# TIME DOMAIN FINITE DIFFERENCE NUMERICAL METHOD OF ANALYSIS OF DIRECT LIGHTNING ELECTROMAGNETIC PULSE AT GROUND HORIZONTAL WIRE

*Xhemali Pejtamalli*
Msc. Electrotechnical Department, Electrical Engineering Faculty, PUT, Albania
*Piro Cipo*
Prof. Dr. Electronic Department, Engineering Faculty  Tirana

**Abstract**
Nowadays ,there are many different studies to analyze the transient process for the spread of the lightning electromagnetic pulse in transmission line with distributed parameters. According to the researcher's interests, works are focused into different directions. This paper presents a model of analysis of the weaning process of spreading the lightning electromagnetic pulse at ground wire located horizontally with the numerical method of finite differences in time zone. Through this study, we can estimate the values of the step voltage, touch voltage, electromagnetic compatibility problems and the over voltage analysis. The mathematical model of ground wire is described by differential equations with partial derivatives of hyperbolic type, whose solution is made with the numerical method of finite differences. Discretiding in space and time is accomplished   with the Lax-Wendroff method.

**Keywords:** Ground horizontal wire, transmission line

## I. Introduction

An electromagnetic pulse (EMP), also sometimes called a transient disturbance, is a short burst of electromagnetic energy. It may occur in the form of a radiated, electric or magnetic field or conducted electric current depending on the source. Electromagnetic pulse is commonly abbreviated EMP, pronounced by spelling the letters separately (E-M-P).

EMP is generally damaging to electronic equipment, and its management is an important branch of Electromagnetic Compatibility  Engineering (EMC). At higher energy levels, an EMP event such as a lightning strike can cause more widespread damage to aircraft components and other objects.

The damaging effects of high-energy EMP have been used to create EMP weapons, both nuclear and non-nuclear. These weapons, both real and fictional, have gained traction in popular culture.

An electromagnetic pulse is a relatively short burst of electromagnetic energy. Its shortness instances means that it will always be spread over a range of frequencies. Pulses are typically characterized by: the type of energy (radiated, electrical, magnetic or conducted); types of EMP divided broadly into natural, man-made and weapons effects. Types of natural EMP event include: Lightning Electro-Magnetic Pulse (LEMP). The discharge is typically an initial huge current flow, at least kilo-amps, followed by a train of pulses of decreasing energy and electrostatic discharge(ESD), as a result of the two charged objects coming into close proximity or even contact. The range of frequencies present pulse envelope or waveform, duration and amplitude. As with any electromagnetic signal, EMP energy may be

transferred in any of the four forms: electric field, magnetic field, electromagnetic radiation and electric conduction.

In general, only radiation acts over long distances, with the others fields, acting only over short distances. There are a few exceptions, such as a solar magnetic flare.

An EMP typically contains energy at frequencies from DC to some upper limits depending on the source. The whole range of concern is sometimes referred to as "DC to daylight", with optical (infrared, visible, ultraviolet) and ionizing (X and gamma rays) ranges being excluded.

Most pulses have a very sharp leading edge, building up quickly to their maximum level. The classic model is a double-exponential curve which increases steeply, by quickly reaching a peak, and then decays more slowly. However, pulses from a controlled switching circuit often take the form of a rectangular or "square" pulse.

EMP events, usually induce a corresponding signal in the victim equipment, due to coupling between the source and the victim. Coupling usually occurs most strongly over a relatively narrow frequency band, leading to a characteristic damped sine wave signal in the victim. Visually, it is shown as a high frequency sine wave growing and decaying within the longer-lived envelope of the double-exponential curve. A damped sine wave typically has much lower energy and a narrower frequency spreading than the original pulse, due to the transfer characteristics of the coupling mode. In practice, EMP test equipment, often injects these damped sine waves directly, rather than attempting to recreate the high-energy threat pulses.

Minor EMP events, and especially pulse trains, cause low levels of electrical noise or interference which can affect the operation of susceptible devices. For example, a common problem in the mid-twentieth century was the interference emitted by the ignition systems of gasoline engines, which caused radio sets to crackle and TV sets to show stripes on the screen. At a higher level, an EMP can induce a spark, for example when fuelling a gasoline-engine vehicle, such sparks have been known to cause fuel-air explosions and consequently precautions must be taken to prevent them.

The direct effect of a very large EMP is to induce high currents and voltages in the victim, damaging electrical equipment or disrupting functions. A very large EMP event, such as a lightning strike is also capable of damaging objects such as trees, buildings and aircraft directly, either through heating effects or the disruptive effects of the very large magnetic field, generated by the current. An indirect effect can be the electrical fires caused by the heating. Most engineered structures and systems require some form of protection against lightning to be designed in. These damaging effects have led to the introduction of EMP weapons. Types of natural EMP events include: Lightning electromagnetic pulse (LEMP). The discharge is typically an initial huge current flow, at least mega-amps, followed by a train of pulses of decreasing energy.

## II CONTENTS
### II.1. Grounding wire modeling. The parameters.

The case that will be taken into consideration is for uniform horizontal earthling (conductor) length $l_p$ area $S_p$ radius $R_{cu}$ and placed into land at depth $h$ (Fig. 1). Specific electric resistance of the soil is $\rho_{tokes}$ and the relative dielectric constant of the soil is. $\varepsilon_r$.
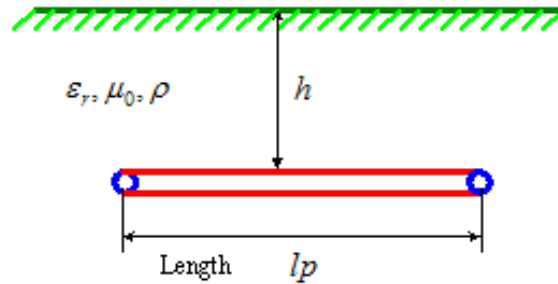
Fig. 1: Ground wire located horizontally

Uniform horizontal grounding system parameters R (resistance), L (inductivity), C (capacity) and G (conductivity) per unit length do not change, i.e. e. are constant. Calculation of parameters per unit length of horizontal grounding system is made uniform by the formulas:

$$R_0 = \rho_{cu} / \pi / (Rcu^2 - (Rcu - \delta)^2) \qquad (1)$$

$$L_0 = (\mu_0 / \pi) * \ln(2h / Rcu) \qquad (2)$$

$$C_0 = \pi\varepsilon_0\varepsilon_r / \ln(2h / Rcu) \qquad (3)$$

$$G_0 = \pi / \rho_{tokes} / \ln(2h / Rcu) \qquad (4)$$

$$\delta = \sqrt{2 * \rho_{cu} / \mu_0 / 2 / \pi / f} \qquad (5)$$

Where $\delta$ is the penetration depth of the wave

## II.2. Solution approximation of hyperbolic partial differential equations with time domain finite differences and with Lax-Wendroff method.

In this study, will be analyzed the spread of the lightning wave, by Lax-Wendroff method, useful to solve various electromagnetic systems in the area of time. This method can be successfully used for solving temporary phenomena in power transmission lines. In our study, Lax-Wendroff method was adapted for the analysis of voltage wave spreading in a uniform horizontal earthling and presents results below, including computer efficiency by using MATLAB's method. This method is used by making discretization in space and time. Line differential equations take the form:

$$-\frac{\partial u}{\partial x} = R * i + L * \frac{\partial i}{\partial t} \qquad (6)$$

$$-\frac{\partial i}{\partial x} = G * u + c * \frac{\partial u}{\partial t} \qquad (7)$$

In matrix form we will have:

$$-\frac{\partial}{\partial x}\begin{bmatrix} i(x,t) \\ u(x,t) \end{bmatrix} = \begin{bmatrix} 0 & G \\ R & 0 \end{bmatrix} * \begin{bmatrix} i(x,t) \\ u(x,t) \end{bmatrix} + \begin{bmatrix} 0 & C \\ L & 0 \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} i(x,t) \\ u(x,t) \end{bmatrix} \qquad (8)$$

where $i(x,t)$ and $u(x,t)$ are respectively the current and voltage wave in line at a point x and time t. To solve these equations (2) the method of Lax-Wendroff's, [1-6] will be used, where time derivatives (t) for j-step time and derivatives position (x) for k-step in space; in the above equations (2), we replace the respective approaches:

$$u(x,t)\big|_{j,k} = \frac{1}{4}(u_{k+1}^j + u_k^j + u_{k+1}^{j-1} + u_k^{j-1}) \qquad (9)$$

$$\frac{\partial u(x,t)}{\partial t}\bigg|_{j,k} \approx \frac{1}{2}(\frac{u_k^j - u_k^{j-1}}{\Delta t} + \frac{u_{k+1}^j - u_{k+1}^{j-1}}{\Delta t}) \qquad (10)$$

$$\frac{\partial u(x,t)}{\partial x}\bigg|_{j,k} \approx \frac{1}{2}(\frac{u_{k+1}^j - u_k^j}{\Delta x} + \frac{u_{k+1}^{j-1} - u_k^{j-1}}{\Delta x}) \qquad (11)$$

Index, respectively k = 1,2, ..., K, and j = 1,2, ..., J, and steps in space and time respectively $\Delta$x = $l_p$ / K and $\Delta$t = T / J and where $l_p$ denotes the length of grounding horizontal (line) and t is the time of the analysis of wave spreading online (earthling). We have selected small equidistant intervals to facilitate our solution by fragmented length and time of analysis, simultaneously.

By substituting (10) in (8), we obtain discretization equations systems in space and time:

$$v_k^j - v_{k+1}^j + A_{vk}(i_k^j + i_{k+1}^j) = -v_k^{j-1} + v_{k+1}^{j-1} + B_{vk}(i_k^{j-1} + i_{k+1}^{j-1}) \qquad (12)$$

$$A_{ik}(v_k^j + v_{k+1}^j) + i_k^j - i_{k+1}^j = B_{ik}(v_k^{j-1} + v_{k+1}^{j-1}) - i_k^{j-1} + v_{k+1}^{j-1}$$

where the coefficients near the currents and voltages are calculated with the formulas, respectively:

$$A_{vk} = -\left[\frac{R}{2} + \frac{L}{\Delta t}\right] * \Delta x \quad , \quad A_{ik} = -\left[\frac{G}{2} + \frac{C}{\Delta t}\right] * \Delta x \qquad (13)$$

$$B_{vk} = \left[\frac{R}{2} - \frac{L}{\Delta t}\right] * \Delta x \quad , \quad B_{ik} = \left[\frac{G}{2} - \frac{C}{\Delta t}\right] * \Delta x \qquad (14)$$

Marking vector of tensions and currents, in the form:

$$V^j = \left[v_1^{jT}, v_2^{jT}, \cdots, v_{K+1}^{jT}\right]^T \ , \ I^j = \left[i_1^{jT}, i_2^{jT}, \cdots, i_{K+1}^{jT}\right]^T \qquad (15)$$

and the composite vector $X^j$, in the form (16) we will have:

$$X^j = \left[v^{jT}, i^{jT}\right]^T \qquad (16)$$

we obtain the solution

$$X^j = A^{-1} * \left(B * x^{j-1} + D^j\right) \qquad (17)$$

Formula (17) provides the composite vector in step (8) (j) of the time by using the values of the step (j-1). Matrices $A$ and $B$ are formed by (13) and (14) the boundary conditions, and the principal vector depends on the values of external resources received in time.

## II.3. Numerical Example

For the analysis of electromagnetic wave spread of the lightning strike, an earthling with horizontal placement is taken on the ground, Fig 2
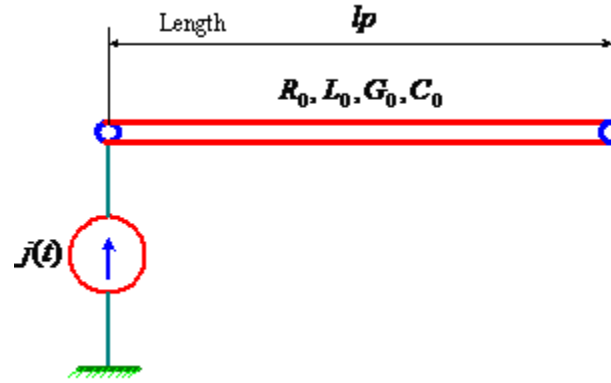
Fig. 2: Lightning current source and horizontal ground wire

Physical parameters are $l_p$, $\varepsilon_r$ and $\rho_{ground}$ which during the simulation scheme variables are taken. Striking wave of lightning is modeled with an ideal source of electricity, of the form:

$$i(t) = 1.04 * \mathrm{Im}\,ax(e^{-t/T1} - e^{-t/T2}) \qquad (18)$$

where

$$T_1 = 0.365434 * TR \qquad (19)$$
$$T_2 = Ts / 2.282835 \qquad (20)$$
$$I_{max} = 150 \ [A] \qquad (21)$$
$$T_S = 2*10^{-6} \ [s] \qquad (22)$$
$$T_R = 77.5*10^{-6} \ [s] \qquad (23)$$
$$f = 1 \ [MHz] \qquad (24)$$

Other parameters

$$\mu_0 = 4\pi * 10^{-7} \ [H/m] \qquad (25)$$
$$\varepsilon_0 = 8.85 * 10^{-12} \ [F/m] \qquad (26)$$

The relative dielectric permeability of the soil
$$\varepsilon_r = 10 \qquad (27)$$

Specific electric resistance of the grounding conductor
$$\rho_{cu} = 16.8 * 10^{-9} \ [\Omega m] \qquad (28)$$
$$\rho_{tokes} = 100 \ [\Omega m] \qquad (29)$$
$$S_p = 50 \ [mm^2] \qquad (30)$$
$$l_p = 20 \ [m] \qquad (31)$$
$$h = 0.5 \ [m] \qquad (32)$$
$$R_{cu} = \sqrt{S_p / \pi / 10^6} \ [m] \qquad (33)$$

### II.3.1.Propagation of current and voltage lightning strike at horizontally ground wire

The first case relates to the analysis simulation of the spread of the current wave of lightning and grounding voltage wave horizontal length [m], at three points: front grounding x = 0 [m], middle grounding x = 10 [m] and its end x = 20 [m].

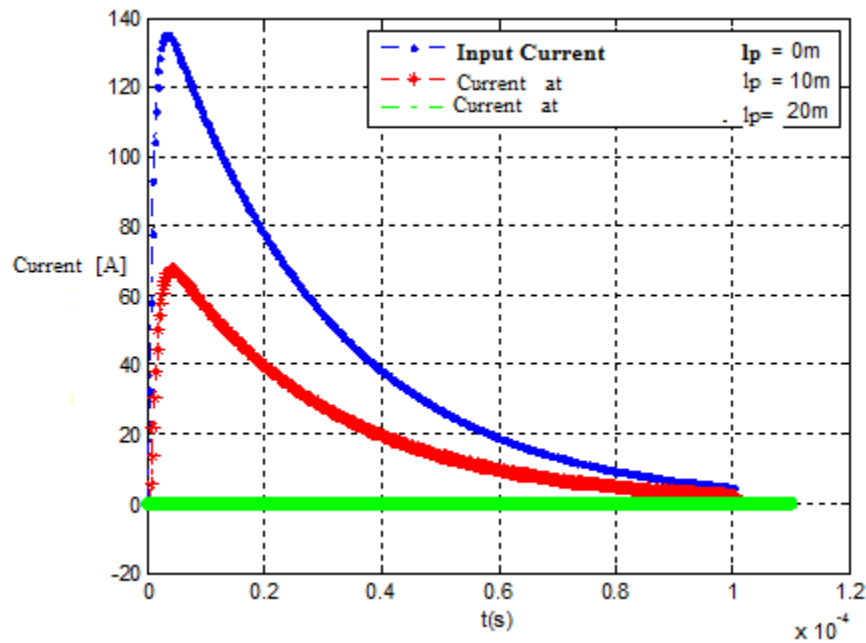The current wave of proliferation caused by thunder lightning, shown in Fig 3:



Fig. 3. Propagation of current lightning wave for $l_p$ =20 [m], $\varepsilon_r$ =10 and $\rho_{grund}$ =100 [Ωm]

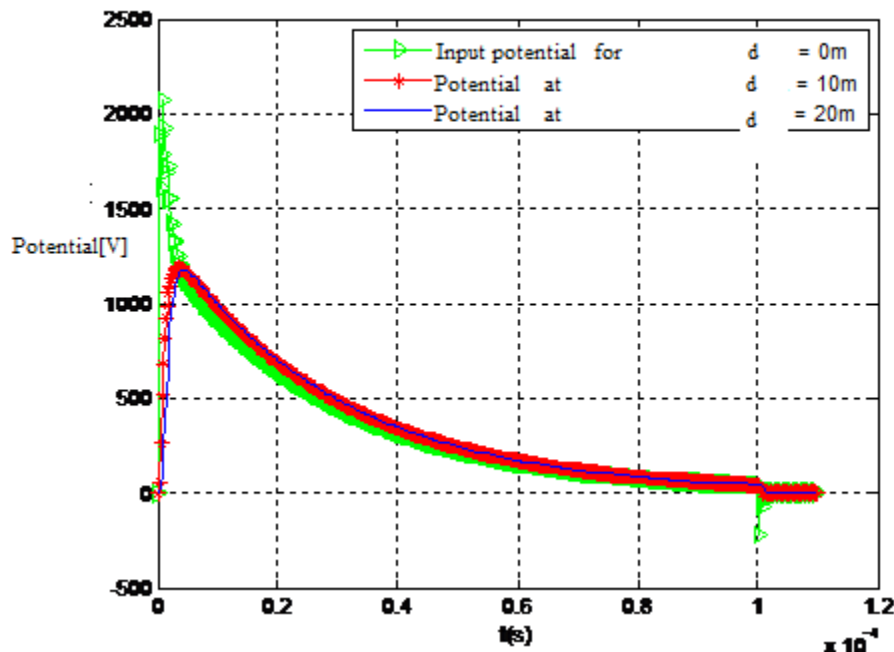Spreading the voltage wave, caused by lightning strike is given in Fig 4:



Fig. 4. Propagation of voltage lightning wave for $l_p$ =20 [m], $\varepsilon_r$ =10 and $\rho_{ground}$ =100 (Ωm)

## II.3.2 The law of the time domain change of input impulsive impedance $Z_{imp}$ as a functions of wire length $l_p$

For the length of the grounding system $l_p$ which vary from 5 to 25 [m] by the analysis, we obtain dependence of resistance impulse $Z_{imp}$ in the function of the length of the grounding system, Fig 5:
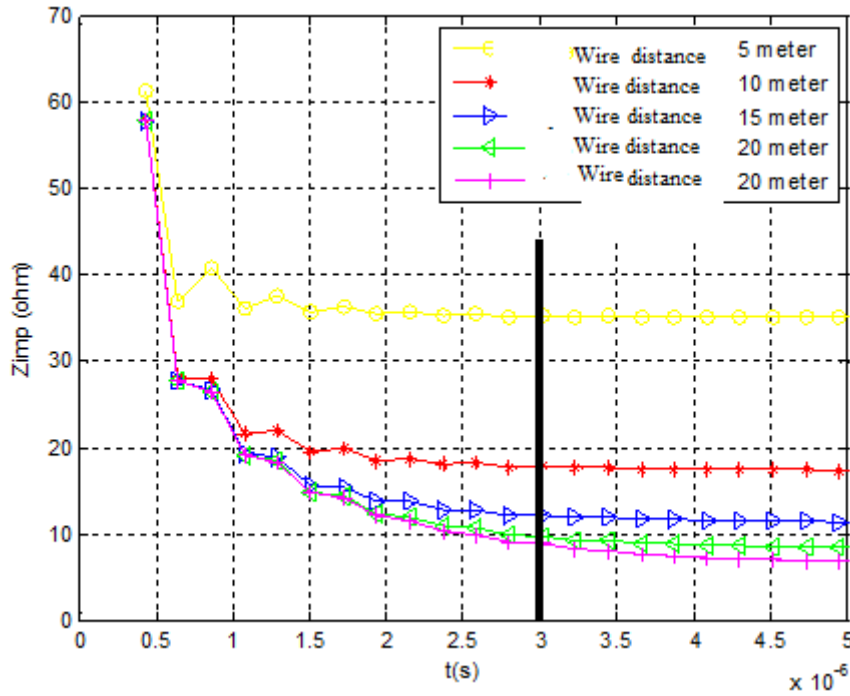
Fig. 5. The law of change of input impulsive impedance for $\varepsilon_r$ =10 and $\rho_{ground}$=100 [Ωm]

Hence, as seen graphically, Fig 5, impulsive resistance entry $Z_{imp}$ is inversely proportional to the length of the grounding system; and it is reduced with the time, up in value $Z_{stabilised}$, stable regime. The time in which $Z_{imp}$ takes values $Z_{stabilised}$ , is called stabilization time wave process of lightning and for our case is $t_{stab}$=3*10$^{-6}$ sec.

### II.3.3 The law of the change of  input impulsive impedance $Z_{imp}$ in time domain as the function of relative dielectric permeability of ground : $\varepsilon_r$

From our analysis of the case for relative dielectric constant of the earth, $\varepsilon_r$ that differs from value 5 to 15, we obtain by simulation, the impendence of $Z_{imp}$ the relative dielectric constant of the soil, wherein is the proper ground,  Fig 6:
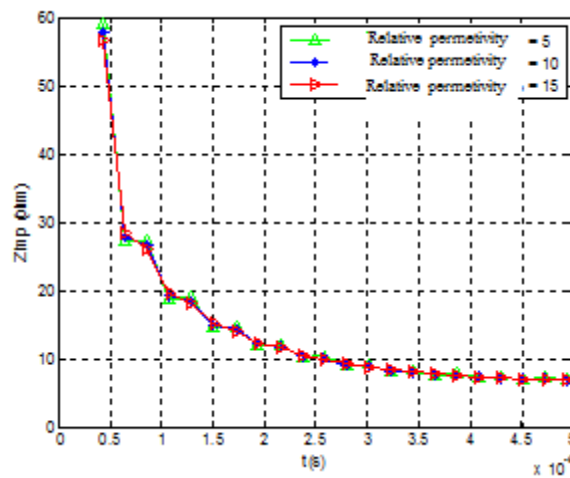


Fig. 6. The Law of the change of input impulsive impedance for $l_p$ =25[m] and $\rho_{ground}$=100 (Ωm)

Consequently, as it turns out graphically in Fig 6, the resistance impulse entry $Z_{imp}$ for our case depends heavily on the relative dielectric constant of the soil $\varepsilon_r$ wherein is the horizontal proper ground

***II.3.4 The law of the time domain change of input impulsive impedance $Z_{imp}$ as a function of electric specific resistance of the ground, $\rho_{ground}$ .***

From the analysis, in our case the specific electric resistance of soil $\rho_{ground}$ which varies from 50 to 150 [Ωm], we take impendence $Z_{imp}$ specific electric resistance of the earth wherein is the proper ground, given in Fig 7.
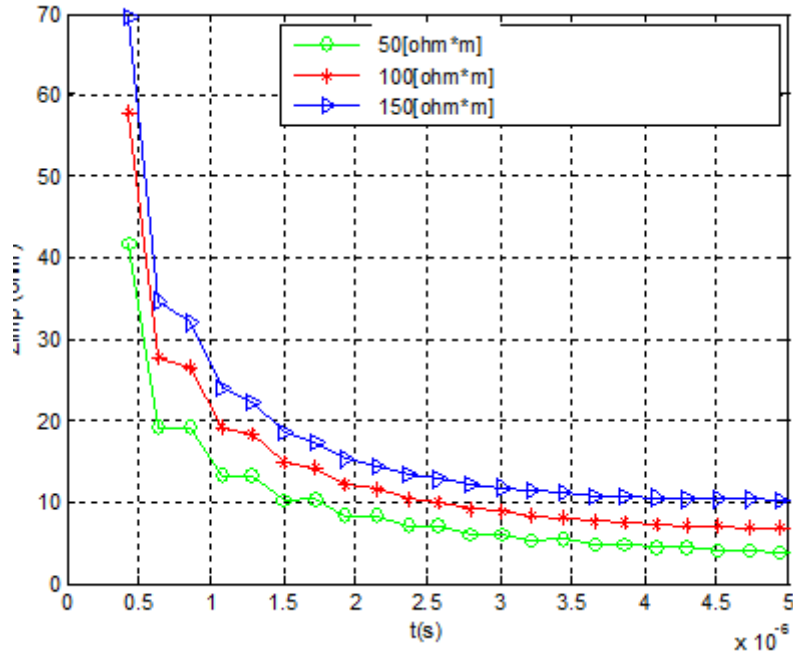
Fig. 7. The law of the change of input impulsive impedance for $l_p$ =25[m] and $\varepsilon_r$ =10

Hence, as it turns out graphically in Fig 7, the resistance impulse entry $Z_{imp}$ is proportional to the specific electric resistance of the soil $\rho_{ground}$ wherein is the proper horizontal ground.

**Conclusion**

Our paper studies a technique for simulating the time zone areas and provide more electromagnetic wave in a transmission line, consisting of a uniform horizontal earthling set in the ground at a certain depth by Lax-Wendroff method. Through a program in MATLAB have managed to analyze the spread of the wave of voltage / current law to analyze the change of resistance of impulse entry $l_p$ , $\varepsilon_r$ , and $\rho_{ground}$ As a result of the graphs obtained, it is clearly observed that the change entry of impulsive resistance $Z_{imp,}$ is inversely proportional to the length of horizontal grounding system $l_p$ and in proportion to the specific electric resistance of the soil, $\rho_{ground}$ where is puted the horizontal ground .

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