INVESTMENT PORTFOLIO REBALANCING DECISION MAKING

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Abstract

Nowadays financial markets' volatility and significant stock prices' fluctuations allow improving investment return actively managing investment portfolio, rather than choosing long term investment strategy. Active portfolio management also allows personal investor's development and gives opportunity to avoid losses in terms of market instability. However active portfolio management is more risky. Rebalancing the investment portfolio investor incurs real costs for expected return, so actively managing the investment portfolio it is crucial to use a good, investor needs meeting portfolio rebalancing method. Dealing with mentioned problem scientific information sources analysis is made and a new portfolio rebalancing method is suggested in the article.

Keywords: Investment portfolio, active portfolio management, portfolio rebalancing

Introduction

Science 1952, when H. Markowitz published his pioneer work "Investment portfolio selection", portfolio selection problem was analysed by many authors. The main aspects of investment portfolio selection problem analysis are: expected return evaluation (prediction) (for example: Willenbrock 2011; Missiakoulis *et al.* 2010; Dzikevicius and Šaranda 2010; Araujo 2010; Erlwein *et al.* 2012), portfolio risk evaluation (Byrne, Lee 2004; Tvaronavicienė, Michailova 2004; Szego 2005), portfolio diversification (for example: Nanda *et al*. 2010; Xidonas *et al*. 2010; Syriopoulos 2011; Thapa, Poshakwale 2012) and portfolio optimisation objectives and methods (technics) selection (for example: Rutkauskas *et al.* 2009; Stasytytė 2011; Xidonas *et al.* 2011; Ustun, Kasimbeyli 2012). All mentioned aspects are very important selecting the best investor needs meeting investment portfolio. However active portfolio management requires frequent portfolio rebalancing, which increases transaction costs. Active portfolio management can be profitable only in cases, when the benefit of active portfolio management exceeds transaction costs. Most scientists analysing portfolio rebalancing problem suggest portfolio rebalancing decision making by evaluating expected return after transaction costs. However actively managing the investment portfolio investor incurs real costs for expected return and the expected return in most cases cannot be precisely evaluated. So portfolio rebalancing decision making only by subtracting transaction costs from expected return cannot ensure best investor needs meeting decisions. Considering the mentioned problem the aim of this work is to suggest a portfolio rebalancing method regarding excess expected return and incurred costs ratio acceptable to the investor.

Theoretical study

Portfolio rebalancing problem is analysed in scientific literature evaluating two main aspects: portfolio rebalancing strategies and portfolio rebalancing algorithms. Dierkes *et al.* (2010) highlights, that the broad range of investment strategies on fundamental classification can be distinguishes between two main categories of investment strategies: forecast-based and forecast-free strategies. Leung (2011) distinguishes three groups of portfolio rebalancing strategies, which can be relatively classified as forecast-free strategies: fixed asset allocations through time; asset allocations that evolve over time according to a fixed and pre-determined schedule; rebalancing rules, under which the allocation at any future time is not pre-determined, but varies according to the actual investment experience up to that time. Jones and Stine (2010) analyses three main forecast-free portfolio rebalancing strategies: buy-hold, constant mix and constant proportion rebalancing separately for bull, bear and trendless markets using Monte Carlo simulation. Cesari and Cremonini (2003), in addition to the above mentioned strategies analysed option based and technical strategies. O'Brien (2006) distinguished four strategies according to the periodicity of portfolio rebalancing: periodic rebalancing, threshold rebalancing, range rebalancing and active rebalancing.

According to Feng *et al.* (2011), Kozat and Singer (2011), frequent portfolio rebalancing can be unprofitable because of portfolio rebalancing costs, so they proposed to rebalance portfolio not every investment interval. Woodside-Oriakhi *et al.* (2013) also indicate that investment horizon has influence on investment results when rebalancing portfolio with transaction costs. Leunberger Kuhn (2010) found that the loss of the uncommon changes of the portfolio composition is very small, and a good portfolio diversification can reduce the negative uncommon portfolio rebalancing effect.

Actively managing investment portfolio, portfolio rebalancing (transactions) costs have a very significant impact on investment results, thus most of researchers dealing with portfolio rebalancing problem (Holden, Holden 2013; Zhang *et al.* 2012, 2011a, 2011b, 2010a, 2010b; Bhattacharyya *et al.* 2011; Fang *et al.* 2006; Feng *et al.* 2011) pays particular attention to the assessment of transaction costs.

Zhang *et al.* (2012, 2011a, 2010b) and Bhattacharyya *et al.* (2011) use formula 1 to evaluate transaction costs:

$$C_t = \sum_{i=1}^{n} c_{t,i} \left| w_{t,i} - w_{t-1,i} \right|, \tag{1}$$

where C_t is the total transaction cost of the portfolio at period t; $c_{t,i}$ - the unit transaction cost of risky asset i at period t; $w_{t,i}$ - the investment proportion of risky asset i at period t.

Zhang *et al.* (2010a, 2011b) proposes more detail formula, which can be used when securities buying and selling costs differ and when new assets can be included in the portfolio:

$$C = \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) + \sum_{j=k+1}^{n} c_j^+ w_j \,, \tag{2}$$

where c_i^+ – the unit transaction cost buying risky asset i; c_i^- – the unit transaction cost selling risky asset i; j – new asset included in the portfolio.

Zhang *et al.* (2012, 2011a, 2011b, 2010a, 2010b) and Bhattacharyya *et al.* (2011) evaluate the expected return of portfolio (which should be maximised or not less than preferred) excluding transaction costs from expected return. If transaction costs are calculated using formula 2, expected return can be calculated using formula 3.

$$E(R) = \sum_{i=1}^{n} E(R_i) w_i - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j \,, \tag{3}$$

where E(R) is expected return after evaluation of transaction costs, $E(R_i)$ – expected return of asset i before evaluation of transaction costs.

Mitchell and Braun (2013) indicate that not only transaction costs, but also market impact costs must be evaluated.

Most of scientists, who deals with portfolio rebalancing problem, prefer fuzzy decision making approach, which usage is broadly analysed in Gupta *et al.* (2013, 2014), Zhang *et al.* (2012, 2011a, 2011b, 2010a, 2010b), Bhattacharyya *et al.* (2011), Fang *et al.* (2006) and Feng *et al.* (2011) publications. However, to propose a portfolio rebalancing decision making method it is not crucial to select one specific approach, more important is the principle of decision making – which objectives are evaluates in portfolio rebalancing decision making. Yu, Lee (2011) analysing portfolio rebalancing problem highlighted short selling possibilities and analysed five multi-objective portfolio rebalancing models: mean-variance model, which does not involve short selling and four models with short selling – mean, variance, and short selling (MVS); mean, variance, short selling, and skewness (MVS_S); mean, variance, short selling, and kurtosis (MVS_K); mean, variance, short selling, skewness, and kurtosis (MVS_SK). The variety of other scientists suggested portfolio rebalancing decision making objectives is presented in Table 1.

Table 1. Portfolio rebalancing objectives

Source	**Objectives, which must be gained making portfolio rebalancing decisions for portfolio rebalancing decisions**
Gupta *et al.* 2014	**Maximise credibility when net return and liquidity are equal or higher than the lower limit on the expected return and liquidity of the portfolio.**
Gupta *et al.* 2013	**Maximise expected return, minimise risk and maximise liquidity.**
Zhang *et al.* 2012	**Minimise the cumulative risk of portfolio and maximize the diversification degree of portfolio, at the same time, the portfolio return at each period must achieve or exceed the given minimum expected level.**
Bhattacharyya *et al.* 2011	**Minimise the risk of portfolio, maximise the expected return and skewness of the portfolio.**
Zhang *et al.* 2011a	$Max \quad U(x) = E(R) - 0{,}005 \times A \times Var$ **, where A is risk tolerance level, Var – risk.**
Zhang *et al.* 2011b	$Min \quad A \times (-E(R)) + Var$
Zhang *et al.* 2010a	**Minimise the cumulative risk of portfolio ensuring not less than given minimum expected portfolio return.**
Zhang *et al.* 2010b	**Minimise the risk and maximise the expected return or** $Min \quad -A \times E(R) + Var$
Fang *et al.* 2006	**Maximize return and minimize risk ensuring that the portfolio liquidity is not less than a given constant.**

As we can see from Table 1, almost all researchers evaluated two main portfolio characteristics – expected return and risk, and only sometimes are mentioned such characteristics like diversification, skewness and liquidity, only Yu, Lee (2011) included kurtosis and short selling. The special attention should be paid to Zhang *et al.* (2011b) work, which also evaluates risk-free lending and borrowing. Expected return in Table 1 given formula is calculated:

$$E(R_p) = \sum_{i=1}^{n} E(R_i) w_i + r_- w_- - r_+ w_+ - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j , \tag{4}$$

where r_- – the interest rate of lending capital; w_- – the amount of lending capital after portfolio rebalancing; r_+ – the interest rate of borrowing capital; w_+ – the amount of borrowing capital after portfolio rebalancing.

Risk-free lending and borrowing cost and return evaluation allows more efficient decisions actively managing investment portfolio with financial leverage.

The suggested investment portfolio rebalancing decision making method

Summarising the analysis of scientific literature dealing with portfolio rebalancing problem, we can see that there are not a lot of works analysing this problem. All these works can be divided into two main groups: scientific works analysing portfolio rebalancing strategies and works which authors suggest direct portfolio rebalancing problem solving solutions. Every investor has to decide himself which portfolio rebalancing strategy meets his needs to e better extent, it is more important to have a method (algorithm) aloving efficient portfolio rebalancing decision making. Evaluating scientific researches made in this field it should be remarked that the most attention in them is paid to transaction costs evaluation and portfolio optimisation after exclusion of transaction costs from expected return. However such portfolio rebalancing approach cannot ensure efficient decision making, because rebalancing investment portfolio investor incurs predetermined costs (losses) and the expected return can be gained or not. It means that in cases when expected benefit of portfolio rebalancing only insignificantly exceeds rebalancing costs, the investor will not be willing to change the composition of the portfolio. The direct application of investment portfolio rebalancing decision making method minimising the cumulative risk of portfolio with not less than given minimum expected portfolio return, proposed by Zhang *et al.* (2010a), cannot ensure efficient decision making because applying this method in some cases investor will refuse higher return portfolio to lower return portfolio. So efficient and investor needs meeting investment portfolio rebalancing method must ensure, that the expected return after transaction costs not only will be higher, but also the difference of expected return between rebalanced and not rebalanced portfolio to some extent will exceed incurred transaction costs.

The proposed investment portfolio rebalancing decision making method is developed using two main portfolio characteristics – expected return and risk, similarly to Zang *et al.* (2011a, 2011b, 2010a, 2010b). No more characteristics like diversification, skewness, liquidity, kurtosis or short selling are included. Skewness and kurtosis are characteristics which also describe risk. Short selling opportunities are allowed not at all financial markets (we cannot use short selling in developing markets). Liquidity must be evaluated selecting securities, which can be included to investment portfolio, because active portfolio management with not liquid securities can be unprofitable due to significant gap between buying and selling prices. Diversification is more important for long term investments, however actively managing the investment portfolio, when investor earns profit from price fluctuations non-diversified investment portfolio can give a higher return than diversified. Wide portfolio diversification can also increase portfolio rebalancing costs.

Let all the amount of money invested in risky assets (for example stocks) of initial investment portfolio (p_0) be equal to w^{t-1}:

$$w^{t-1} = \sum_{i=1}^{n} w_i^{t-1}, \qquad (5)$$

where w_i^{t-1} – amount of money invested in i asset at t-i period.

If the portfolio is not rebalanced its expected return at t period is calculated using formula:

$$E(R_{p_0}) = \sum_{i=1}^{n} w_i^{t-1} E(R_i^t), \qquad (6)$$

where $E(R_i^t)$ – experted return of i asset at t period.

Evaluating risk-free lending return and borrowing costs expected return of not rebalanced portfolio at t period is equal to:

$$E(R_{p_0}) = \sum_{i=1}^{n} w_i^{t-1} E(R_i^t) + r_- w_-^{t-1} - r_+ w_+^{t-1}. \qquad (7)$$

The expected return of rebalanced portfolio (p), when transaction costs are evaluated using formula 2, is equal to:

$$E(R_{p_0}) = \sum_{i=1}^{n} w_i^t E(R_i^t) + r_- w_-^t - r_+ w_+^t - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j . \tag{8}$$

Portfolio risk (variation) is calculated:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{M} w_i w_j COV(R_i R_j) . \tag{9}$$

Portfolio optimisation is made maximising investors utility function:

$$\max U(w) = \sum_{i=1}^{n} w_i^t E(R_i^t) + r_- w_-^t - r_+ w_+^t - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j - A \sum_{i=1}^{N} \sum_{j=1}^{M} w_i w_j COV(R_i R_j). \tag{10}$$

or, in cases when investor has predetermined maximal acceptable risk level $\hat{\sigma}_p^2$ – $\sigma_p^2 \le \hat{\sigma}_p^2$, maximizing expected return:

$$\max E(R_p) = \sum_{i=1}^{n} w_i^t E(R_i^t) + r_- w_-^t - r_+ w_+^t - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j , \tag{11}$$

when $w_i^t, w_-^t, w_+^t \ge 0$

$$\sum_{i=1}^{n} w_i^t + w_-^t - w_+^t - \sum_{i=1}^{k} (c_i^+ w_i^+ + c_i^- w_i^-) - \sum_{j=k+1}^{n} c_j^+ w_j = W , \tag{12}$$

if risk-free borrowing possibilities are limited, must be satisfied condition $w_+^t \le w_+^{\max}$, if minimal amount of risk-free lending is pre-determined, must be satisfied condition $w_-^t \ge w_-^{\min}$ and if minimal risky assets trading volume is pre-determined, must be satisfied condition $w_i^t \ge w_i^{\min}$, where A – risk tolerance level, W – total amount of money, which can be invested; $w_+^{\max}$ - maximal amount of money, which can be borrowed with risk-free interest rate; $w_-^{\min}$ - minimal amount of risk-free lending; $w_i^{\min}$ - minimal amount of investment in i risky asset.

Portfolio rebalancing decision making is based on:

- expected return of initial (not rebalanced) investment portfolio;
- expected return of optimal portfolio;
- transaction costs.

Portfolio must be rebalanced when is satisfied condition:

$$K = f(E(R_{p_0}); E(R_p); C) \ge K_{\min} , \tag{13}$$

where K – value of portfolio rebalancing decision making criterion; K_{min} – the minimal value of criterion, which must be gained to make a decision to rebalance portfolio.

The minimal value of criterion K_{min} is individual for each investor depends on forests accuracy (how reliable forecasting programs and tools uses investor) and investors attitude to portfolio rebalancing.

The specification of formula 13 can be also individual for each investor. One of possible formula 13 specification versions could be excess expected return and incurred costs ratio. Portfolio is rebalanced when is satisfied condition:

$$\frac{E(R_p) - E(R_{p_0})}{C} \ge K_{\min} . \tag{14}$$

If investor chooses the minimal value of criterion K_{min} to be equal to 1, it means, that investor will make decision to rebalance investment portfolio only if the expected increase of return after transaction costs will exceed transaction costs (the difference between optimized portfolio return before transaction costs and initial portfolio return is twice more than transaction costs).

Using the suggested method investment portfolio would be rebalanced not every period, but only under certain conditions, so this method will ensure implementation of Feng *et al.* (2011), Kozat and Singer (2011) approach to portfolio rebalancing.

Considering the suggested investment portfolio rebalancing decision making method it is important to mention, that it, like and other scientists proposed methods, gives opportunity tu evaluate only expediency of option to choose the new optimal portfolio, however it cannot be used for partial portfolio rebalancing. Partial portfolio rebalancing decision could be made based on marginal change of expected return and marginal transaction costs. In this case, investor should sell worst assets and buy best optimal portfolio assets until is satisfied condition:

$$\frac{\Delta E(R)}{\Delta C} \geq K'_{\min}, \tag{15}$$

where ΔE(R) – marginal change of expected return; ΔC – marginal transaction costs; $K'_{\min}$ - the minimal value of marginal change of expected return and marginal transaction costs ratio criterion, which must be gained to make a decision to rebalance portfolio.

The application of marginal change of expected return and marginal transaction costs approach allows evaluating and making investment portfolio rebalancing decisions, which have the most impact to expected portfolio return, it ensures that no transactions will be made, when the change of expected return only insignificantly exceeds transaction costs. In view of the fact that partially rebalancing the investment portfolio investor chooses not optimal portfolio, which can less or more risky than the level of risk acceptable by investor, portfolio risk can be modified by changing the intensity of the use of financial leverage.

Conclusion

Scientific literature analysis and investigation made preparing this article allow to make some conclusions:

1. The analysis of scientific literature shows that portfolio rebalancing problem is not broadly analysed by scientists, there are two main aspects of mentioned problem investigated: portfolio rebalancing strategies and portfolio rebalancing algorithms.
2. The scientific literature usually propagates the application of the portfolio rebalancing decision making method (algorithm) evaluating expected return after transaction costs, however such an approach cannot ensure best investor needs meting decisions, because rebalancing investment portfolio incurs real costs for expected return.
3. Investment portfolio rebalancing decision making method based on expected return change and actually experienced transaction costs ratio suggested in the article allows portfolio rebalancing not every period, but only under certain, investor's pre-determined conditions.
4. Even in cases, when using suggested portfolio rebalancing method it is not beneficial to rebalance portfolio, initial portfolio can have assets, which should be sold and purchased new assets from optimal portfolio. Partial portfolio rebalancing should be made based on proposed marginal change of expected return and marginal transaction costs ratio.
5. Partially rebalanced investment portfolio risk can be adjusted by changing the intensity of the use of financial leverage.

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