# A TWO-STAGE GROUP SAMPLING PLAN BASED ON TRUNCATED LIFE TESTS FOR A EXPONENTIATED FRÉCHET DISTRIBUTION

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**Abstract**

A two-stage group acceptance sampling plan is proposed for truncated life tests when the life time of an item follows exponentiated Fréchet distribution. The decision about the lot acceptance can be made in the first or second stage according to the number of failures from each group. In this paper, we determined number of groups required for each of two stages for the underlying lifetime distribution so as to minimize the average sample number under the constraints of satisfying the producer's and consumer's risks simultaneously. Single-stage group sampling plans are also considered as special cases of the proposed plan and compared with the proposed plan in terms of the average sample number and the operating characteristics.

**Keywords:** Exponentiated Fréchet distribution; average sample number; consumer’s risk; operating characteristic

## Introduction

Acceptance sampling is a methodology commonly used in quality control. The aim is to make an inference about the quality of a batch/lot of product from a sample. Depending on what is found in the sample the whole lot is either accepted or rejected and rejected lots can then be scrapped or re-worked. A lot is accepted if the number of failures during the test time does not exceed the acceptance number. The acceptance sampling plans based on truncated life tests are proposed by many authors including, Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam *et al.* (2001), Baklizi (2003), Baklizi and El Masri (2004), Rosaiah and Kantam (2005), Rosaiah *et al.* (2006), Tsai and Wu (2006), Balakrishnan *et al.* (2007), Aslam (2007) and Rao *et al*. (2008).

Generally in these plans a single item will be tested in a tester. In practical situations, there may be a tester in which multiple items can be installed at the same time. Hence, a group acceptance sampling plan should be used and designed. This group sampling plan may be more desirable than the ordinary sampling plan in terms of the test time because the former can test a larger number of items in a given test time. One simple decision rule in a group sampling plan is to accept the lot if the number of failures from each group does not exceed the specified number. The group acceptance sampling plans will be useful for sudden death testing. These sampling plans are proposed by many authors like Balasooriya (1995), Pascual and Meeker (1998), Wu *et al*. (2001), Jun *et al*. (2006), Aslam and Jun (2009a, 2009b) and Rao (2009, 2010).

However, in many applications, the small percentile of lifetime is required to meet engineering design purpose. Lately, Balakrishnan *et al.* (2007), Lio *et al*. (2009), Lio *et al*. (2010), Rao and Kantam (2010) and Rao (2013) developed the acceptance sampling plans for the lifetime percentiles. They argued that mean lifetime of the product may not satisfy the requirement of engineering design consideration. Acceptance sampling plans developed using mean life could pass a lot which has a low percentile below the required specification. Moreover, most of the employed life distributions are not symmetric. In viewing Marshall and Olkin (2007), the mean life may not be adequate to describe the central tendency of the skewed distribution. Furthermore, exponentiated Fréchet distribution is skewed distribution, thus we study the two-stage grouped sampling plans based on percentiles. To the best of our knowledge, two-stage group sampling plans based on the exponentiated Fréchet distribution using percentiles have not ascertained in literatures. The purpose of this article is to develop the two-stage group plans for the exponentiated Fréchet distribution percentiles.

All the aforementioned works in the area of acceptance sampling plans were developed for single sampling plan in terms of the sample size. It has been well known that a double sampling plan performs better than a single sampling plan. Therefore, Aslam *et al.* (2010) rightly pointed out that, there is a need of developing a version of double sampling plan for a life test using groups, which they called as two-stage group sampling plan. The two-stage group sampling plan indeed further reduce the sample size. Recently, Aslam *et al.* (2010) studied a time truncated two-stage group sampling plan for Weibull distribution. Aslam *et al.* (2011) developed two-stage group acceptance sampling plan based on truncated life tests for a general distribution. Rao (2013) formulated a two-stage group sampling plan based on truncated life tests for a M-O extended exponential distribution.

The purpose of this paper is to develop a two-stage group sampling plan for the truncated life tests when the lifetime of a product follows the exponentiated Fréchet distribution introduced by Nadarajah and Kotz (2003). A recent book by Kotz and Nadarajah (2000), which describes Fréchet distribution and lists over fifty applications of this distribution in various fields. The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of the exponentiated Fréchet distribution respectively, are given by

$$f(\mathrm{t};\sigma,\lambda,\alpha)=\frac{\alpha\lambda}{\sigma}\left(\frac{\sigma}{t}\right)^{\lambda+1}\exp\left\{-\left(\frac{\sigma}{t}\right)^{\lambda}\right\}\left[1-\exp\left\{-\left(\frac{\sigma}{t}\right)^{\lambda}\right\}\right]^{\alpha-1} \quad (1)$$

And $F(\mathrm{t};\sigma,\lambda,\alpha)=1-\left[1-\exp\left\{-\left(\frac{\sigma}{t}\right)^{\lambda}\right\}\right]^{\alpha}, t>0, \lambda,\sigma>0$ and $\alpha$>0. (2)

Where $\sigma$ is a scale parameter and $\lambda$ and $\alpha$ respectively shape parameters. The standard Fréchet distribution is the particular case of (2) for $\alpha$ =1.

The 100$q$-th percentile of the exponentiated Fréchet distribution is given as:

$$t_q=\sigma\eta, \quad \text{where } \eta=\left(-\ln\left(1-(1-q)^{1/\alpha}\right)\right)^{-1/\lambda} \quad (3)$$

In this paper, we propose a two stage group acceptance sampling plan for truncated life tests when the life time of the product is assumed to follow an exponentiated Fréchet distribution. We construct the tables for finding the number of groups required for each stage of the proposed plan so as to minimize the average sample number under the constraints of satisfying the producer's and consumer's risks simultaneously. The design of two-stage plan is given in Section 2. The comparison with the single-stage group sampling plan is given in Section 3. Methodology is illustrated with industrial application in Section 4 and some conclusions are given at Section 5.

**Two-stage group sampling plan**

In this section, we develop a two-stage group sampling plans for exponentiated Fréchet distribution which was introduced by Aslam *et al.* (2010). The authors proposed the following two-stage group sampling plan, where testers having the group size of *r* are assumed to be prefixed.

*First stage*: Draw the first random sample of size $n_1$ from a lot, allocate $r$ items to each of the $g_1$ groups (or testers) so that $n_1 = r\,g_1$ and put them on test for the test time $t_0$. Accept the lot if the number of failures from each group is $c_1$ or less. Truncate the test and reject the lot as soon as the number of failures in any group is larger than $c_2$ before $t_0$. Otherwise, go to the second stage.

*Second stage*: Draw the second random sample of size $n_2$ from a lot, allocate $r$ items to each of $g_2$ groups, so that $n_2 = r\,g_2$ and put them on test for $t_0$. Accept the lot if the number of failures in each group is $c_1$ or less. Truncate the test and reject the lot if the number of failures in any group is larger than $c_1$ before $t_0$.

Many sampling plans are the special cases of the present group sampling plan. When $r = 1$, the present plan becomes a special scheme of a double sampling plan. The present two-stage group sampling plan with $c_1 = c_2$ reduces to a (single-stage) group sampling plan. The major design parameters of the present plan will be the number of groups in each of two stages. The acceptance numbers of $c_1$ and $c_2$ can be determined as well, but it is found that the plan with $c_1$=0, $c_2$=1 minimizes the average sample number (ASN). The two-stage sampling plan with $c_1$=0, $c_2$=1 can be practically useful because lower acceptance numbers are often preferred by customers.

The lot acceptance probability from the first stage in the two-stage group sampling plan will be given by

$$P_a^{(1)} = \sum_{i=0}^{c_1} \binom{rg_1}{i} p^i (1-p)^{rg_1-i} \qquad (4)$$

where $p$ is the probability that an item in a group fails by time $t_0$. It would be convenient to determine the termination time $t_0$ as a multiple of the specified percentile $t_{q_0}$ such that $t_0 = \delta_q^0 t_{q_0}$ for a constant $\delta_q^0$. Then, the probability of a failure occurs during the termination time $t_0$ denoted by $p = F(t_0;\sigma,\lambda,\alpha)$, is obtained as

$$p = 1 - \left[1 - \exp\left\{-\left(\frac{t_0}{\sigma}\right)^{-\lambda}\right\}\right]^{\alpha} = 1 - \left[1 - \exp\left\{-\left(\frac{\eta\delta_q^0}{\left(t_q/t_{q_0}\right)}\right)^{-\lambda}\right\}\right]^{\alpha} \qquad (5)$$

The lot rejection probability from the first stage is obtained by

$$P_r^{(1)} = 1 - \sum_{i=0}^{c_2} \binom{rg_1}{i} p^i (1-p)^{rg_1 - i} \qquad (6)$$

The lot acceptance probability from the second stage will be

$$P_a^{(2)} = \left[1 - \left(p_a^{(1)} + p_r^{(1)}\right)\right]\left[\sum_{i=0}^{c_1} \binom{rg_2}{i} p^i (1-p)^{rg_2 - i}\right] \qquad (7)$$

Therefore, the lot acceptance probability in the proposed two-stage group sampling plan is given by

$$L(p) = P_a^{(1)} + p_a^{(2)} \qquad (8)$$

For the case of $c_1 = 0,\ \mathrm{c}_2 = 1,$ the acceptance probability of Equation (8) Reduces to $L(p) = (1-p)^{rg_1} + rg_1 p\left(1-p\right)^{rg_1 - 1}\left(1-p\right)^{rg_2}$ (9)

When the quality level based on the percentile ratio $t_q / t_{q_0}$ between the true percentile $t_q$ and targeted percentile $t_{q_0}$, the two-point approach of finding the design parameters is to determine the minimum number of groups, $g_1$ and $\mathrm{g}_2$, to satisfy the following two inequalities

$$L\left(p / t_q / t_{q_0} = \delta_1\right) \le \beta \qquad (10)$$

$$L\left(p / t_q / t_{q_0} = \delta_2\right) \ge 1 - \gamma \qquad (11)$$

where, $\delta_1$ is the percentile ratio at the consumer's risk and $\delta_2$ is the percentile ratio at the producer's risk. In this study, the ratio $\delta_1$ is setting as 1. Let $p_1$ and $p_2$ are the failure probabilities of corresponding to consumer's and producer's risks, respectively.

Where

$$p_1 = 1 - \left[1 - \exp\left\{-\left(\eta\delta_q^0\right)^{-\lambda}\right\}\right]^{\alpha} \ and \ p_2 = 1 - \left[1 - \exp\left\{-\left(\frac{\eta\delta_q^0}{\left(t_q / t_{q_0}\right)}\right)^{-\lambda}\right\}\right]^{\alpha} \qquad (12)$$

There may exist a multiple solutions of design parameters satisfying equations (10) and (11), so we need to select them to minimize the ASN for our two-stage group sampling plan. The ASN for the two-stage sampling plan is obtained by

$$\mathrm{ASN} = rg_1 + rg_2\left(1 - p_a^{(1)} - p_r^{(1)}\right) \qquad (13)$$

where $p_a^{(1)}$ and $p_r^{(1)}$ are evaluated at $p = p_2$. Therefore, the design parameters for the proposed two-stage group sampling plan can be obtained by the solution from the following optimization:

$$\text{Minimize } \mathrm{ASN}(\mathrm{p}_2) = rg_1 + rg_2\left(1 - p_a^{(1)} - p_r^{(1)}\right) \qquad (14a)$$

Subject to

$L(p_1) \le \beta$ (14b)

$L(p_2) \ge 1-\gamma$ (14c)

$1 \le g_2 \le g_1$ (14d)

$0 \le c_1 < c_2$ (14e)

$g_1,\ g_2,\ c_1,\ c_2$ are integers (14f)

The constraint (14d) is specified because it may not be desirable if the number of groups in the second stage is larger than that in the first stage.

Table 1: The minimum number of groups required in the two-stage sampling plan $c_1$=0, $c_2$=1 for exponentiated Fréchet distribution with for 50th percentile.

| $\lambda$ | $\alpha$ | $\beta$ | $t_q/t_{q_0}$ | r =3 | | | | r=5 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $g_1$ | $g_2$ | ASN | $L(p_2)$ | $g_1$ | $g_2$ | ASN | $L(p_2)$ |
| 2 | 1.5 | 0.25 | 2 | 1 | 1 | 4.125 | 0.9912 | 1 | 1 | 5.781 | 0.9761 |
| | | 0.25 | 4 | 1 | 1 | 4.125 | 0.9985 | 1 | 1 | 5.781 | 0.9997 |
| | | 0.25 | 6 | 1 | 1 | 4.125 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.25 | 8 | 1 | 1 | 4.125 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.10 | 2 | 2 | 1 | 6.281 | 0.9772 | 1 | 1 | 5.781 | 0.9761 |
| | | 0.10 | 4 | 2 | 1 | 6.281 | 0.9985 | 1 | 1 | 5.781 | 0.9997 |
| | | 0.10 | 6 | 2 | 1 | 6.281 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.10 | 8 | 2 | 1 | 6.281 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.05 | 2 | 2 | 1 | 6.281 | 0.9772 | 1 | 1 | 5.781 | 0.9761 |
| | | 0.05 | 4 | 2 | 1 | 6.281 | 0.9985 | 1 | 1 | 5.781 | 0.9998 |
| | | 0.05 | 6 | 2 | 1 | 6.281 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.05 | 8 | 2 | 1 | 6.281 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.01 | 2 | 3 | 1 | 9.053 | 0.9589 | - | - | - | - |
| | | 0.01 | 4 | 3 | 1 | 9.053 | 0.9985 | 2 | 1 | 10.049 | 0.9998 |
| | | 0.01 | 6 | 3 | 1 | 9.053 | 0.9999 | 2 | 1 | 10.049 | 0.9999 |
| | | 0.01 | 8 | 3 | 1 | 9.053 | 1.0000 | 2 | 1 | 10.049 | 1.0000 |
| 2 | 2 | 0.25 | 2 | 1 | 1 | 4.125 | 0.9975 | 1 | 1 | 5.781 | 0.9930 |
| | | 0.25 | 4 | 1 | 1 | 4.125 | 0.9985 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.25 | 6 | 1 | 1 | 4.125 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.25 | 8 | 1 | 1 | 4.125 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.10 | 2 | 2 | 1 | 6.281 | 0.9934 | 1 | 1 | 5.781 | 0.9930 |
| | | 0.10 | 4 | 2 | 1 | 6.281 | 0.9985 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.10 | 6 | 2 | 1 | 6.281 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.10 | 8 | 2 | 1 | 6.281 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.05 | 2 | 2 | 1 | 6.281 | 0.9934 | 1 | 1 | 5.781 | 0.9930 |
| | | 0.05 | 4 | 2 | 1 | 6.281 | 0.9985 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.05 | 6 | 2 | 1 | 6.281 | 0.9999 | 1 | 1 | 5.781 | 0.9999 |
| | | 0.05 | 8 | 2 | 1 | 6.281 | 1.0000 | 1 | 1 | 5.781 | 1.0000 |
| | | 0.01 | 2 | 3 | 1 | 9.053 | 0.9877 | 2 | 1 | 10.049 | 0.9919 |
| | | 0.01 | 4 | 3 | 1 | 9.053 | 0.9985 | 2 | 1 | 10.049 | 0.9999 |
| | | 0.01 | 6 | 3 | 1 | 9.053 | 0.9999 | 2 | 1 | 10.049 | 0.9999 |
| | | 0.01 | 8 | 3 | 1 | 9.053 | 1.0000 | 2 | 1 | 10.049 | 1.0000 |
| 1.068 | 0.924 | 0.25 | 2 | - | - | - | - | - | - | - | - |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.25 | 4 | 1 | 1 | 3.448 | 0.9674 | - | - | - | - |
| | | 0.25 | 6 | 1 | 1 | 3.107 | 0.9983 | 1 | 1 | 5.289 | 0.9951 |
| | | 0.25 | 8 | 1 | 1 | 3.023 | 0.9999 | 1 | 1 | 5.063 | 0.9998 |
| | | 0.10 | 2 | - | - | - | - | - | - | - | - |
| | | 0.10 | 4 | - | - | - | - | - | - | - | - |
| | | 0.10 | 6 | 2 | 1 | 6.206 | 0.9954 | 1 | 1 | 5.289 | 0.9951 |
| | | 0.10 | 8 | 2 | 1 | 6.045 | 0.9998 | 1 | 1 | 5.063 | 0.9998 |
| | | 0.05 | 2 | - | - | - | - | - | - | - | - |
| | | 0.05 | 4 | - | - | - | - | - | - | - | - |
| | | 0.05 | 6 | 2 | 1 | 6.206 | 0.9954 | 1 | 1 | 5.289 | 0.9951 |
| | | 0.05 | 8 | 2 | 1 | 6.045 | 0.9998 | 1 | 1 | 5.063 | 0.9998 |
| | | 0.01 | 2 | - | - | - | - | - | - | - | - |
| | | 0.01 | 4 | - | - | - | - | - | - | - | - |
| | | 0.01 | 6 | 3 | 1 | 9.297 | 0.9914 | 2 | 1 | 10.544 | 0.9873 |
| | | 0.01 | 8 | 3 | 1 | 9.068 | 0.9996 | 2 | 1 | 10.125 | 0.9994 |

The cells with hyphens (-) indicate that parameters cannot be found to satisfy the conditions.

Table 2: The number of groups in the two-stage sampling plan $c_1$=0, $c_2$=1 for exponentiated Fréchet distribution with for 25th percentile.

| $\lambda$ | $\alpha$ | $\beta$ | $t_q/t_{q_0}$ | r =3 | | | | r=5 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $g_1$ | $g_2$ | ASN | OC | $g_1$ | $g_2$ | ASN | OC |
| 2.0 | 1.5 | 0.25 | 2 | 3 | 1 | 9.676 | 0.9998 | 2 | 1 | 10.939 | 0.9998 |
| | | 0.25 | 4 | 3 | 1 | 9.676 | 0.9999 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 6 | 3 | 1 | 9.676 | 0.9999 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 8 | 3 | 1 | 9.676 | 1.0000 | 2 | 1 | 10.939 | 1.0000 |
| | | 0.10 | 2 | 4 | 1 | 12.380 | 0.9997 | 3 | 1 | 15.334 | 0.9997 |
| | | 0.10 | 4 | 4 | 1 | 12.380 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 6 | 4 | 1 | 12.380 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 8 | 4 | 1 | 12.380 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |
| | | 0.05 | 2 | 5 | 1 | 15.201 | 0.9995 | 3 | 1 | 15.334 | 0.9997 |
| | | 0.05 | 4 | 5 | 1 | 15.201 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 6 | 5 | 1 | 15.201 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 8 | 5 | 1 | 15.201 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |
| | | 0.01 | 2 | 7 | 1 | 21.050 | 0.9995 | 4 | 1 | 20.106 | 0.9995 |
| | | 0.01 | 4 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 6 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 8 | 7 | 1 | 21.050 | 1.0000 | 4 | 1 | 20.106 | 1.0000 |
| 2.0 | 2.0 | 0.25 | 2 | 3 | 1 | 9.676 | 0.9998 | 2 | 1 | 10.939 | 0.9998 |
| | | 0.25 | 4 | 3 | 1 | 9.676 | 0.9999 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 6 | 3 | 1 | 9.676 | 0.9999 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 8 | 3 | 1 | 9.676 | 1.0000 | 2 | 1 | 10.939 | 1.0000 |
| | | 0.10 | 2 | 4 | 1 | 12.380 | 0.9998 | 3 | 1 | 15.334 | 0.9998 |
| | | 0.10 | 4 | 4 | 1 | 12.380 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 6 | 4 | 1 | 12.380 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 8 | 4 | 1 | 12.380 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |
| | | 0.05 | 2 | 5 | 1 | 15.201 | 0.9998 | 3 | 1 | 15.334 | 0.9998 |
| | | 0.05 | 4 | 5 | 1 | 15.201 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 6 | 5 | 1 | 15.201 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 8 | 5 | 1 | 15.201 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.01 | 2 | 7 | 1 | 21.050 | 0.9998 | 4 | 1 | 20.106 | 0.9998 |
| | | 0.01 | 4 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 6 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 8 | 7 | 1 | 21.050 | 1.0000 | 4 | 1 | 20.106 | 1.0000 |
| 1.068 | 0.924 | 0.25 | 2 | 3 | 1 | 9.676 | 0.9809 | 2 | 1 | 10.939 | 0.9721 |
| | | 0.25 | 4 | 3 | 1 | 9.676 | 0.9998 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 6 | 3 | 1 | 9.676 | 0.9999 | 2 | 1 | 10.939 | 0.9999 |
| | | 0.25 | 8 | 3 | 1 | 9.676 | 1.0000 | 2 | 1 | 10.939 | 1.0000 |
| | | 0.10 | 2 | 4 | 1 | 12.380 | 0.9702 | 3 | 1 | 15.334 | 0.9504 |
| | | 0.10 | 4 | 4 | 1 | 12.380 | 0.9998 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 6 | 4 | 1 | 12.380 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.10 | 8 | 4 | 1 | 12.380 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |
| | | 0.05 | 2 | 5 | 1 | 15.201 | 0.9578 | 3 | 1 | 15.334 | 0.9504 |
| | | 0.05 | 4 | 5 | 1 | 15.201 | 0.9998 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 6 | 5 | 1 | 15.201 | 0.9999 | 3 | 1 | 15.334 | 0.9999 |
| | | 0.05 | 8 | 5 | 1 | 15.201 | 1.0000 | 3 | 1 | 15.334 | 1.0000 |
| | | 0.01 | 2 | - | - | - | - | - | - | - | - |
| | | 0.01 | 4 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 6 | 7 | 1 | 21.050 | 0.9999 | 4 | 1 | 20.106 | 0.9999 |
| | | 0.01 | 8 | 7 | 1 | 21.050 | 1.0000 | 4 | 1 | 20.106 | 1.0000 |

The cells with hyphens (-) indicate that parameters cannot be found to satisfy the conditions.

Therefore, the design parameters of the proposed plan $g_1$ and $g_2$ are determined for a given $\gamma$ and $\beta$, $\delta_1$ the percentile ratio, at the consumer's risk and the percentile ratio, $\delta_2$, at the producer's risk, such that the is minimized ASN($p_2$) and inequalities (14b) and (14c) are satisfied simultaneously for specified values of shape parameters, $\lambda$ and $\alpha$, termination ratio $\delta_q^0$ and the number of testers, *r*. Tables 1 and 2, shows the minimum numbers of groups required for the two-stage group sampling plan according to values of percentile ratios $\delta_2$=2,4,6,8 and $\delta_1$=1 when $r$ = 3 and $r$ = 5 at the four levels of the consumer's risks such as $\beta$ = 0.25, 0.10, 0.05 and 0.01 with known parameters $\lambda = 2$ and 1.068 and $\alpha$ =0.924,1.5 and 2. As mentioned earlier, $(c_1, c_2)$ were determined as (0, 1) in all cases and $\delta_q^0$ =1.

It is observed from these tables that the numbers of groups required decrease as the group size increases from r=3 to r=5 when other parameters remain the same and also the ASN increases marginally. The sample size ($rg_1$ or $rg_2$) also decreases as the group size increases, which indicates that a larger group size may be more economical. Another interesting observation from tables is the number of groups not influenced by shape parameters. When percentile values decreases from 5o$^{th}$ percentile to 25$^{th}$ percentile, the number of groups increases.

**Comparisons with single-stage group sampling plans**

As a special case of the proposed two-stage group sampling plan, we consider a single-stage group sampling plan (having group size $r$) and prepare the table for designing the plan. As mentioned earlier, this will be the case when $c_1 = c_2 = c$ in the two-stage group sampling plan. The lot acceptance probability under this plan will be given by

$$P_a = \sum_{i=0}^{c} \binom{rg}{i} p^i \left(1-p\right)^{rg-i} \qquad (15)$$

where $g$ is the number of groups required. Obviously, the ASN is obtained by $r$ times $g$.

Similarly as in the two-stage group sampling plan, a table for determining the number of groups and acceptance number required can be prepared according to the value of the specified
unreliability at a given consumer's risk. Tables 3 and 4, shows the minimum numbers of groups required for the single stage group sampling plan with $c$ = 0 or $c$ = 1 when $r$ = 3 and $r$ = 5 according to values of percentile ratios $\delta_2$ =2,4,6,8 and $\delta_1$=1 at $\beta$ = 0.25, 0.10, 0.05 and 0.01 with known parameters $\lambda = 2$ and 1.068 and $\alpha$ =0.924,1.5 and 2.

Table 3: The number of groups in the single-stage sampling plan for exponentiated Fréchet distribution with for $50^{th}$ percentile

| $\beta$ | $t_q/t_{q_0}$ | $\lambda=2,\alpha=1.5,r=3$ | | | | | | $\lambda=2,\alpha=1.5,r=5$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Single with c=0 | | | Single with c=1 | | | Single with c=0 | | | Single with c=1 | | |
| | | g | ASN | OC | g | ASN | OC | g | ASN | OC | g | ASN | OC |
| 0.2 | 2 | - | - | - | 2 | 6 | 0.989 | - | - | - | 1 | 5 | 0.992 |
| 0.2 | 4 | 1 | 3 | 0.999 | 2 | 6 | 0.999 | 1 | 5 | 0.999 | 1 | 5 | 0.999 |
| 0.2 | 6 | 1 | 3 | 0.999 | 2 | 6 | 0.999 | 1 | 5 | 0.999 | 1 | 5 | 0.999 |
| 0.2 | 8 | 1 | 3 | 1.000 | 2 | 6 | 1.000 | 1 | 5 | 1.000 | 1 | 5 | 1.000 |
| 0.1 | 2 | - | - | - | 3 | 9 | 0.975 | - | - | - | 2 | 10 | 0.969 |
| 0.1 | 4 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.1 | 6 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.1 | 8 | 2 | 6 | 1.000 | 3 | 9 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.0 | 2 | - | - | - | 3 | 9 | 0.975 | - | - | - | 2 | 10 | 0.969 |
| 0.0 | 4 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.0 | 6 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.0 | 8 | 2 | 6 | 1.000 | 3 | 9 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.0 | 2 | - | - | - | 4 | 12 | 0.957 | - | - | - | - | - | - |
| 0.0 | 4 | 3 | 9 | 0.999 | 4 | 12 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.0 | 6 | 3 | 9 | 0.999 | 4 | 12 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.0 | 8 | 3 | 9 | 1.000 | 4 | 12 | 1.000 | 2 | 10 | 1.000 | 3 | 15 | 1.000 |
| | | $\lambda=2,\alpha=2.0,r=3$ | | | | | | $\lambda=2,\alpha=2.0,r=5$ | | | | | |
| 0.2 | 2 | 1 | 3 | 0.956 | 2 | 6 | 0.996 | - | - | - | 1 | 5 | 0.997 |
| 0.2 | 4 | 1 | 3 | 0.999 | 2 | 6 | 0.999 | 1 | 5 | 0.999 | 1 | 5 | 0.999 |
| 0.2 | 6 | 1 | 3 | 0.999 | 2 | 6 | 0.999 | 1 | 5 | 0.999 | 1 | 5 | 0.999 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 8 | 1 | 3 | 1.000 | 2 | 6 | 1.000 | 1 | 5 | 1.000 | 1 | 5 | 1.000 |
| 0.1 | 2 | - | - | - | 3 | 9 | 0.992 | - | - | - | 2 | 10 | 0.991 |
| 0.1 | 4 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.1 | 6 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.1 | 8 | 2 | 6 | 1.000 | 3 | 9 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.0 | 2 | - | - | - | 3 | 9 | 0.975 | - | - | - | 2 | 10 | 0.991 |
| 0.0 | 4 | 2 | 6 | 0.999 | 3 | 9 | 0.992 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.0 | 6 | 2 | 6 | 0.999 | 3 | 9 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.0 | 8 | 2 | 6 | 1.000 | 3 | 9 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.0 | 2 | - | - | - | 4 | 12 | 0.987 | - | - | - | 3 | 15 | 0.980 |
| 0.0 | 4 | 3 | 9 | 0.999 | 4 | 12 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.0 | 6 | 3 | 9 | 0.999 | 4 | 12 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.0 | 8 | 3 | 9 | 1.000 | 4 | 12 | 1.000 | 2 | 10 | 1.000 | 3 | 15 | 1.000 |
| | | $\lambda = 1.068, \alpha = 0.924, r = 3$ | | | | | | $\lambda = 1.068, \alpha = 0.924, r = 5$ | | | | | |
| 0.2 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.2 | 4 | - | - | - | 2 | 6 | 0.959 | - | - | - | 1 | 5 | 0.972 |
| 0.2 | 6 | 1 | 3 | 0.964 | 2 | 6 | 0.997 | - | - | - | 1 | 5 | 0.998 |
| 0.2 | 8 | 1 | 3 | 0.992 | 2 | 6 | 0.999 | 1 | 5 | 0.987 | 1 | 5 | 0.999 |
| 0.1 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.1 | 4 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.1 | 6 | - | - | - | 3 | 9 | 0.995 | - | - | - | 2 | 10 | 0.993 |
| 0.1 | 8 | 2 | 6 | 0.984 | 3 | 9 | 0.999 | 1 | 5 | 0.987 | 2 | 10 | 0.999 |
| 0.0 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 4 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 6 | - | - | - | 3 | 9 | 0.995 | - | - | - | 2 | 10 | 0.993 |
| 0.0 | 8 | 2 | 6 | 0.984 | 3 | 9 | 0.999 | 1 | 5 | 0.987 | 2 | 10 | 0.999 |
| 0.0 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 4 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 6 | - | - | - | 4 | 12 | 0.991 | - | - | - | 3 | 15 | 0.986 |
| 0.0 | 8 | 3 | 9 | 0.977 | 4 | 12 | 0.999 | 2 | 10 | 0.974 | 3 | 15 | 0.999 |

The cells with hyphens (-) indicate that parameters cannot be found to satisfy the conditions.

Table 4: The number of groups in the single-stage sampling plans for exponentiated Fréchet distribution with for $25^{th}$ percentile

| $\beta$ | $t_q / t_{q_0}$ | $\lambda = 2, \alpha = 1.5, r = 3$ | | | | | | $\lambda = 2, \alpha = 1.5, r = 5$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Single with c=0 | | | Single with c=1 | | | Single with c=0 | | | Single with c=1 | | |
| | | g | AS | OC | g | AS | OC | g | AS | OC | g | AS | OC |
| 0.2 | 2 | 2 | 6 | 0.991 | 4 | 12 | 0.999 | 1 | 5 | 0.993 | 2 | 10 | 0.999 |
| 0.2 | 4 | 2 | 6 | 0.999 | 4 | 12 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.2 | 6 | 2 | 6 | 0.999 | 4 | 12 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.2 | 8 | 2 | 6 | 1.000 | 4 | 12 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.1 | 2 | 3 | 9 | 0.987 | 5 | 15 | 0.999 | 2 | 10 | 0.986 | 3 | 15 | 0.989 |
| 0.1 | 4 | 3 | 9 | 0.999 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 6 | 3 | 9 | 0.999 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 8 | 3 | 9 | 1.000 | 5 | 15 | 1.000 | 2 | 10 | 1.000 | 3 | 15 | 1.000 |
| 0.0 | 2 | 4 | 12 | 0.983 | 6 | 18 | 0.999 | 3 | 15 | 0.979 | 4 | 20 | 0.999 |
| 0.0 | 4 | 4 | 12 | 0.999 | 6 | 18 | 0.999 | 3 | 15 | 0.999 | 4 | 20 | 0.999 |
| 0.0 | 6 | 4 | 12 | 0.999 | 6 | 18 | 0.999 | 3 | 15 | 0.999 | 4 | 20 | 0.999 |
| 0.0 | 8 | 4 | 12 | 1.000 | 6 | 18 | 1.000 | 3 | 15 | 1.000 | 4 | 20 | 1.000 |
| 0.0 | 2 | 6 | 18 | 0.975 | 8 | 24 | 0.999 | 4 | 20 | 0.972 | 5 | 25 | 0.999 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 4 | 6 | 18 | 0.999 | 8 | 24 | 0.999 | 4 | 20 | 0.999 | 5 | 25 | 0.999 |
| 0.0 | 6 | 6 | 18 | 0.999 | 8 | 24 | 0.999 | 4 | 20 | 0.999 | 5 | 25 | 0.999 |
| 0.0 | 8 | 6 | 18 | 1.000 | 8 | 24 | 1.000 | 4 | 20 | 1.000 | 5 | 25 | 1.000 |
| | | $\lambda = 2, \alpha = 2.0, r = 3$ | | | | | | $\lambda = 2, \alpha = 2.0, r = 5$ | | | | | |
| 0.2 | 2 | 2 | 6 | 0.996 | 4 | 12 | 0.999 | 1 | 5 | 0.996 | 2 | 10 | 0.999 |
| 0.2 | 4 | 2 | 6 | 0.999 | 4 | 12 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.2 | 6 | 2 | 6 | 0.999 | 4 | 12 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.2 | 8 | 2 | 6 | 1.000 | 4 | 12 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.1 | 2 | 3 | 9 | 0.994 | 5 | 15 | 0.999 | 2 | 10 | 0.993 | 3 | 15 | 0.999 |
| 0.1 | 4 | 3 | 9 | 0.999 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 6 | 3 | 9 | 0.999 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 8 | 3 | 9 | 1.000 | 5 | 15 | 1.000 | 2 | 10 | 1.000 | 3 | 15 | 1.000 |
| 0.0 | 2 | 4 | 12 | 0.992 | 6 | 18 | 0.999 | 3 | 15 | 0.990 | 4 | 20 | 0.999 |
| 0.0 | 4 | 4 | 12 | 0.999 | 6 | 18 | 0.999 | 3 | 15 | 0.999 | 4 | 20 | 0.999 |
| 0.0 | 6 | 4 | 12 | 0.999 | 6 | 18 | 0.999 | 3 | 15 | 0.999 | 4 | 20 | 0.999 |
| 0.0 | 8 | 4 | 12 | 1.000 | 6 | 18 | 1.000 | 3 | 15 | 1.000 | 4 | 20 | 1.000 |
| 0.0 | 2 | 6 | 18 | 0.988 | 8 | 24 | 0.999 | 4 | 20 | 0.987 | 5 | 25 | 0.999 |
| 0.0 | 4 | 6 | 18 | 0.999 | 8 | 24 | 0.999 | 4 | 20 | 0.999 | 5 | 25 | 0.999 |
| 0.0 | 6 | 6 | 18 | 0.999 | 8 | 24 | 0.999 | 4 | 20 | 0.999 | 5 | 25 | 0.999 |
| 0.0 | 8 | 6 | 18 | 1.000 | 8 | 24 | 1.000 | 4 | 20 | 1.000 | 5 | 25 | 1.000 |
| | | $\lambda = 1.068, \alpha = 0.924, r = 3$ | | | | | | $\lambda = 1.068, \alpha = 0.924, r = 5$ | | | | | |
| 0.2 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.2 | 4 | 2 | 6 | 0.983 | 4 | 12 | 0.999 | 1 | 5 | 0.986 | 2 | 10 | 0.999 |
| 0.2 | 6 | 2 | 6 | 0.999 | 4 | 12 | 0.999 | 1 | 5 | 0.999 | 2 | 10 | 0.999 |
| 0.2 | 8 | 2 | 6 | 1.000 | 4 | 12 | 1.000 | 1 | 5 | 1.000 | 2 | 10 | 1.000 |
| 0.1 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.1 | 4 | 3 | 9 | 0.975 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 6 | 3 | 9 | 0.998 | 5 | 15 | 0.999 | 2 | 10 | 0.999 | 3 | 15 | 0.999 |
| 0.1 | 8 | 3 | 9 | 1.000 | 5 | 15 | 1.000 | 2 | 10 | 1.000 | 3 | 15 | 1.000 |
| 0.0 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 4 | 4 | 12 | 0.966 | 6 | 18 | 0.998 | 3 | 15 | 0.958 | 4 | 20 | 0.998 |
| 0.0 | 6 | 4 | 12 | 0.998 | 6 | 18 | 0.999 | 3 | 15 | 0.998 | 4 | 20 | 0.999 |
| 0.0 | 8 | 4 | 12 | 0.999 | 6 | 18 | 1.000 | 3 | 15 | 0.999 | 4 | 20 | 1.000 |
| 0.0 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| 0.0 | 4 | 6 | 18 | 0.950 | 8 | 24 | 0.997 | - | - | - | 5 | 25 | 0.997 |
| 0.0 | 6 | 6 | 18 | 0.997 | 8 | 24 | 0.999 | 4 | 20 | 0.997 | 5 | 25 | 0.999 |
| 0.0 | 8 | 6 | 18 | 0.999 | 8 | 24 | 1.000 | 4 | 20 | 0.999 | 5 | 25 | 1.000 |

The cells with hyphens (-) indicate that parameters cannot be found to satisfy the conditions.

We notice from Tables 3 and 4 that the number of groups required for the single-stage group sampling plan increases rapidly when the acceptance number changes from $c$=0 to $c$=1. When the group size increases, the number of groups required for the single-stage plan decreases. Moreover, the proposed two-stage group sampling plan performs better than the single-stage group sampling plan in terms of the ASN and the OC values . Whereas, comparing Tables 1 and 2 with Tables 3 and 4, we observed that the ASN for the single-stage group sampling plan with $c$=0 is smaller than that of the two-stage group sampling plan with $c_1$=0, $c_2$=1 at the same value of the parameters, furthermore the ASN for the single-stage group sampling plan with $c$=1 is much larger than that for the two-stage group sampling plan with $c_1$=0, $c_2$=1. If we consider the ASN and OC values at the same time, the two-stage group sampling plan seems to be better than the single-stage group sampling plan.

**Industrial Applications**

In this section, we use a real data set to show that the exponentiated Fréchet distribution can be a suitable model. The data set represents an active repair times (hours) for an airborne communication transceiver reported by Balakrishnan *et al*. (2009), which was originally given by Chhikara and Folks (1989). To be self-contained, this data set is reproduced as follows:

0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0, 1.0, 1.1, 1.3, 1.5, 1.5, 1.5,

1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3,22.0, 24.5.

Using exploratory data analysis and then goodness-of-fit , Balakrishnan *et al*. (2009) showed that the inverse Gaussian (IG) distribution is a better fit to this dataset. We show a rough indication of the goodness of fit for our model by plotting the superimposed for the data shows that the EFD is a good fit in Figure 1 and also goodness of fit is emphasized with QQ plot, displayed in Figure 2. The maximum likelihood estimates of the two-parameter EFD for the active repair times (hours) for an airborne communication transceiver are $\hat{\lambda}$=1.0680 and $\hat{\alpha}$=0.9237 and the Kolmogorov-Smirnov test and found that the maximum distance between the data and the fitted of the EFD is 0.0969 with p-value is 0.7804. Therefore, the two-parameter EFD provides reasonable fits for lifetimes of items.

Suppose that an experimenter would like to use the proposed two-stage sampling plan to establish the true unknown 25$^{th}$ percentile lifetime for the product is at least 2 hours and experiment will be stopped after 2 hours. Further, suppose that in the laboratory the experimenter has facility to install five items on a tester. This information leads to $\delta_q^0$=1. Let $\beta$ = 0.10 and $t_q / t_{q_0}$ =2 with $\gamma$=0.05 for this experiment. The above data is well fitted to the two-parameter EFD with $\hat{\lambda}$=1.0680 and $\hat{\alpha}$=0.9237. So, from Table 2, the two-stage acceptance sampling plan parameters at r= 3 are $g_1$=4, $g_2$=1, $c_1$=0 and $c_2$=1. The two-stage acceptance sampling plan is implemented as follows: randomly select 12 items and distribute 3 items into each of 4 tester and accept the product if no failure from each tester in 2 hours and reject the product if more than 1 failure from any tester before 2 hours. If one failure is observed from any tester then go to the second stage, then randomly select another 3 items from the lot and distribute 3 items into each tester. If the total number of failure items from the two-stage testing within 2 hours for each stage is less than 0ne then the lot is accepted; otherwise, the lot is rejected. The probability of acceptance for this plan is 97.02% and ASN is 12.38.

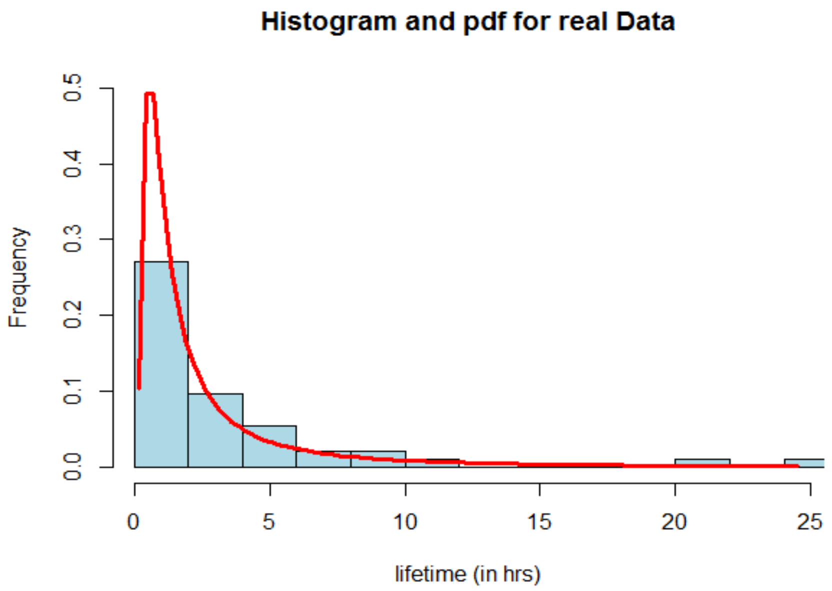


Figure 1. Histogram with estimated pdf for the active repair times (hours).

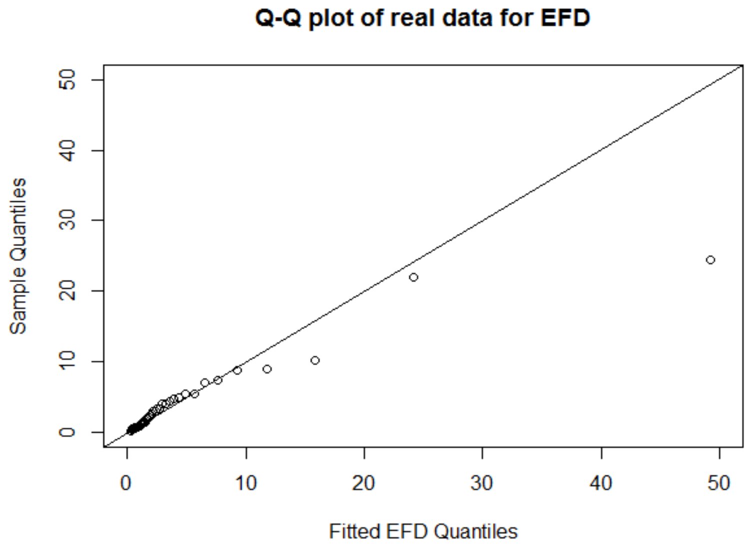


Figure 2. QQ plots for the active repair times (hours).

## Conclusion

In this study, a two-stage grouped acceptance sampling plan is developed when the lifetime of a product follows exponentiated Fréchet distribution percentiles with known shape parameters. The plan parameters like the number of groups in each stage were determined so as to minimize the ASN subject to satisfying the consumer's and the producer's risks simultaneously under a variety of conditions. Tables for the plan parameters were constructed under various combinations such as known shape parameters, group sizes, consumer's and the producer's risks and so on. An industrial example has been presented to illustrate the applications of the proposed two-stage grouped sampling plan. We made the comparison between the proposed two-stage grouped acceptance sampling plan and single-stage acceptance sampling plan. We observed from tables that the numbers of groups required decrease as the group size increases from 3 to 5 when other parameters remain the same and also the ASN increases marginally. The sample size also decreases as the group size increases, which indicates that a larger group size may be more economical. When percentile values decreases from 5o$^{th}$ percentile to 25$^{th}$ percentile, the number of groups increases. Finally it should be mentioned from tables that the proposed two-stage grouped acceptance sampling plan performs better in terms of the average sample number and the operating characteristics than single-stage grouped acceptance sampling plan.

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