EFFECTS OF VISCOUS DISSIPATION AND JOULE HEATING ON MHD FLOW OF A FLUID WITH VARIABLE PROPERTIES PAST ASTRETCHING VERTICAL PLATE

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Abstract

An analysis is presented to study the combined effects of viscous dissipation and Joule heating on MHD free-convective flow of a fluid with variable properties near a linearly stretching isothermal vertical sheet. The flow is subjected to a uniform transverse magnetic field, with high temperature differences between the plate and ambient fluid. The Boussinesq approximation is neglected due to the large temperature differences between the plate and the ambient fluid. The resulting coupled nonlinear differential equations are solved numerically. The effects of magnetic parameter, Grashof number, Eckert number and other involved parameters on the velocity and temperature functions are investigated.

Keywords: Astretching Vertical Plate, Viscous Dissipation

Introduction

The interest in studying the effect of viscous and Joule heating on the MHD flow and heat transfer is due to the important role of this flow in various devices which are subjected to large variations of gravitational force, its application on heat exchanger designs, wire and glass fiber drawing and its application in nuclear engineering in connection with the cooling of reactors.

The flow of anelectrically conducting fluid past a continuously moving plate was initiated in 1969 by Sakiadis and then extended by Erickson et al. to include blowing or suction at the plate. After thatmany investigations have focused on the problems of a stretched sheet oriented in the horizontal direction with a linear velocity and a different thermal boundary layer conditions (see for instance Crane and Gupta and Gupta). Vayjravelu and Hadyinicolaou studied the convective heat transfer in an electrically conducting fluid near an isothermal stretching sheet with uniform free stream. Chamkha studied the problem of steady free-convective laminar flow over a vertical porous surface in the presence of a magnetic field with heat absorption or generation.

Viscous dissipation effects plays animportant role in natural convection in various devices which are subjected to large variations of gravitational force or which operate at high speeds. The work of Brinkman appears to be the first theoretical work dealing viscous dissipation. The temperature distribution in the entrance region of a circular pipe at the wall of which was maintained at either the constant temperature of the entering fluid or constant heat-flux was examined. The highest temperatures were, not surprisingly, discovered to be localized in the wall region. Lin studied laminar heat-transfer to a non-Newtonian Couette flow with pressure gradient using the power-law model. The effects of pressure gradient and viscous dissipation on the heat transfer were discussed. Using a functional analysis method, Lahjomri analytically studied thermally-developing laminar Hartman flow through a parallel-plate channel, with a prescribed transversal uniform magnetic field, including both viscous dissipation, Joule heating and axial heat-conduction with uniform heat-flux. In a recent study, Nield et al. investigated the thermal development of forced convection in a parallel plate channel filled by a saturated porous medium, with walls held at a uniform temperature, and with the effects of axial conduction and viscous dissipation included. Davaa et al. numerically studied fully-developed laminar heat transfer to non-Newtonian fluids flowing between parallel plates with the axial movement of one of the plates with an emphasis on the viscous-dissipation effect. Increasing the Brinkman number increased the heat-transfer rates at the heated wall when the movement direction of the upper plate was the same as the direction of the main flow, while the opposite is true for the movement of the upper plate in the opposite direction.Aboeldahab et al studied the viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents.

The case where the temperature difference between the plate and the fluid is large fluid's physical properties such as its viscosity and thermal conductivity cannot be taken as constant. Also, in this case, the Boussinesq approximation can no longer be used.

Some recent studies for radiating fluids have taken into account variations of the physical properties with temperature. For example, Aboeldahab studied radiation and variable density effects on the free convective flow of a gas past a semi-infinite vertical plate, Jaber studied the effect of Hall currents, radiation and variable viscosity on free convective flow past a semi-infinite, Jaber studied the effect of Hall currents and variable fluid properties on MHD flow past a continuously stretching vertical plate in the presence of radiation. They showed that for high-temperature differences the Boussinesq approximation leads to substantial errors in velocity and temperature distributions. Also, they showed that the flow characteristics are markedly affected by the variation of the fluid physical properties.

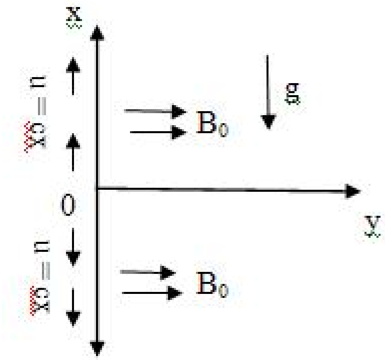


Fig. 1. Sketch of the physical model.

The present work is devoted to study the viscous and Joule heating on the free convection flow of an electrically conducting fluid with variable fluid properties near a vertical plate. The plate is subjected to a uniform transverse magnetic field. Numerical solutions are obtained for the flow and temperature fields for several values of the material properties.

Mathematical Formulation

Consider the steady, two-dimensional, laminar free convective boundary layer flow of an incompressible, electrically conducting fluid with variable fluid properties, adjacent to a vertical sheet coinciding with the plane y=0, where the flow is confined to y>0. The sheet is stretched with fixed origin by two equal and opposite forces along the x-axis. A uniform magnetic field of strength B_0 is applied along the y-axis. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field can be neglected. The viscous and joule heating are taken into account. The density is assumed to vary exponentially with temperature as follows:

$$\rho = \rho_\infty e^{-\beta(T-T_\infty)} \qquad (1)$$

where

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (2)$$

The fluid thermal conductivity is assumed to vary as a linear function of temperature in the form

$$K = k_{\infty}\left[1 + b(T - T_{\infty})\right] \quad (3)$$

Where b is a constant depending on the nature of the fluid. In general, b> 0 for fluids such as water and air, while b <0 for fluids such as lubricating oils.

The fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature in the form (see Lai and Kulacki)

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}}\left[1 + \gamma(T - T_{\infty})\right] \quad (4) \quad \text{or}$$

$$\frac{1}{\mu} = a\left[T - T_r\right] \quad (5)$$

Where

$a = \dfrac{\gamma}{\mu_{\infty}}$ and $T_r = T_{\infty} - \dfrac{1}{\gamma}$ are constants and their values depend on the reference state and the thermal property of the fluid γ. In general a > 0 for liquids and a < 0 for gases.

Then the steady laminar two-dimensional free-convective flow is governed by the following boundary- layer equations:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (6)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\rho_{\infty}\left(1 - e^{-\beta(T - T_{\infty})}\right) - \sigma_o B_0 u \quad (7)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q(T - T_{\infty}) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma_o B_0^2 u^2 \quad (8)$$

The physical problem suggests the following initial and boundary condition

u = v = 0, $T = T_{\omega}$ at y= 0 ; u $\longrightarrow$ 0, $T \longrightarrow T_{\infty}$ as y $\longrightarrow \infty$ (9)

By using equation (1),(3) and (5) , equation (7) and (8) become

$$e^{-\beta(T - T_{\infty})}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{1}{\rho_{\infty}}\frac{\partial}{\partial y}\left(\frac{1}{a(T - T_r)}\frac{\partial u}{\partial y}\right) + g(1 - e^{-\beta(T - T_{\infty})}) - \sigma_o B_o u \quad (10)$$

$$e^{-\beta(T - T_{\infty})}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{C_P}\frac{\partial}{\partial y}\left[\left\{1 + b(T - T_{\infty}\right\}\frac{\partial T}{\partial y}\right] + \frac{Q}{C_P}(T - T_{\infty})$$
$$+ \frac{1}{a(T - T_r)C_P}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{C_P}\sigma_0 B_0^2 u^2 \quad (11)$$

Introducing the following dimensionless variables

$$\psi = 4\upsilon_\infty C X^{\frac{3}{4}} f(\xi,\eta), \quad \xi = X^{\frac{1}{2}} L^{\frac{-1}{2}} \quad , \eta = C X^{\frac{-1}{4}} \int_0^y \frac{\rho}{\rho_\infty} \qquad (12)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad , \quad C^4 = \frac{g(1-e^{-n})}{4\upsilon^2{}_\infty}$$

The continuity equation is satisfied by

$$u = \frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\rho_\infty}{\rho}\frac{\partial \psi}{\partial x} \qquad (13)$$

From (14) and (15) we find that

$$u = 4\upsilon_\infty C^2 X^{\frac{1}{2}} f^{/} \quad , \quad v = -\frac{\rho_\infty}{\rho}\upsilon_\infty C X^{-\frac{1}{4}}(3f + 2\xi\frac{\partial f}{\partial \xi} - \eta f^{/}) \quad (14)$$

Using the above transformation the governing equations are reduced to:

$$2f^{/2} - 3ff^{//} + 2\xi[f^{/}\frac{\partial f^{/}}{\partial \xi} - f^{//}\frac{\partial f}{\partial \xi}] = \frac{\theta_r}{(\theta_r - \theta)^2} e^{-n\theta} f^{//}\theta^{/}$$
$$+\frac{\theta_r}{\theta_r - \theta} e^{-n\theta}(f^{///} - n f^{//}\theta^{/}) - (\frac{1-e^{n\theta}}{1-e^{-n}}) = 0 \qquad (15)$$

$$e^{-n\theta}\theta^{//} + Se^{-n\theta}\theta^{/2} + 3p_r f\theta^{/} + \gamma \Pr e^{n\theta}\theta + 2\xi P_r[\theta^{/}\frac{\partial f}{\partial \xi} - f^{/}\frac{\partial \theta}{\partial \xi}]$$
$$\Pr Ec\left[\frac{\theta_r}{\theta_r - \theta} Gr\, e^{-n\theta} f^{//2} + M\sqrt{Gr}\, f^{/2}\right] = 0 \qquad (16)$$

The boundary conditions are transformed into

$$\eta = 0: \quad f = f' = 0, \qquad \theta = 1$$
$$\eta \longrightarrow \infty \quad : f^{\backslash} \longrightarrow o, \qquad \theta \longrightarrow 0 \qquad (17)$$

Where n= $\beta(T_w - T_\infty)$ is the density temperature parameter, S= $b(T_w - T_\infty)$ is the thermal conductivity parameter, $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$ is the viscosity temperature parameter, $G_r = \frac{4g(1-e^{-n})}{\upsilon_\infty^2}$ is the Grashof number, $Ec = \frac{u^2}{C_P(T_W - T_\infty)}$ is the Eckert number, $M = \frac{4\sigma B_0^2}{\rho_\infty u}x$ is the Magnetic number and $P_r = \frac{\upsilon_\infty}{\alpha}$ is the Prandtl number.

Also, primes denote differentiation with respect to η only

The most important characteristics of the flow are shearing stress at the plate

$$\tau_w = -\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = -4\upsilon_\infty C^2 X^{1/4} f^{//}(\xi,0) \qquad (18)$$

And the rate of heat transfer at the plate (Nusselt number)

$$N_u = (T_w - T_\infty) C X^{-1/4} e^{-n} \theta^{/}(\xi,0) \qquad (19)$$

Results and discussion

The governing boundary layer equations 15 and 16 are coupled non-linear partial differential equations, which possess no similarity or closed form solution. Therefore, a numerical solution of the problem under consideration using the fourth-order Runge-Kutta method is performed. Figures 2-13 represent purely numerical results which illustrate the effects of various physical parameters on the flow and heat transfer aspects of the present problem, within the boundary layer for Pr = 0.72. To study the effects of the involved parameters, standard values are assigned as follow the magnetic parameter M=1, density temperature parameter n = 0.5, thermal conductivity parameter s = 0.2, Eckert number Ec = 1,viscosity temperature parameter$\theta_r = 20$ and Grashof number Gr = 1.

Figs. 2 and 3 show the effects of the magnetic parameter M on the velocity and temperature profiles within the boundary layer. It is noticed that the increasing of the magnetic field parameter M decreases the velocity and increases the temperature. In general, applying of a transverse magnetic field normal to the flow direction has a tendency to induce a flow-resistive force in the x-direction. This force tends to slow down the motion of the fluid upwards along the plate. Accordingly, increases the temperature in the boundary layer.

Figs. 4 and 5 show typical profiles for the fluid's velocity and temperature for various values of theEckert number Ec. The increasing in values of the Eckert number Eccausesincreasing in the velocity and temperature in the boundary layer. It is obvious that the velocity f' and the temperature increase owing to the increase inthe Gamma parameter as shown in Figures 6 and 7. Also, velocity and temperature in the boundary layer increase as the Grashof number Gr increases and as the thermal conductivity number s increases as shown in Figures 8 – 11. Figures 12 and 13 show that the velocity increases due to the increase in the thermal conductivity parameter $\theta_r$and the density temperature parameter n respectively. This yields to enhance the buoyancy forces due to mass density variations. It is also observed that the increasing in the density temperature parameter slightly decreases the temperature in the boundary layer,so no figure for this variable is presented herein.

Table 1 illustrates the effects of the Eckert Ec, magnetic field M, Grashof number Gr, γ , thermal conductivity θ_r , density temperature n and the thermal conductivity s parameters on the shear stressf "(0) and rate of heat transferθ'(0) at the wall for Prandtl number Pr = 0.72. The increasing of the Eckert Ec, magnetic field M, Grashof number Gr, γ, density temperature n and thermal conductivity s parameters tend to increase the shear stress and wall-temperature gradient. The increasing of the thermal conductivity $\theta_r$yields to increase theshear stress and to decrease the wall-temperature gradient. It is worth mentioning that the negative values of wall-temperature gradient indicate that the heat flows from the sheet surface to the ambient fluid.

Concluing remarks

In the present work, the combined effects of viscous and Joule heatingon the MHD free-convection steady laminar flow of a fluid with variable physical properties have been studied, for high temperature differences. The fluid is assumed to be electrically conducting flowing past an isothermal electrically conducting semi-infinite vertical plate.

It is observed that:

1- The fluid velocity f', fluid temperature θ, the shear stress f"(0) and the rate of heat transfer θ'(0) at the wall increase as the Eckert number Ec, Grashof number Gr, γ , thermal conductivity θ_r , density temperature parameter n and the thermal conductivity parameter s increase.
2- The increasing of the magnetic field parameter M tends to increase the fluid temperature θ , the rate of heat transfer θ'(0) and the shear stress at the wallf"(0), while the fluid velocity f'decreasesdue to the increase in the magnetic field strength.

Table 1: Variation of dimensionless wall-velocity gradient and dimensionless rate of heattransferat the plate with the dimensionless parameters M, Ec, Gr, θ_r , n, s and γ forPr = 0.72

M	Ec	Gr	n	s	θ_r	γ	f "(0)	θ'(0)
1	1	1	0.5	0.2	20	0.1	0.314839	-0.30054
5	1	1	0.5	0.2	20	0.1	0.334754	-0.208537
15	1	1	0.5	0.2	20	0.1	0.425935	0.154811
1	3	1	0.5	0.2	20	0.1	0.363958	0.102509
1	4	1	0.5	0.2	20	0.1	0.40339	0.389824
1	1	3	0.5	0.2	20	0.1	0.354375	0.0540638
1	1	5	0.5	0.2	20	0.1	0.420827	0.571153
1	1	1	0.1	0.2	20	0.1	0.300322	-0.438718
1	1	1	0.7	0.2	20	0.1	0.449053	-0.183945
1	1	1	0.5	1	20	0.1	0.359803	-0.108385
1	1	1	0.5	2	20	0.1	0.418309	0.0809971
0.1	1	1	0.5	0.2	1.8	0.1	0.706093	-0.296673
0.1	1	1	0.5	0.2	5	0.1	1.09185	-0.302718

0.1	1	1	0.5	0.2	20	-1	0.224934	-1.13554
0.1	1	1	0.5	0.2	20	0.4	0.393435	0.178101

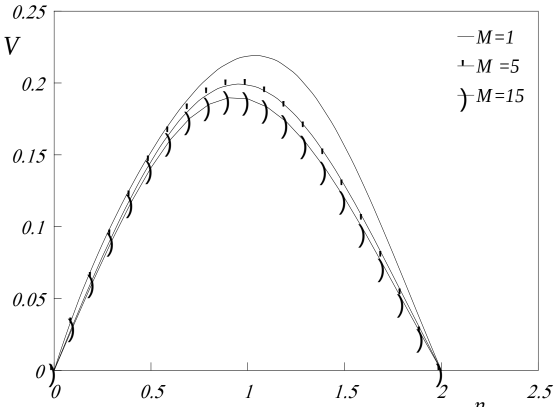
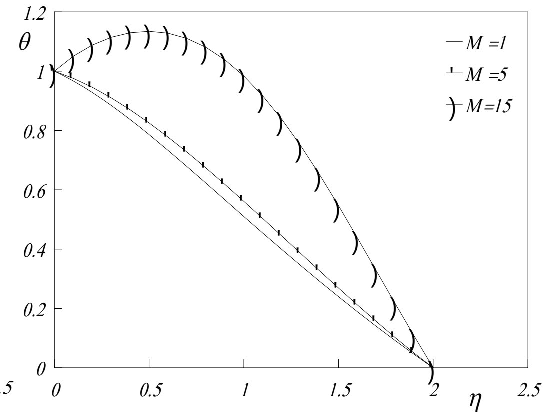


Fig. 2.Velocity profile for various values of magnetic number M



FF Fig. 3.Temperature profile for various values of magnetic number M

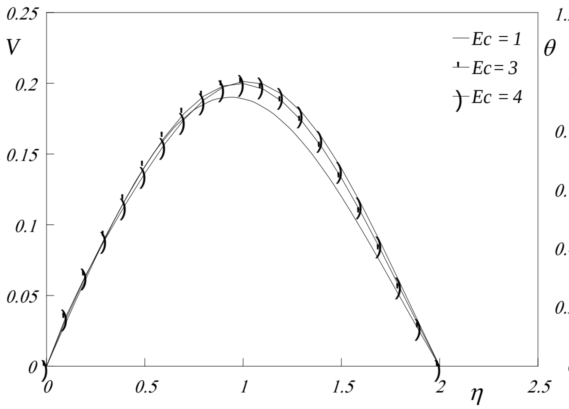


Fig. 4.Velocity profile for various values of Eckert number Ec

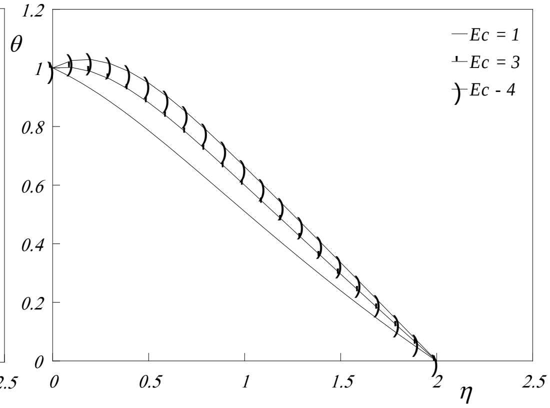


Fig. 5.Velocity profile for various values of Eckert number Ec

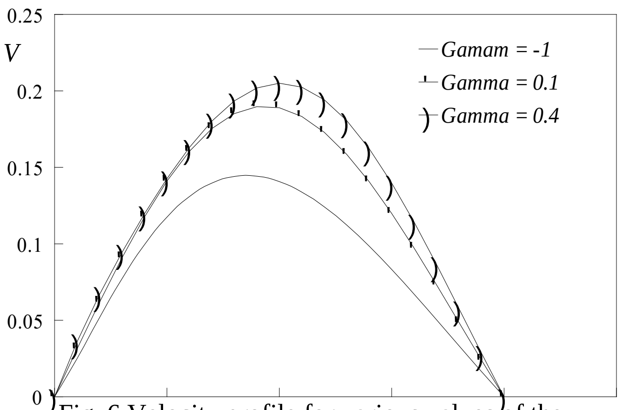


Fig. 6.Velocity profile for various values of the Gamma parameter γ

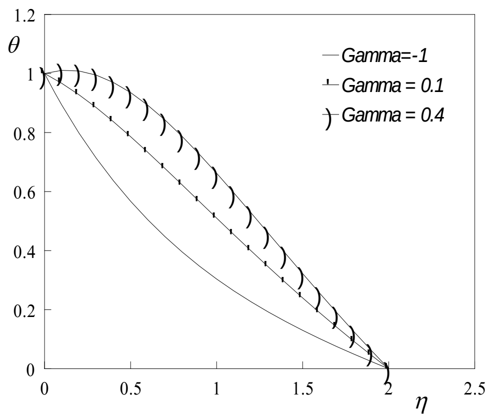


Fig. 7.Temperature profile for various values of the Gamma parameter γ

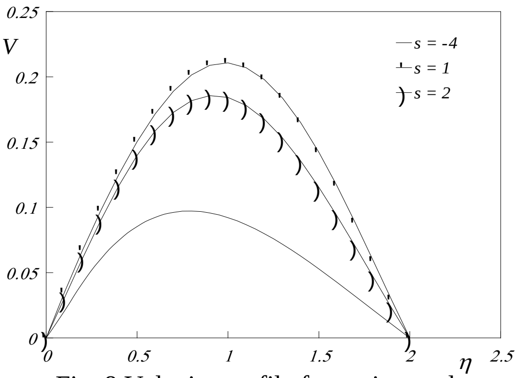


Fig. 8. Velocity profile for various values of the thermal conductivity parameter s

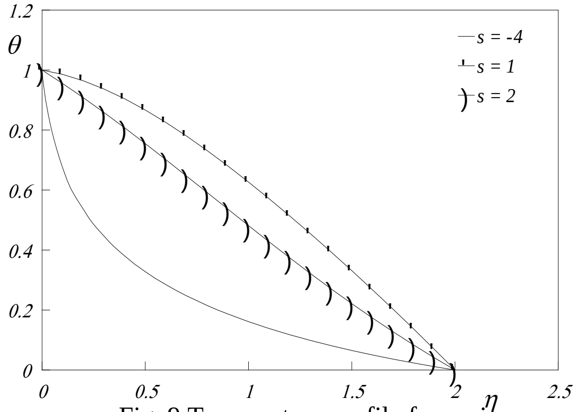


Fig. 9. Temperature profile for various values of the thermal conductivity parameter s

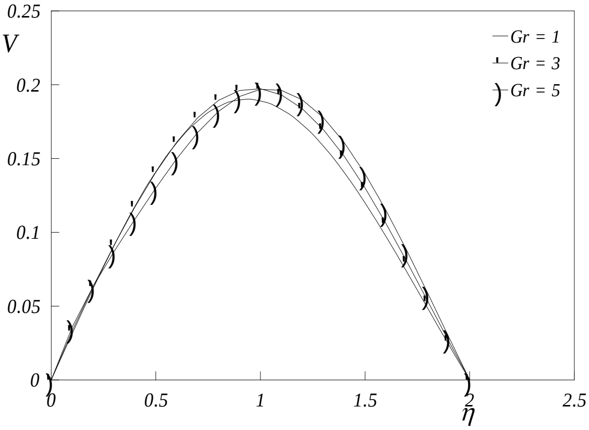


Fig. 10. Velocity profile for various values of the Grashof number Gr

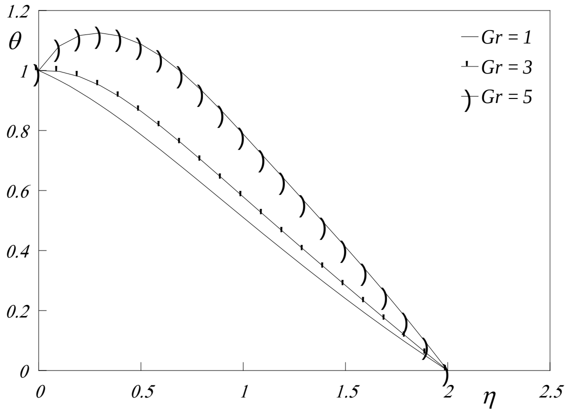


Fig. 11. Temperature profile for various values of the Grashof number Gr

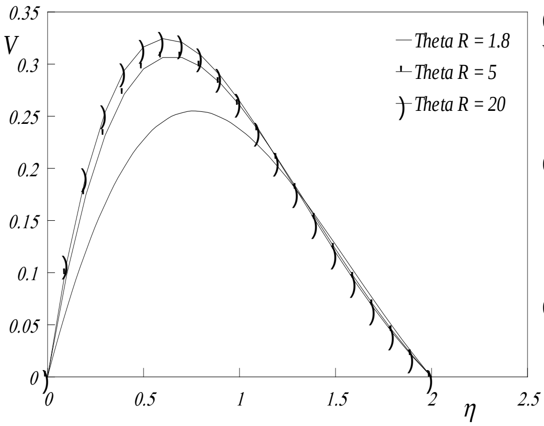


Fig. 12. Velocity profile for various values of the viscosity temperature parameter θ_r

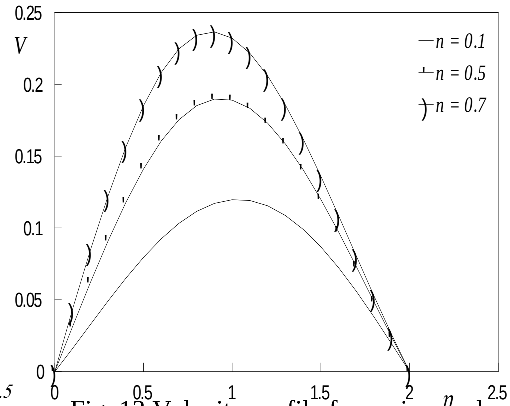


Fig. 13. Velocity profile for various values of the density temperature parameter n

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