FRACTAL CORRELATIONS ON CONTENT AND COGNITIVE DOMAINS AND MATHEMATICS PERFORMANCE ACROSS COUNTRIES

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Abstract

Fractal correlations on content and cognitive domains and mathematics performance based on TIMSS 2011 across countries was the focus of this study. Fractal dimensions as reflected by the fractograms and correlations of the variables for each factor were analyzed. Results showed that Mathematics performance is highly influenced by content and cognitive domains if taken jointly. But if variables for each factor are analyzed independently, cognitive domains such as Knowing, Applying, and Reasoning have high positive relationship to Mathematics performance. Content domains like Algebra, Geometry and Number Sense have also high positive relationship except Data and Chances. The findings revealed that Content and Cognitive domains have significant bearings in Math performance of the participating countries.

Keywords: Fractal correlations analysis, fractogram, TIMSS mathematics scores, content domain, cognitive domain

Introduction

The Trends in International Mathematics and Science Study (TIMSS 2011) is an international study directed by the International Association for the Evaluation of Educational Achievement (IEA) as cited by Thomson, Hillman and Wernert (2012). It summarized the fourth and eighth grade students' math achievement of the 63 countries and 14 benchmarking countries (Mullis, 2012). The goal of TIMSS is to provide comparative information about educational achievement across countries to improve teaching and learning in mathematics and science.

TIMSS (2011) results present the content and cognitive domains across participating counties. The content domains described the content that is intended to be assessed; while the cognitive domain described the cognitive abilities and behaviors as they engage with the mathematics content. The content domains were found in the curricula of the participating countries, and are the subject of the major international and national reports for TIMSS. Grade eight students are assessed in *number, algebra, geometry and data and chance*. TIMSS 2011 Mathematics Framework identified the three (3) cognitive domains which are; *knowing* which covers the facts, procedures and concepts students need to know; *applying* which focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions; and *reasoning* – which goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts and multi-step problems.

At the eighth grade, students should have developed number sense and computational fluency, understand the meanings of operations and how they relate to one another, and be able to use numbers and operations to solve problems. It includes whole numbers, fractions and decimals, integers, ratio and proportion and percent, number patterns and relationships (TIMSS, 2011).

As noted in TIMSS 2011, in algebra, functional relationships and their uses for modeling and problem solving are of prime interest. The algebra content domain includes recognizing and extending patterns, using algebraic symbols to represent mathematical situations, and developing fluency in producing equivalent expressions and solving linear equations. The major topic areas in algebra are: patterns, algebraic expressions and equations/formulas and functions.

Geometric shapes, geometric measurement and location and movement are contained in the content domain in geometry. As presented in TIMSS 2011 Mathematics Framework, this also includes understanding coordinate representations and using spatial visualization skills to move between two- and three-dimensional shapes and their representations. Students are expected to use symmetry and apply transformation to analyze mathematical situations.

The cognitive range extends from making drawings and constructions to mathematical reasoning about combinations of shapes and transformations. Students describe, visualize, draw, and construct a variety of geometric figures, including angles, lines, triangles, quadrilaterals, and other polygons. Students are asked to combine, decompose, and analyze compound shapes.

The data and chance content domain includes data organization and representation, data interpretation and chance. Students describe and compare characteristics of data (shape, spread, and central tendency), and draw conclusions based on data displays. They have to identify trends in data, make predictions based on data, and evaluate the reasonableness of interpretations. Eighth-grade students' appreciation of chance (elementary probability) also includes the occurrence of familiar events as certain; as having greater, equal, or less likelihood; or as impossible, and should extend to using data from experiments or knowledge of equally likely outcomes to predict the chance of a given outcome.

Problem solving is central to the applying domain, but the problem settings are more routine than those aligned with the reasoning domain, being rooted firmly in the implemented curriculum. The routine problems will typically have been standard in classroom exercises designed to provide practice in particular methods or techniques. Though they range in difficulty in TIMSS 2011, each of these types of "textbook" problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned facts, concepts, and procedures.

According to Thomson (2006) developing reliable and valid achievement scales for the cognitive domains is not as straightforward, and differences among students across and within countries in their mathematical knowledge and problem solving skills make it difficult to know which cognitive abilities students are using to solve a particular mathematical or scientific problem. Thus, this study attempts to examine the findings about the TIMSS 2011 math cognitive domains, content domains and math performance across countries via fractal correlations.

Fractals are the latest development in statistics. An appreciation of the properties of fractals is changing the most basic ways to analyze and interpret date from experiments and is leading to new insights into understanding the cognitive and content domains and the math performance of the grade eight (8) students across countries.

Related Literature

Using mathematics or reasoning about mathematical situations depends to a large extent on the students' mathematical knowledge and familiarity with mathematical concepts. The greater the student's automatic recall of facts, procedures and concepts, the greater the ability to make extensions beyond their existing knowledge and to apply their skills to more complex problems The "Mathematics Report" provides a rich array of information which describes the educational contexts for mathematics, including home environment support, students' backgrounds and attitudes toward mathematics, the mathematics curriculum, teachers' education and training, classroom characteristics and activities, and school contexts for mathematics learning and instruction. The TIMSS 2003 Assessment

Frameworks (Mullis et al, 2003) as cited by Thomson (2006) elaborates this cognitive domain which highlights that TIMSS assesses a range of problem-solving situations within mathematics, with about two-thirds of the items requiring students to use applying and reasoning skills. The cognitive domains are the same for both grades, but with a shift in emphasis. Compared to the fourth grade, the eighth grade has less emphasis on the knowing domain and greater emphasis on the reasoning domain

The space-time fluctuation pattern in dynamical systems was shown to have a self-similar or fractal structure in 1970s (Mandelbrot, 1975). The larger scale fluctuation consists of smaller scale fluctuations identical in shape to the larger scale. Fractal systems extended over many scales and so cannot be characterized by a single characteristic average number (Liebovitch and Scheurle, 2000). Further, the self-similar fluctuations imply long-range space-time correlations or interdependence. Therefore, the Gaussian distribution will not be applicable for description of fractal data sets. However, the bell curve still continues to be used for proximate quantitative characterization of data which are now identified as fractal space-time fluctuations.

Method

Fractal correlation, which is nonconventional statistical method, was used to analyse the scores in TIMSS 2011. This method allows us to see how the ruggedness (λ_1) of a certain variable dictates the ruggedness (λ_2) of another variable. In this particular study, two variables in TIMSS scores that need to be analysed in terms of fractal dimension (λ_i) were identified at a time. The data sets (scores) in the two identified variables were labelled as *x* and *y*, and the following steps were then followed: **(1)** plot *diagram 1* out of the given data sets (*x* and *y*); **(2)** plot *diagrams 2* and *3* by setting $x = 0$ and $y = 0$ respectively; **(3)** combine the three diagrams resulting to *diagram 4*; **(4)** cluster the dots in *diagram 4* as reflected in *diagrams 2* and *3* by making use of horizontal and vertical lines respectively; **(5)** use the software ***FrakOut!*** to determine the fractal dimensions (λ, λ_y, and λ_x) of *diagram 4* as reflected in *diagrams 1, 2,* and *3*; and **(6)** determine the fractal correlation ($P_{\lambda y \lambda y}$) by utilizing the fractal dimensions generated in step (5). To determine the fractal correlation, the following formula was used:

$$(P_{\lambda y \lambda y}) = \quad (\lambda - 1) \exp\{½ \log_e(\lambda x + \lambda y)\}$$

Results and Discussion

Fractal dimensions (λ) describe how fractal objects fill the spaces they occupy. Fractal analysis refers to a statistical analysis of fractal observations based on graphical presentation of all factors namely; math performance, cognitive domains and content domains. Fractal dimensions are determined and correlations for the variables for each factor are analyzed. The result of correlations of the factors mentioned is presented in table 1 and the fractogram is shown in figure 1.

Table 1
Fractal Correlations on Math Performance, Content and Cognitive Domains

Variable	Fractal Dimension	Fractal Correlation
1. Cognitive Dimensions = (x)	$\lambda x = 0.970$	
2. Math Performance = (y)	$\lambda y = 1.02$	$P\lambda x\lambda y = 1.000$
3. Cognitive Dimensions and Math Performance	$\lambda = 1.00$	
1. Content Dimensions = (x)	$\lambda x = 0.950$	
2. Math Performance = (y)	$\lambda y = 0.973$	$P\lambda x\lambda y = 1.000$
3. Content Dimensions and Math Performance	$\lambda = 1.00$	
1. Cognitive Dimensions = (x)	$\lambda x = 0.982$	
2. Content Dimensions = (y)	$\lambda y = 0.954$	$P\lambda x\lambda y = 1.000$
3. Cognitive and Content Dimensions	$\lambda = 1.000$	

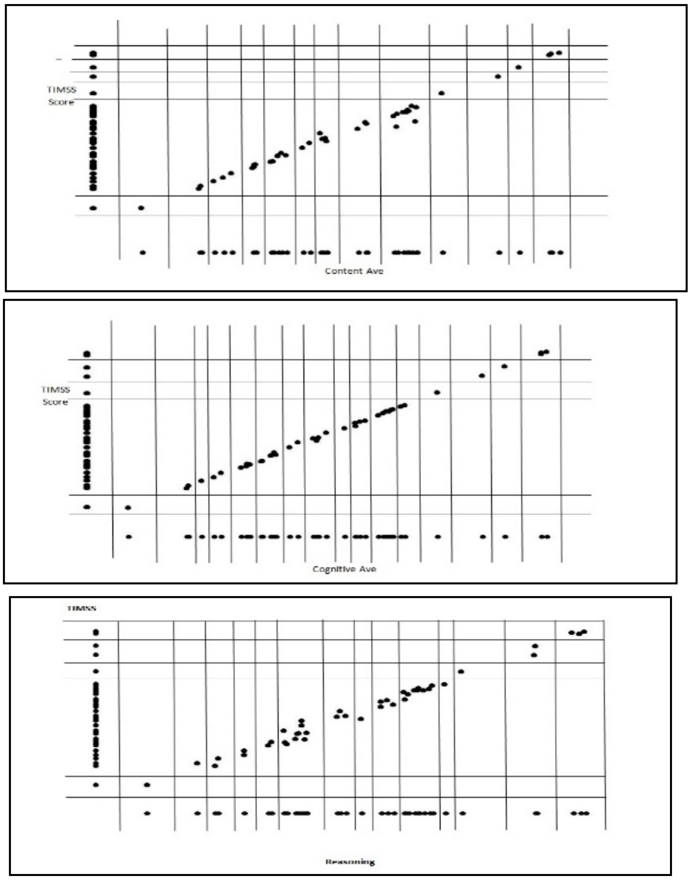

Figure 1. Fractogram of Mathematics Performance and Cognitive and Content Domains

Fractal correlations of math performance based on TIMSS 2011 and the cognitive and content domains is equal to 1. In the fractogram it tends to fall a straight line which suggests a very high correlation or strong positive relationship. This means that factors in cognitive domains are absolutely dictating the behavior of the factors in the content domains in terms of students' Mathematics performance.

Table 2 Fractal Correlations on Math Performance and Cognitive Domains

COGNITIVE DOMAINS		
1.Knowing = (x) 2.Math Performance = (y) 3.Knowing and Math Performance	$\lambda x = 0.948$ $\lambda y = 1.07$ $\lambda = 1.07$	$P\lambda x\lambda y = 0.606841$
1.Applying = (x) 2.Math Performance = (y) 3.Applying and Math Performance	$\lambda x = 0.894$ $\lambda y = 0.890$ $\lambda = 0.970$	$P\lambda x\lambda y = 1.000$
1.Reasoning = (x) 2.Math Performance = (y) 3.Reasoning and Math Performance	$\lambda x = 0.988$ $\lambda y = 0.961$ $\lambda = 1.06$	$P\lambda x\lambda y = 0.608871$

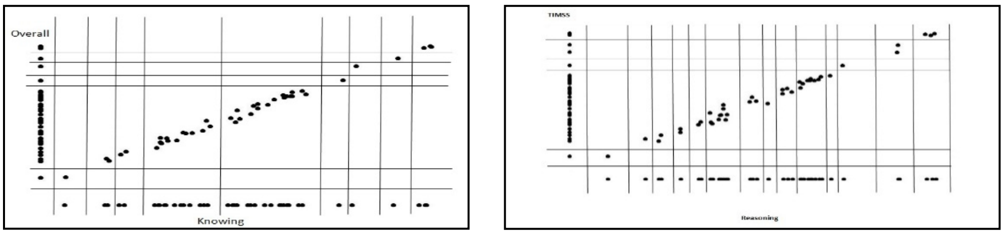

Figure 2. Fractogram of Mathematics Performance and Cognitive Domains

Fractograms show that the data tend to cluster and form a line but with few outliers. Consequently, fractal correlations of math performance based on TIMSS 2011 and cognitive domains namely Knowing, Applying and Reasoning are 0.61, 1.0 and 0.61 respectively which suggest a strong to very strong relationship. Among the factors in the cognitive domain, "Applying" is influencing more the students' performance than the other factors. This means that the students' skill and ability to apply knowledge and conceptual understanding to solve problems or answer questions is more developed and established.

Table 3 Fractal Correlations on Math Performance and Content Domains

CONTENT DOMAINS		
1.Number Sense = (x) 2.Math Performance = (y) 3.Number Sense and Math Performance	λx = 0.972 λy = 0.998 λ = 1.15	Pλxλy = 0.474369
1.Algebra = (x) 2.Math Performance = (y) 3.Algebra and Math Performance	λx = 0.951 λy = 0.970 λ = 1.12	Pλxλy = 0.499477
1.Geometry (Ave) = (x) 2.Math Performance = (y) 3.Geometry and Math Performance	λx = 0.900 λy = 1.02 λ = 1.1	Pλxλy = 0.449935
4.Data and Chance = (x) 5.Math Performance = (y) 4.Data and Chance and Math Performance	λx = 0.998 λy = 1.000 λ = 1.24	Pλxλy = 0.389750

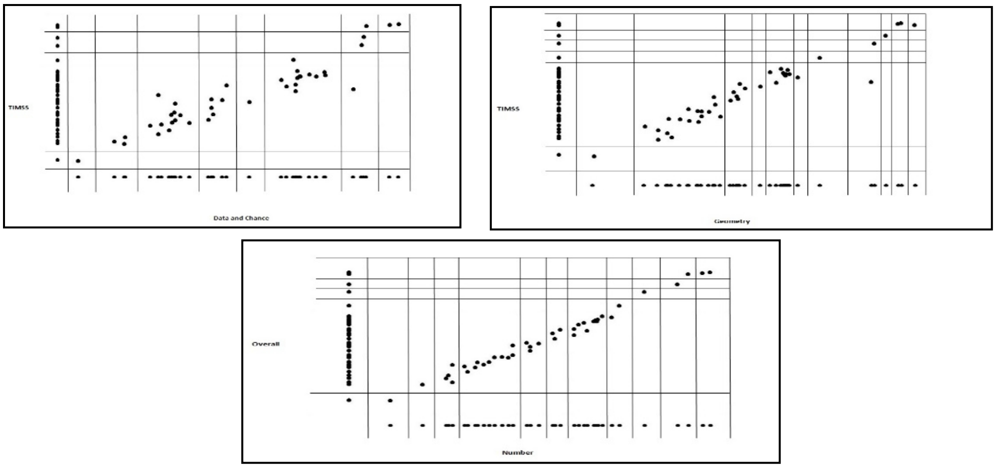

Figure 3. Fractogram of Mathematics Performance and Content Domains

Table 3 and Figure 3 shows fractal correlations of math performance and content domains namely Number sense, Algebra, Geometry and Data and Chance are 0.47, 0.50, 0.45 and 0.39 respectively. The correlations suggest a moderate to strong relationship of these variables. It implies that the factors in content domain influence moderately the students' performance. Based on the figures, Number sense is dictating the students' performance by 47%, Algebra by 50%, Geometry by 45% and Data and Sense by 39%.

The research conducted to correlate Grade eight students' Math performance based on TIMSS 2011 and the cognitive and content domains show that their performance is highly influenced by the cognitive and content domains. But when the variables for each domain were taken independently, a moderate to high positive relationship is observed.

Among the variables in the cognitive domains, *applying* greatly influences the students' math performance which could be attributed to the teacher's emphasis on the application of the concept to real life situations. *Knowing* or *reasoning* about mathematical situations depends on a large extent on the student's mathematical knowledge and familiarity with mathematical concepts. The greater the student's automatic recall of facts, procedures and concepts, the greater the ability to make extensions beyond their existing knowledge and to apply their skills to more complex problems.

Reasoning mathematically involves the capacity for logical, systematic thinking (Thomson, 2006). It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems which are unfamiliar to students. This makes cognitive demands on them over and above even when the skills needed have been learned. Problems may be mathematical or set in real-life situations, and both involve transfer of knowledge and skills to new situations.

The variables of the content domain when taken singly moderately influence the students' math performance. The cognitive domains on *knowing* and *reasoning* support the mathematical knowledge on the contents of *number sense*, *algebra*, *geometry* and *data and chances*. Among the content domains, the content on *data and chances* has the least influence compared to *geometry*, *number sense* and *algebra*. The results may be attributed to the importance of good teaching inputs into the educational system as cited by Pogoy and Montalbo (2010). Mathematics teacher preparation and qualification improve the mathematics skills and performance of the students.

Conclusion and Recommendations

Students' math performance based on TIMSS 2011across forty-two countries is highly influenced by the cognitive and content domains which could be attributed to mathematics teacher's preparation and content knowledge. Thus, teacher quality in the educational system needs to establish excellent pre-service and in-service programs for mathematics teachers.

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