MATHEMATICAL-STATISTICAL MODELS IN INSURANCE

Zuzana Kratka, Mgr., Ing., PhD
Slovak University of Technology in Bratislava, Slovak Republic

Abstract

The insurance companies use many different models especially when they look for the optimal solutions of problems in pricing and development of product, complete risk profile of the company, risk quantification, cession of risks, capital adequacy, the location of assets, administration and management of capital and in many other areas. The big world-wide insurance companies invest huge amounts to development of models focused on right pricing insured risks and also risks which are very difficult quantified and non-insurable nowadays, but probably they will influence the insurance and reinsurance market markedly in future. Mentioned risks are risk of global ecological disasters, risk of global climatic conditions, energetic risks, risk of information and communication technologies, etc. The climatic changes bring new and statistic uncharted risks. The insurance companies have to give up reliance only upon risks models strictly based on historical data. The models are simplified images of complicated real systems and modeling is often sole instrument for their understanding. The key to successful modeling is also accession to right information. The insurance companies have to develop database of information and also technologies for specialists responsible for decision making. The problem is not sufficient data of loss experience in some countries. The modeling, if it should be exact, has to work with sufficient data. Because of decrease of costs connected to records archive and database making, it will be not problems in future.

Keywords: Model, insurance, risk management, Solvency II

Introduction

The signification of modeling is essential in insurance and it incidentals to its stochastic character. Accidental nature of events and theirs unpredictability are basic attributes of insurance. It consists in occurrence indeterminacy of specific unfavorable situation and also in its temporal indefiniteness. It is problematical for individual subject to estimate, if the

appropriate risk appears and measure of loss caused by accidental event. There is possible to estimate occurrence and measure of accidental events for group of participants by statistical-mathematical models.

The insurance companies use many different models especially when they look for the optimal solutions of problems in pricing and development of product, complete risk profile of the company, risk quantification, cession of risks, capital adequacy, the location of assets, administration and management of capital and in many other areas.

The big world-wide insurance companies invest huge amounts to development of models focused on right pricing insured risks and also risks which are very difficult quantified and non-insurable nowadays, but probably they will influence the insurance and reinsurance market markedly in future. Mentioned risks are risk of global ecological disasters, risk of global climatic conditions, energetic risks, risk of information and communication technologies, etc. The climatic changes bring new and statistic uncharted risks. The insurance companies have to give up reliance only upon risks models strictly based on historical data. There is necessary to use results of academic studies and prognosis for adequate valuation future risks and mainly the insurance companies should work out their own scenarios about potential development of risk. That should not be only simply projections, but more detailed presentation of alternative futures. The responsible access by these scenarios allows analysis in advance the potential consequences of interplay of several variables by insurance companies and allows take measured and complex decisions.

The development of new technologies and informatization has significant influence on performance of insurance companies and also on making of models. This process allows increase the requests in creation of the statistical-mathematical models, but there are also increased requests in understanding of processes and results of models. The capacity and rapidity computer techniques allow construct such large models which were not considered before.

The deterministic and stochastic approach to modeling

The models are simplified images of complicated real systems and modeling is often sole instrument for their understanding. The model should retain only these aspects which are relevant during solving proper problem and other should be abandoned. This is the reason why the model can abstract from reality, but reversely, it is its main task. The quality of model is measured by its explanatory value, and not according to its abnormalities from reality. Despite of many models are built on anti realistic premises, but their application leads to excellent estimative solutions. At first the models are created as very simplified picture of reality. In case of need there is

possible to achieve detailed description of real system by their subtilization. This process is called the method of decrease abstraction.

In real world, in which the future is not known, the human behavior depends on expectations. The models are simplified images of reality and their main advantage is possibility to express indefiniteness of real world by random variables. The models are used to search for common and generally relevant patterns and relationships of real systems. It is meaningful and adequate reduction of reality.

Other advantages of modeling, which is often only tool for study of real system, are following:

- Premise of creation of any model of examined system is structuralization of system, which leads to generally knowing of system, all of its basic factors, relations and coherences. The construction of model is specific method for observation of reality, which would be very multiform for deduction of required conclusions (often numerical), whereas the model is confined on sufficient segment of reality built by properly chosen system of premises;
- The analysis of system specifies possible variants of estate of modeled system (there can be infinite amount of estates).
- The models allow the analysis of system behavior in significantly shorter time;
- It is possible to experiment and realize different operations by changes of parameters in models. The creators or users can express their different projections of reality by setting up the parameters;
- There are always the costs of model realization lower than costs of experiments with real system;
- The work with model allows avoid the real experimentation which can lead to unacceptable estate.

The modeling is process with several phases. At first is necessary to specify problem which has to be solved. The formulated model is literal and numerical picture of problem. This model is translated into mathematical model which is solved by standard procedures. The solution of mathematical model is solved by available software. Very important phase is interpretation of solutions and their verification. Then it is possible to implement the model within the analyzed real system. The single formulation of problem needs some measure of simplification in sense split it off the real system. The model has to be assembled like simplified image of real system with essential items and relations between them. There should be defined the tools for solving this problem. The real system is very complicated for model and often the formulated problem has not connections with all of areas in real

system. There is very important to make right decision about the aspects which are necessary to make provision for creating of model in actual situation. The most important issue of decision making is the purpose of using the model. The included issues and excluded issues have relation on purpose of model. The model has to respect the goal of analysis. We should know the goal of real system when we are creating the model. The model has to include:

- Description of processes which are the part of real system – we understand the process as each activity of system which can influence final status,
- Description of elements which have influence processes – each process passes with proper intensity and its realization is influenced by many elements which have to be respected,
- Description of connections – correlative connections and relations among processes of real system, its elements and goals.

The formulated model is literal and numerical description of problem. Before resolution of problem is necessary to transfer this problem to mathematical model which is solved by standard procedures. The mathematical model follows the formulated model in every detail, but the formulation is different. The mathematical language is clear. The specific mark of mathematical model is the abstraction of aspects which are not connected with its quantitative and space forms and relationships. The solution of mathematical model is sort of technical job. There is many of software tools nowadays, which effective solve the typical tasks of method of operation analysis. There was created software system ICFRS (Interactive Claims Reserving Forecasting System) in Australia. There are made the software for modeling of insurance claims and other software developed and used by actuarial consulting companies. The interpretation of results is important part of application of models of operational analysis. The difference from above-mentioned step, which is sort of routine issue, is difficulty in interpretation at the beginning. The interpretation is not satisfactory, the results of models are necessary to verify. The analysis of results confirms also the right formulation of model and mathematical model. When the verification of system runs correctly, there is possible to implement the model into the analyzed real system. The successful implementation should contribute toward improvement of functioning of modeled system in relation to goal defined in model.

In theory and practice are used the deterministic and stochastic models which are connected with two different approach to modeling of real systems. The deterministic approach predicts the stability when accidental variations of system can be neglected. All of future decisions flow from fundamentals of system. There are all of variables given in the deterministic

model or there are easy calculated. The presence and future are interlacing only on former times. This type of modeling is useful, but it is not always adequate in connection to uncertainty of surrounding of insurance companies. The stochastic approach assumes at least one of variables has values which we not know in advance. The changes occur with certain probability. In comparison to simple deterministic models, the stochastic models present dynamic approach of modeling. The main tool is accidental processes which can be defined as set of accidental quantities depending on certain number of parameters and each of parameters is defined on set of real numbers. The procedure of accidental process is always different. There are many processes with one parameter defined on discrete or linear sets in practical applications. There is often time this parameter.

The modeling – important tool for risk management in insurance companies

The mathematical-statistical models are part of risk management, make support for its processes, in spite of they are not the risk management. The models present valuable tool for management of company. The financial reports present retrospective view of financial position in specific period, but model´s approach allows consider to the management prospective possible future financial conditions of insurance company like dynamic business item. Quantitative data about the borrower's credit quality and data warehouses have allowed the creation of tools for portfolio analysis as well as active asset management (Jagric & Kracun & Jagric, 2011).

More insurance companies revaluate their present strategy for identification and quantification of risks. They are very well conscious of good system of risk management focused on early and complex identification and quantification markedly reduces possibility of malfunction and uncertainty connected with the achievement of goals in insurance companies, because it allows understand potential negative or positive consequences of factors which can have influence on it. The concept of sophisticated risk management and its modeling play more important role nowadays. The mathematical-statistical models support more the processes of risk management in all of areas. Mainly, they allow better define and quantify the level of risks or groups of risks influencing activities of insurance companies according to its individual need.

The effect of reinsurance is possible to model for both sides – for insurer and also for reinsurer, e.g. during the processing of aggregate model about performance of insurance company must be always included the structure of reinsurance program. This model can include comparison of different reinsurance programs in different categories as price, capacity and accessibility.

The mathematical-statistical models are used in every phase of insurance activity and also in economical activities of insurance companies nowadays. There are models used in investment and also in insurance activities of insurance companies. In final phase the management has to has the ambition to see the company as on integral body. The identification and next quantification of risks have to be in progress on individual and also on consolidated basic. This complex perception of activities and connected risks allows see different interconnections and influence among modeled risks areas. The obtained outputs are as good as inputs connected with risks for each class insurance or investment. There are not unificated opinions what is more useful the simply modeling or complex modeling in practice in spite of their advantages. The advantages of simple models are lower costs of realization, bigger transparency in comparison to more complex models and lower number of mistakes. There is recommended to begin with simply model with less number of demands. There is recommended also to improve the level of models from time to time, recover the aspects which were not implemented yet by current control of existing results. On the other side there are opinions that the complex reality needs complex models from the beginning and during whole process.

Internal models in the new regulation Solvency II

The models of risks identification and quantification can create sole insurance company – they are internal models or they can rely on external models. It is in competence of management.

Internal models are developed by insurance companies or specialized software companies cooperated with insurance companies and developed the models according to needs and demands of insurance companies. In case the insurance company makes decision to developed own model, it has to enough capacities of material, human resources, technical and technological sources especially at the beginning of process. The internal models allow identify and quantify risks corresponding to risks profile of existing insurance company. The conclusions of analysis well-founded by quality internal models are used many years in insurance industry, more advanced internal models are used at last years and they are used especially in big global insurance companies which insure risks potentially caused the global disasters. The problems of internal models are data intensity, demandingness on specialists and costingness. The smaller companies are conscious of their competitive disadvantage in area of internal models and this is the reason of their cooperation, e.g. „data pooling“ or „data exchange“ that means merging and changing of information. Other trend is „data mining“ that means revision of old data and aspiration of understanding not yet detected relations which is possible to use for growth of competition.

The internal models allow better understanding of risks profile of the insurance company than external models. In external models are lump sum risks factors appropriate average risk rate of market, but not of the insurance company. The results of external model should not be relevant for the insurance company and can lead to wrong decisions.

The new legal regulation of the insurance market of the European Union – called Solvency II – supports internal models. Solvency II project places emphasis on the modeling and management of risks of the insurance companies. This requires further improvement in actuarial methods and their application in insurance practice (Pacáková & Šoltés & Linda, 2014). In accordance with the risk-oriented approach to the capital requirement, it should be possible, in specific circumstances, to use partial or full internal models for the calculation of that requirement rather than the standard formula. Insurance companies may use partial internal models for the calculation of one or more risk modules, or sub-modules, the capital requirement for operational risk, etc. In addition, partial modeling may be applied to the whole business of insurance companies, or only to one or more major business units.

In order to provide policy holders and beneficiaries with an equivalent level of protection, such internal models should be subject to prior supervisory approval on the basis of harmonized processes and standards. Insurance companies shall demonstrate that the internal model is widely used in and plays an important role in their system of governance and that the internal model continues to appropriately reflect the risk profile of the insurance company concerned. Insurance companies belonging to a group should be able to apply for the approval of an internal model to be used for the solvency calculation at both group and individual levels.

For insurance companies using a partial or full internal model the risk management shall cover the following additional tasks (European Parliament & Council, 2009):

- to design and implement the internal model;
- to test and validate the internal model;
- to document the internal model and any subsequent changes made to it;
- to analyze the performance of the internal model and to produce summary reports thereof;
- to inform the administrative, management or supervisory body about the performance of the internal model, suggesting areas needing improvement, and up-dating that body on the status of efforts to improve previously identified weaknesses.

The internal model, and in particular the calculation of the probability distribution forecast underlying it, shall comply with the following criteria (European Parliament & Council, 2014):

- The methods used to calculate the probability distribution forecast shall be based on adequate, applicable and relevant actuarial and statistical techniques and shall be consistent with the methods used to calculate technical provisions. The methods used to calculate the probability distribution forecast shall be based upon current and credible information and realistic assumptions. Insurance companies shall be able to justify the assumptions underlying their internal model to the supervisory authorities.
- Data used for the internal model shall be accurate, complete and appropriate. Insurance companies shall update the data sets used in the calculation of the probability distribution forecast at least annually.
- No particular method for the calculation of the probability distribution forecast shall be prescribed. Regardless of the calculation method chosen, the ability of the internal model to rank risk shall be sufficient to ensure that it is widely used in and plays an important role in the system of governance of insurance companies, in particular their risk management system and decision making processes, and capital allocation. The internal model shall cover all of the material risks to which insurance companies are exposed. Internal models shall cover at least the following risks: non-life underwriting risk, life underwriting risk, health underwriting risk, market risk, counterparty default risk.
- As regards diversification effects, insurance companies may take account in their internal model of dependencies within and across risk categories, provided that supervisory authorities are satisfied that the system used for measuring those diversification effects is adequate.
- Insurance companies may take full account of the effect of risk-mitigation techniques in their internal model, as long as credit risk and other risks arising from the use of risk-mitigation techniques are properly reflected in the internal model.
- Insurance companies shall accurately assess the particular risks associated with financial guarantees and any contractual options in their internal model, where are material. They shall also assess the risks associated with both policy holder option s and contractual options for insurance companies. For that purpose, they shall take account of the impact that future changes in

financial and nonfinancial conditions may have on the exercise of those options.

- In their internal model, insurance companies may take account of future management actions that they would reasonably expect to carry out in specific circumstances.
- In their internal model, insurance companies shall take account of all payments to policy holders and beneficiaries which they expect to make, whether or not those payments are contractually guaranteed.

Supervisory authorities may require insurance companies to run their internal model on relevant benchmark portfolios and using assumptions based on external rather than internal data in order to verify the calibration of the internal model and to check that its specification is in line with generally accepted market practice.

Insurance companies shall review, at least annually, the causes and sources of profits and losses for each major business unit. They shall demonstrate how the categorization of risk chosen in the internal model explains the causes and sources of profits and losses. The categorization of risk and attribution of profits and losses shall reflect the risk profile of the insurance companies.

Insurance companies shall have a regular cycle of model validation which includes monitoring the performance of the internal model, reviewing the ongoing appropriateness of its specification, and testing its results against experience. The model validation process shall include an effective statistical process for validating the internal model which enables the insurance companies to demonstrate to their supervisory authorities that the resulting capital requirements are appropriate. The statistical methods applied shall test the appropriateness of the probability distribution forecast compared not only to loss experience but also to all material new data and information relating thereto. The model validation process shall include an analysis of the stability of the internal model and in particular the testing of the sensitivity of the results of the internal model to changes in key underlying assumptions. It shall also include an assessment of the accuracy, completeness and appropriateness of the data used by the internal model.

Insurance companies shall document the design and operational details of their internal model. The documentation shall provide a detailed outline of the theory, assumptions, and mathematical and empirical bases underlying the internal model. The documentation shall indicate any circumstances under which the internal model does not work effectively. Insurance companies shall document all major changes to their internal model. The use of a model or data obtained from a third party shall not be

considered to be a justification for exemption from any of the requirements for the internal model set out in Directive.

Conclusion

There are some negative issues which have influence on modeling, in spite of importance of modeling in insurance industry. There is problem with adequate group of authors (their qualification and expertise). It can be strong advantage, but also weakness when the creation of fine model can fall in beginning. The strong project team is as much important as IT and this is the reason why companies have to invest to education of their employees or to achieve new specialists because of intensity of calculations. Other possibility is support from external surrounding (services of consultants and actuaries who are not employees of company) or buying of external software. Relatively long time necessary to line up model to practice is also restricting factor. The modeling can help to decrease uncertainty, but managers have to take into account that building of right model can take three or four years that means the reinsurers have to include time period to the modeling.

There is necessary to remember that the key to successful modeling is also accession to right information. The insurance companies have to develop database of information and also technologies for specialists responsible for decision making. The problem is not sufficient data of loss experience in some countries. The modeling, if it should be exact, has to work with sufficient data. Because of decrease of costs connected to records archive and database making, it will be not problem in future.

Last but not least two connected problems in implementation of models are leak of consensus with top management which goes together with risk that model will not bring applicable outputs. Many factors of models rely on choice of expert (this is e.g. selection of probability of models), because to explain the positive effects of model implementation to top management (especially then they are „non-actuaries“) can be very difficult in consequence of complicated interpretation for experts. The costs for creation and improvement of models are high, and if investments to development have to refund, management has to understand the models that they can implement results in practice. This is the reason why actuaries do not rely on method Monte Carlo. They aware that other classic stochastic simulation techniques can present many simulations and then experts reach the number which is not relevant for insurance company. The accent is put on development of techniques which allow more definite exploration of problems with detailed analysis of data.

None of models is perfect and calculation of risk will be always combination of science and personal deduction. The effusively reliance on models can caused the overloading by information and ineffective exploiting

of scarce resources. The models are not black boxes containing all of information for effective maintenance of company.

References:

ČERMÁKOVÁ, H. (2002). *Modelování efektivnosti řízení technologických procesů, popis matematických metod*. Liberec: Technická univerzita v Liberci.

EUROPEAN PARLIAMENT & COUNCIL OF THE EUROPEAN UNION. (2009). *Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)*. Brussels: Official Journal of the European Union. Retrieved December 9, 2014, from http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:02009L0138-20140523&qid=1421162957182&from=SK

EUROPEAN PARLIAMENT & COUNCIL OF THE EUROPEAN UNION. (2014). *Directive 2014/51/EU of the European Parliament and of the Council of 16 april 2014 amending Directives 2003/71/EC and 2009/138/EC and regulations (EC) No 1060/2009, (EU) No 1094/2010 and (EU) No 1095/2010 in respect of the powers of the European Supervisory Authority (European Insurance and Occupational Pensions Authority) and the European Supervisory Authority (European Securities and Markets Authority).* Brussels: Official Journal of the European Union. Retrieved December 9, 2014, from http://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32014L0051&qid=1421164239138&from=SK

FABOZZI, F. J. & FOCARDI, S. M. (2004). *The Mathematics of Financial Modeling and Investment management*. New Persey: Wiley. ISBN 0-471-46599-2.

JAGRIC, V. & KRACUN, D. & JAGRIC, T. (2011). *Does Non-linearity Matter in Retail Credit Risk Modeling?* Czech Journal of Economics and Finance (Finance a úvěr), 61(4), 384-402. ISSN: 0015-1920.

PACÁKOVÁ, V. & ŠOLTÉS, E. & LINDA, B. (2014). *Multidimensional credibility model and its application*. E+M Ekonomie a Management, 17(3), 170-184. DOI: 10.15240/tul/001/2014-2-013