WAVELET TRANSFORM APPLIED IN ECG SIGNAL PROCESSING

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Abstract

In this paper, we have introduced the compression of electrocardiogram (ECG) using a wavelet transformation. We will treat ECG as a signal and will implement a matched filter; and more precisely, the Wiener filter which is proportional to the signal itself. Also, we will use this filter to detect the positions of the heart beats. In this application, we will be using the white noise. However, the matched filter will not be proportional to the signal itself. Other results we have computed for this ECG signal are the mean difference of the heart beats and the heart rate. All these results are well–established diagnostic tools for cardiac diseases.

Keywords: Wavelet transform, filter, signal processing, noise

1. Introduction

The electrocardiogram (ECG) provides information about the heart. ECG is a biological signal which generally changes its physiological and statistical property with respect to time, tending to be a non-stationary signal. In studying such types of signals, wavelet transforms is very useful. The most striking waveform when considering the ECG is QRS wave complex which gives the R wave peak which is time–varying. In this paper, we will describe the detection of QRS complex using wavelet transform. This detector is reliable to QRS complex morphology and properties which changes with time and with the noise in the signal.

During a single cardiac cycle, there are different feature points known as the P wave, QRS complex, and the T wave. Specifically, QRS wave is used to detect arrhythmias and identify problems in the regularity of the heart rate. It is complicated to detect the R wave which is the highest point of the QRS complex. This is because it is changing with time, corrupted with noise, and is subject to baseline wandering due to different patient conditions. Sometimes, in the ECG signal, QRS complexes may not always be the prominent waves because they change their structure with respect to time at different conditions. However, they may not always be the strongest signal sections in the ECG signal. Also, the ECG signal can be affected and degraded by other sources such as noise in a clinical environment like patient condition, baseline wandering due to respiration, patient movement, interference of the input power supply, contraction and twitching of the muscles, and the weak contact of the ECG electrodes.

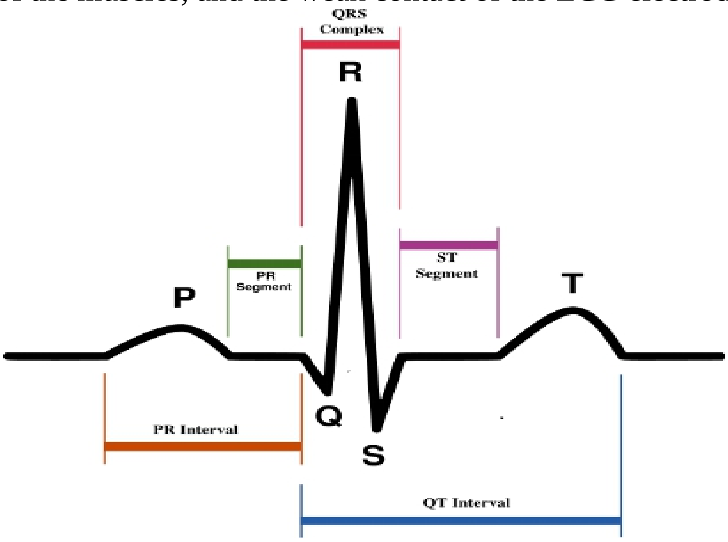

Figure 1. ECG Waveform and its Components

Therefore, it is determinative for the QRS detector to avoid the noise interference and correctly detect QRS complexes even when the ECG signal varies with respect to time. Also, the chances of getting human error are high if the ECG is monitored visually. It is a complicated task and it increases chances of losing important clinical related information. Therefore, lot of efforts has been made to avoid this problem by developing various analog and digitized systems for ECG analysis. Digitized systems have proved to be more efficient as compared to analog systems. This makes it possible to retrieve information rapidly for the storage of important data and techniques to present that data, which is prominent for clinical usage. Many approaches used or proposed in the past have been complicated and have used a great deal of time. Real time approaches on the other hand, can be used to monitor the R wave complexes and in determining the correct heart rate.

2. Wavelet Transform

A representation of functions with respect to wavelets is known as a wavelet transform. A continuous time signal is distributed into different scale components using a mathematical function called wavelet. The "*mother wavelet*" is a fixed length waveform which is scaled and thus translated into "*daughter wavelets*". Wavelet transforms represents functions with discontinuities, sharp peaks, and it exhibits accuracy in the reconstruction of signals which are non-stationary, non-periodic, and finite in nature. Thus, it is advantageous over Fourier Transforms in such cases.

The discrete wavelet transform represent a digital signal with respect to time using various filtering techniques. Various cutoff frequencies as multiple scales are used to analyze the signal. Filters perform the functions in processing the signal. Scaling the filters in iterations produces wavelets. Scales are determined using the up and down sample method. The use of filter provides the information in the signal. Therefore, this uses the low and high pass filters over a digitized input signal.

Let us consider a discrete signal s, corresponding to the ECG, and mixed with EMG (Electromyogram artifacts) noise $n : x = s + n\,; x, s, n \in R^N$. The procedure of de-noising contains two steps which can be described as follows:

The signal-noise mixture $x = s + n$ decomposed in W_1 wavelet domain; the wavelet coefficients y_1 which is shrinked using wavelet domain filter H_{SH}; the estimate of the signal is calculated by inverse wavelet transform of the shrinked wavelet coefficients $\hat{y}_1$; and finally, the coefficients estimate $\hat{y}_{21}$ in W_2 wavelet domain is obtained:

$$\hat{y}_{21} = W_2 W_1^{-1} H_{SH} W_1 x \qquad (1)$$

The wavelet coefficients of the signal-noise mixture in W_2 domain y_2 and their estimate $\hat{y}_{21}$ obtained in (1) are used to design an optimal in MSE (*Mean Square Error*) sense Wiener Filter H_{WF}

$$H_{WF}(j,k) = \frac{\hat{y}_{21}(j,k)^2}{\hat{y}_{21}(j,k)^2 + \hat{\sigma}(k)^2}, \qquad (2)$$

Where j is the time position and k is scale position. The denoised signal is obtained by inverse wavelet transform of the filtered by H_{WF} wavelet coefficients $\hat{y}_2$:

$$\hat{s} = W_2^{-1} H_{WF} W_2 x. \qquad (3)$$

Here, H_{WF} is a diagonal matrix containing $H_{WF}(j,k)$ in the main diagonal; and H_{SH} is a diagonal matrix containing *time – frequency* dependent threshold.

3. Applications of Wiener Filter

Traditional noise reduction is based on standard filter processing, either by low – pass filter or high – pass filter. Wiener filter is noise filtering approach used in this paper. Wiener filter is a well-developed class of optimal filters which uses the signal and noise characteristics that are available. Winer filter theory is based on the minimization of the differences between the filtered output and the desired output. However, Gaussian white noise is used as a general noise source and added to the ECG signal.

3.1 Implementing a Wiener Filter in a Sine-wave

Let us numerically implement a Weiner filter to recover a sine-wave of the form $f(t) = \sin(2t)$. Here, we assumed that the frequency is 4 Hz, the function is sampled at 100 Hz, and the signal is corrupted by white Gaussian noise, $\sigma = 0.4$.

Figure 2 shows the original signal, noisy signal, and reconstructed signal for the case of white Gaussian noise.

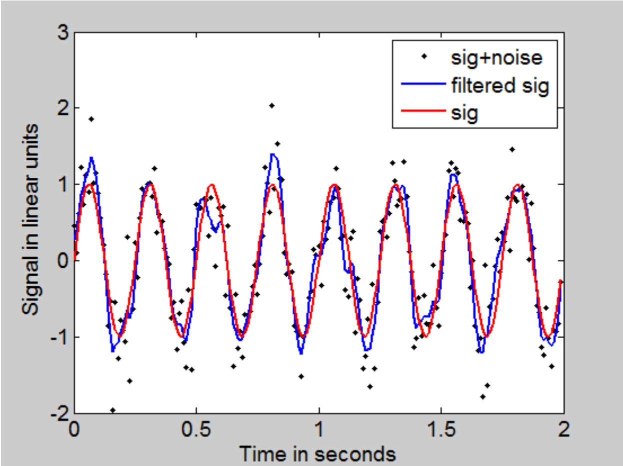

Figure 2. The graphical representation of the original signal, noisy signal, and reconstructed signal for the case of white Gaussian noise, $\sigma = 0.4$

As seen, the filter recovers the original signal fairly well. Thus, the code outputs the fractional variance left in the residual signal:

Fractional variance of residuals is (white): 0.0737

Fractional variance of residuals is (colored): 0.2485

However, the graphical and numerical results came out using the script in MATLAB shown in Appendix 1.

Furthermore, we re-implemented the Weiner reconstruction increasing the correlation length of the noise. Also, we used the colored noise and observed what happens to the reconstructed signal. The colored noise is represented by the following formulae:

$$w_1[n] = \frac{w[n-1] + w[n] + w[n+1]}{\sqrt{3}}.$$

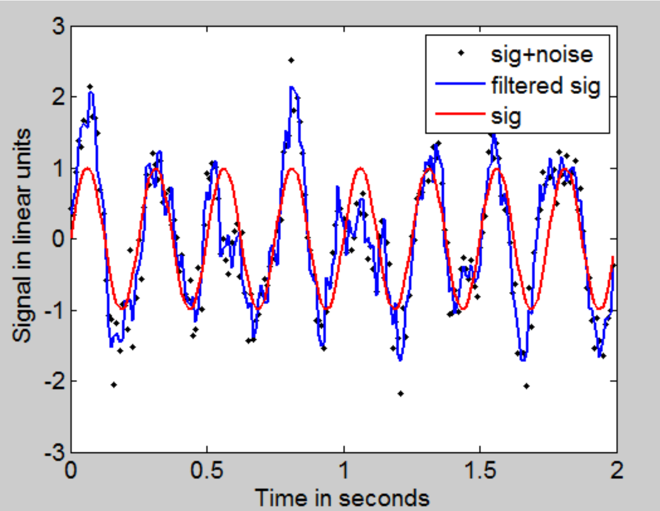

Figure 3. The graphical representation of the original signal, noisy signal, and reconstructed signal for the case of colored Gaussian noise

We can see that in case of white noise, the residual signal has a variance < 10% of the original signal showing that we have reconstructed the signal well in the presence of noise. In the case of the colored noise, we have a variance of ~ 25% in the residuals. This is because of the fact that as the noise correlation length increases, the difference in the covariance matrix of the signal and the noise reduces, and the filter is unable to separate one from the other. Thus, the reconstruction is clearly worse. If we increase the correlation length of the noise, then we would get worse reconstructions as expected.

3.2 Implication of Wiener Filter in ECG

In this paragraph, we will see the implication of Wiener filter in a real wave. ECG is a time-domain recording of a human heart beat (electrocardiogram). In the previous problem, we only knew the correlation matrix of the signal. Here, we know the signal itself, and hence, we will implement a matched filter. Using this filter, we will detect the heart beats in the cardiogram and calculate the heart rate.

Furthermore, we will assume that the noise is white and that each sample has noise with the same variance. Hence, the noise covariance matrix is diagonal with the same entry for each diagonal element. Consequently, the matched filter is proportional to the signal itself. To find the positions of the heart beats, we can use a matched filter that is simply given by a copy of the signal. Note that if the noise is not white, and/or if different samples have different noise levels, the matched filter will not be proportional to a copy of the signal itself. Hence, it will depend on the noise covariance matrix.

The matched filtered output is shown in the Figure 4. We do not scale the ***y*** axis to the right values, since it is inconsequential in determining the heart rate. (Note: we will not plot the observed ECG signal here)

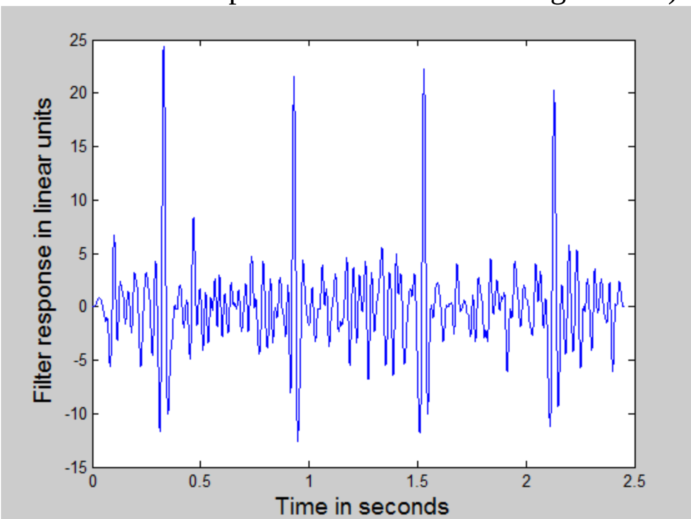

Figure 4. Wiener filter in the ECG signal

The beats are detected at timed 0.3288, 0.93, 1.529, and 2.129. The mean difference between them is 0.6 seconds. This corresponds to a heart rate of ~ 100 beats per minute, which is within the normal heart rate range from 60 – 100 beats per minute. The numerical and graphical results are done using the script shown in Appendix 2 (code 2).

4. Conclusion

It is well known that modern clinical systems require the storage, processing, and transmission of large quantities of ECG signals. ECG signals are collected both over long periods of time and at high resolution. This creates substantial volumes of data for storage and transmission. Data compression seeks to reduce the number of bits of information required to store or transmit digitized ECG signals without significant loss in signal quality.

The wavelet decomposition splits the analyzing signal into average and detail coefficients using finite impulse response digital filters. In this paper, we implemented a Wiener filter to the ECG signal to detect the heart beats and determine the correct heart rate. As discussed in the introductory part of this

study, it is a hard task which provides a cleaner image as the consequence of these transformations.
Wavelet technique is the obvious choice for ECG signal compression because it is localized, and has a non-stationary property of the wavelets to see through signals at different resolutions.

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Appendix

1)

```
function Code3()
close all; clear all; clc
dt = 1/100; % Sampling rate
t = [0:dt:(200-1)*dt]'; % Time vector
sig = sin(2*pi*4*t); % Signal
noise = 0.4*randn(size(sig)); % White noise
signoise = sig+noise; % sig+ noise
% Construct the signal covariance matrix
sig2 = xcorr(sig,sig);
Csig = zeros(200,200);
for a=1:200
   for b = 1:200
      Csig(a,b) = sig2(200-(a-b));
   end
end
Cnoise = (0.4^2)*eye(200); % Noise covariance for white noise
F = inv(Csig+Cnoise)*Csig; % Weiner filter for white noise case
figure(1);
plot(t,signoise,'k.'); hold on;
plot(t,F*signoise,'b','linewidth',2);
plot(t,sig,'r','linewidth',2);
hold off
xlabel('Time in seconds','fontsize',14);
```

```
ylabel('Signal in linear units','fontsize',14);
set(gca,'fontsize',14); legend('sig+noise','filtered sig','sig');
print('no_corr.png','-dpng','-r200');
%
Ncor = 3; % Correlation length = 2*Ncor+1
noise1 = noise; % new colored noise vector (initialize memory)
Cnoise1 = Cnoise; % New colored noise covariance matrix (initialize memory)
% Construct the colored noise and its covariance matrix
for icor = -Ncor:Ncor
   if(icor~=0)
      noise1 = noise1+circshift(noise,icor)/sqrt(2*Ncor+1);
      Cnoise1 = Cnoise1+diag(0.4^2*(Ncor+1)/(2*Ncor+1).*ones(200-abs(icor),1),icor);
      if(icor>0)
         Cnoise1 = Cnoise1+diag(0.4^2*(Ncor+1)/(2*Ncor+1).*ones(abs(icor),1),200-icor);
      else
         Cnoise1 = Cnoise1+diag(0.4^2*(Ncor+1)/(2*Ncor+1).*ones(abs(icor),1),-200-icor);
      end
   end
end
signoise1 = sig+noise1; % Sig + colored noise
F1 = inv(Csig+Cnoise1)*Csig; % New weiner filter
figure(2); % plot results for colored noise case
plot(t,signoise1,'k.'); hold on;
plot(t,F1*signoise1,'b','linewidth',2);
plot(t,sig,'r','linewidth',2);
hold off
xlabel('Time in seconds','fontsize',14);
ylabel('Signal in linear units','fontsize',14);
set(gca,'fontsize',14); legend('sig+noise','filtered sig','sig');
print(sprintf('corr_%d.png',Ncor),'-dpng','-r200');
end
```

2) Code 2

```
function Code2()
close all;
clear all;
clc
l=load ('heart_beat.mat');
npts_filt = length(l.beat_profile); % Num of points in the matched filter
npts_sig = length(l.ecg); % Num of points in the ECG
dt = 1/l.fs; % Sampling interval
time = 0:dt:(npts_sig+npts_filt-1)*dt;
filt_sig = xcorr(l.beat_profile,l.ecg);
filt_sig = filt_sig(1:npts_sig+npts_filt);
figure(1);
plot(time, filt_sig);
xlabel('Time in seconds','fontsize',14);
ylabel('Filter response in linear units','fontsize',14);
end
```