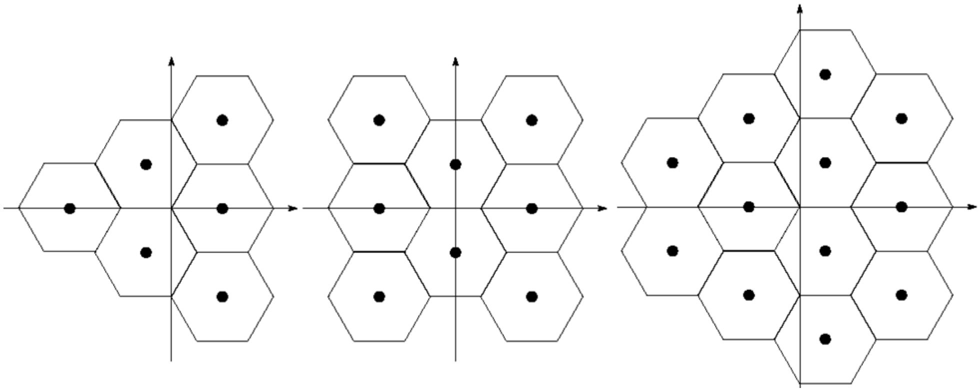


Fig. 4: 6QAM, 8QAM, and 12QAM Hexagonal constellations.

An exhaustive search for the optimal mapping is only possible when the number of constellation points is small as the computational cost increases exponentially with the number of points. Previously, we used the following design principle to search for good mappings. Starting from a bit, the constellation points are divided into two clusters. Similarly, starting from a trit, the constellation points are divided into three clusters. Then, ‘0’ and ‘1’ (for a bit) or ‘0’, ‘1’ and ‘2’ (for a trit) are arbitrarily assigned to each of the clusters. For the remaining bits or trits, binary or ternary numbers are assigned to the points in one cluster first, and in turn to the points in the other clusters. The same number is assigned to neighboring points in different clusters as much as possible. The mapping of the three mentioned constellation is represented in figure 5 which shows that some bits/trits are more protected by the constellation from other bits/trits. This robustness of some bits/trits than others is shown, for example, by the number of transitions from ‘0’ to ‘1’ and vice versa for any bit and from ‘0’ to ‘1’ and/or to ‘2’ for any trit represented by the transition boundaries in figure 5.

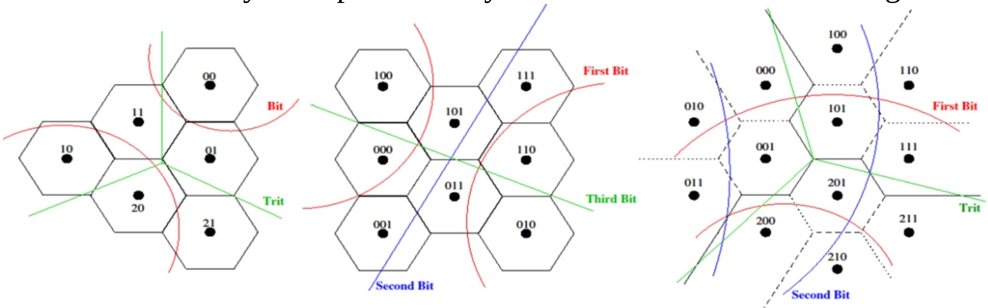
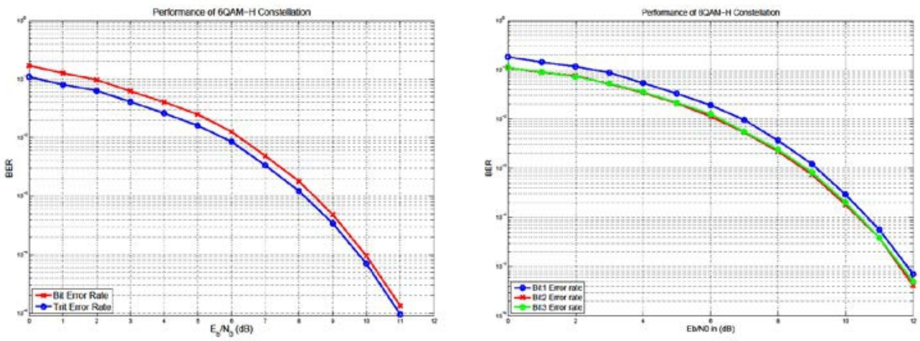



Fig. 5: Mapping and transition boundaries for Hex- 6QAM, 8QAM, and 12QAM.

Based to the transitions number, we can remark simply, for example, the first bit of the 8QAM-H constellation is less protected by the mapping since it has two transitions thus with MLC this bit should be more protected than the other. We have the same remark for the bit in the 6QAM-H constellation. In the 12QAM-H constellation, the first and second bits have slightly the same protection which is less than the protection of the trit. The figure 6 shows the simulation results of these constellations, where we present the bit/trit error rate of each bit/trit in these constellation, these results confirm our conclusions about the mapping protection of each bit/trit. For example, we can see that the error rate of the trit is smaller than the one of the bit for all SNR in the 6QAM-H constellation, and the error rate of the first bit is bigger than the one of the both other bits for all SNR in the 8QAM-H constellation, and finally in the 12QAM-H constellation the error rate of the trit is better than the two bits.



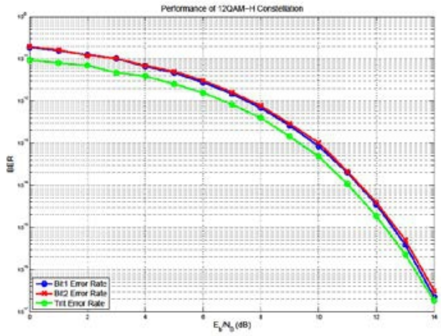



Fig. 6: Bit/Trit Error Rate Simulations results of the considered constellations

From the results shown in figure 6, we decide to protect the bits/trits unequally, where we will change the coding rate of each level with respect to the protection degree of each bit/trit, without the changing the total coding rate which should be near 1/2 to compare it the half rate coding without multilevel.

**Level Encoder**

Convolutional coding has been widely used in wireless systems because of the implementation simplicity and good performances. A binary/ternary convolutional code (BCC/TCC) can be represented by the parameters (*n*; *k*; *m*) where *k* and *n* are the numbers of input and output bits, respectively, and *m* is the encoder memory size. The code rate is *k*/*n*. The TCC encoder was designed previously in our work. At the receiver, the received coded bit stream is decoded to recover the original message bit stream. The Viterbi algorithm follows the maximum-likelihood decoding approach and is widely used in practice because of the low implementation complexity.

Here, we will use at each level a binary or ternary convolutional encoder with respect to the bit or trit input. These convolutional encoders will have different coding rate to compensate the difference in protection by the mapping, with the only condition of obtaining a total effective coding rate for all level encoders equal to half. Figure 7 shows the structure of the three encoders used in our project, two for binary encoding and the other one for ternary encoding. The variation of coding rate in each level is done using a puncturing; the puncturing pattern differs from one rate to another.

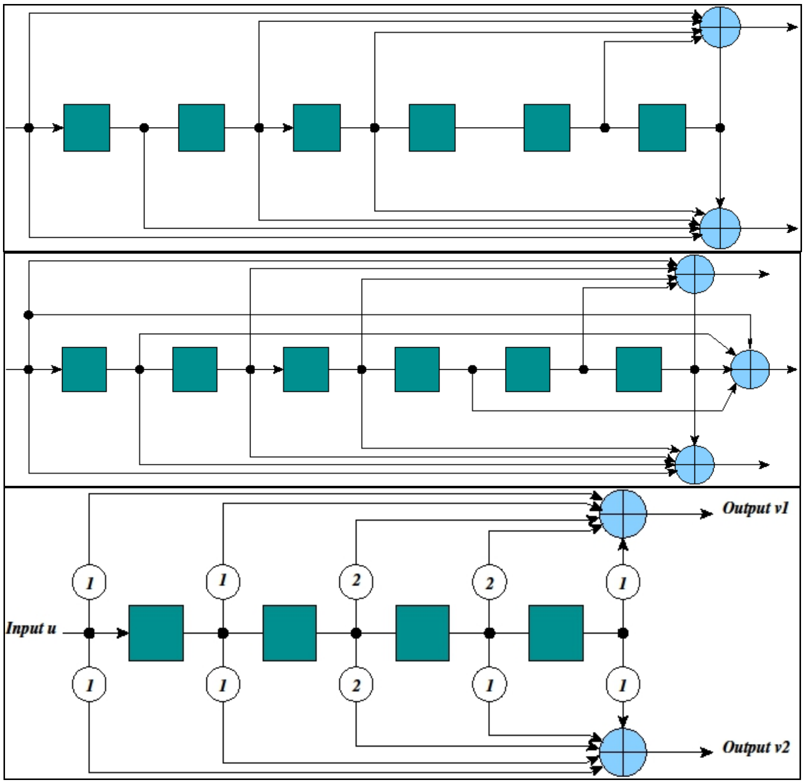



Fig. 7: Binary/ternary convolutional level encoders.

The use of interleaver is desirable to separate the burst errors. So, bits or trits are interleaved within a packet at each level. The same interleaver is used for all level.

## System Architecture

With respect to the constellations, the messages may be divided into two or three queues (in our cases) equal to the number of bits/trits presented in the labeling, i.e., with 6QAM-H constellation we need two queues one for bits and the other for trits but with 12QAM-H three queues are necessary, 2 for bits and 1 for trits.

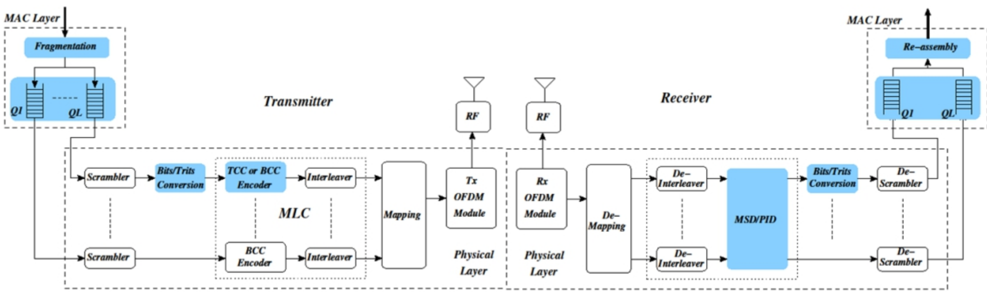



Fig. 8: The proposed system architecture.

The block diagram of the transmitter will contain simultaneously: a fragmentation block which divide the message into two or three queues, with respect to the constellation, a bit to trit conversion block which is connected to the output of the trits queue (if used), two or more convolutional encoder, after that each stream is interleaved and entered to the modulator block, and finally a multiple access and RF block which is used to carry the message on a certain carrier frequency corresponding to the standard used. In the block diagram of the receiver, we will see especially the Multi-Stage Decoder or the Parallel Independent Decoder, and after that queues will be re-assembled.

Figure 8 shows the system architecture, this proposed non-binary multi-level encoded communication system is compatible with conventional systems and does not require additional communication overhead.

## Simulation Results and Analysis

A key performance index to evaluate the capacity-approaching is the BER given a received SNR over an AWGN channel. We consider a received SNR from 0 to 11 dB and examine the BER. The received signal to noise ratio is considered here as $E_b/N_0$ where $E_b$ is the received energy per bit, and $N_0$ is the noise power spectral density. Monte Carlo simulation by MatLab is used to obtain the results shown in the following figures. For the evaluation, a rate 1/2 one level BICM for was compared to rate approximately equal to 1/2 multi-level BICM for hexagonal modulation. The code chosen are represented in figure 7 and with 64 states for Binary and 81 states for ternary code. The polynomial generators are for the ternary code [11221; 11211], for the rate 1/2 binary code [1011011; 1111001], and for the rate 1/3 binary code [1011011; 1111001; 1100101]. The rate 1/3 binary code is used with puncturing to obtain certain necessary coding rates.

## 6QAM-H Modulation

For the 6QAM-H constellation, which contains one bit and one trit, we cannot make more than two-level coded modulation, one for bits and the other one is for trits. As shown in figure 6, the trit error rate is better than the bit error rate, so we will choose to protect the bits with a stronger error correction code. Here we will encode the bits stream with rate 2/5 binary code (the 1/3 rate binary code is used here with puncturing), and the trits stream with rate 3/5 ternary code. We can choose other rates without changing the total encoding rate which is here 0.55, but these two rate gave the best performances.

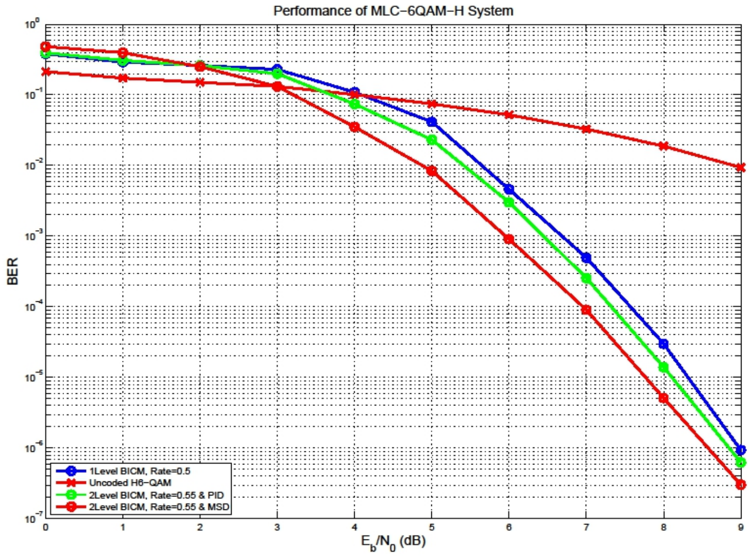



Fig. 9: Conventional BICM vs. Multi-Level coded schemes for 6QAM-H.

In figure 9, we show the performance of the uncoded, coded in one-level, and coded in two-level 6QAM-H modulation, in addition to the performance of two decoding techniques the PID and the MSD, presented previously, used with two-level coded modulation. The results show that the performance of coded 6QAM-H is much better than the uncoded one. When we compare the one-level and the two-level coded, we can remark that the two-level coded modulation improves the performance whatever the decoding technique used, this improvement is bigger when we used the Multi-Stage Decoding, and the gain at BER=$10^{-4}$ is approximately 0.2 dB for the Parallel Independent Decoding and it is up to 0.6 dB for the MSD, we should not forget here that these improvement are obtained in despite that the total encoding rate is 0.55 bigger than for the one-level case, which indicates that with two-level encoding we obtain an improvement in the probability of error and in the total throughput.

## 8QAM-H Modulation

For the 8QAM-H constellation, which contains three bits and no trits, we can make two-level or three-level coded modulation. In the case of two-level coded modulation, we can choose the first level for the LSB bit protected by a stronger error correction code and the second one is for the two remaining bits which have better performance, as shown in figure 6, and can less protected by another code. Figure 6 shows also that the performance of the second bit is slightly better that the MSB bit, so we can make three-level coded modulation and give to each level a different coding rate. Better performance is obtained when we choose three-level coding, so here we will encode the LSB bits stream with rate 2/5 binary code, the second bits stream with rate 4/7 binary code, and the third bits stream with rate 1/2 binary code, which give a total encoding rate approximately equal to 0.5.

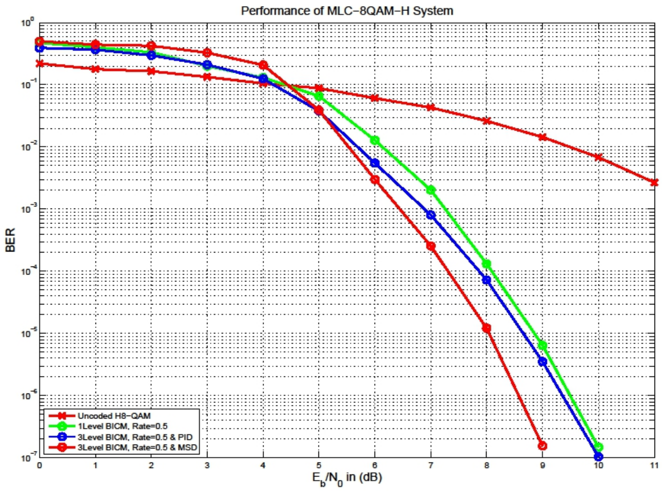



Fig. 10: Conventional BICM vs. Multi-Level coded schemes for 8QAM-H.

Again, we show the performance of the uncoded, coded in one-level, and coded in three-level 8QAM-H modulation in figure 10, with the performance of two decoding techniques the PID and the MSD. The results show that the performance of coded 8QAM-H is much better than the uncoded one, and also the three-level coded modulation improves the performance whatever the decoding technique used. The PID technique has 0.2 dB gain at $10^{-4}$, but this performance converges to the one-level coded modulation at high SNR. The MSD technique has better performance equal to approximately 1 dB at BER=$10^{-4}$ and this gain is slightly increasing with the SNR.

## 12QAM-H Modulation

Finally, with respect to the 12QAM-H constellation, which contains two bits and one trit, we also can design our system with two-level or three-level coded modulation. In the case of two-level coded modulation, we can choose the first level for the trit and the second one is for the two remaining bits. Figure 6 showed that the two bits are less protected from the modulation than the trit, so we could choose a relatively high coding rate for the trits stream and a lower coding rate for the bits stream. Also, we can remark in figure 6 that the performance of the first bit is slightly better that the second one, so we will design our system with three-level coded modulation and give to each level a different coding rate since better performance will be obtained. The trits stream will be encoded with 4/7 ternary code, the first bits stream will be encoded with 1/2 binary code, and the second bits stream with 3/7 binary code. The total encoding rate given with these choices is slightly bigger that 0.5 and it is equal to 0.507.

Figure 11 presents the same cases shown in figures 9 and 10 but for 12QAM-H modulation. The results, presented in figure 11, show a big improvement of coded 12QAM-H with respect to the uncoded one, which is improved again with the three-level coded modulation whatever the decoding technique used. With the PID technique we obtain a gain equal to 0.4 dB at BER=$10^{-4}$, but again this performance converges to the one-level coded modulation at high SNR. With the MSD technique a better performance is obtained and the gain is increased until 0.9 dB at BER=$10^{-4}$ and this gain is slightly increasing with the SNR.

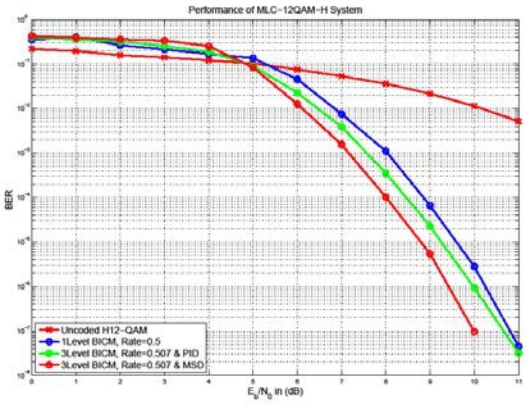



Fig. 11: Conventional BICM vs. Multi-Level coded schemes for 12QAM-H.

Like we have seen from the simulations results the MLC system improves, in general, the performance whatever the decoding scheme used. This improvement increase or decrease with respect to the decoding technique. Moreover, when the PID is used the performance converges to the performance of one level coded system at high SNR, this is because the limitation of PID technique and the fact that it didn’t used the information of the previous decoded bit, which is the case with the MSD technique, where we obtained an increasing gain whatever the SNR is increased.

## Conclusion

In this paper, the design and implementation of a multi-level encoding (MLC) system for modulation with hexagonal constellations was considered which employs both binary and ternary error control coding. The proposed system deviates from the conventional one level bit interleaved coded modulation, although it is compatible with existing systems. Further, the decoding scheme can be implemented using two different techniques; The Multi-Stage decoding (MSD) technique, which is more complex but improves the BER performances of the MLC system, and the Parallel independent decoding (PID) technique that reduces the complexity of the receiver but unfortunately decrease the improvement of the MLC system. The proposed system can be implemented by simply cloning the coding scheme into several levels, here two- or three-level. Three hexagonal modulation schemes were examined as examples, but others are possible and the design should depend on the particular application. Finally, even though MLC systems with hexagonal modulation have not received much attention yet, we showed here their elegance and efficiency, which permit us to predict their necessity in the future.

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