QUASI-CLASSICAL MODEL FOR ONE-DIMENSIONAL POTENTIAL WELL

Guangye Chen, (Associate Professor)

Shanghai Jiaotong University, Shanghai, China

Abstract

Particle in one dimensional potential well demonstrates typical quantum mechanical behaviors determined by the related Schrodinger equation. The problem is solved here with a quasi-classical particle model that qualifies for quantum wave description. Our results cover majority of previous conclusions and reveal new mechanisms that, closure of the particle's trajectory leads to quantization and lateral projections of particle's three dimensional trajectories generate the Schrodinger wave functions for different energies. We explain that, Schrodinger wave functions represent different modes of particle's motion and their amplitudes might measure the probability density that the particle is to be observed, but their interference invalids the probabilistic implication.

Keywords : Potential well, Quantization, Schrodinger wave, Probability interpretation

Introduction

Quantum mechanics is far not as successful in interpretation of fundamental concepts as in prediction of some properties of atomic systems. Due to the essence of its study object is in a cloud of wave-particle duality, concomitant are other weird properties and unsolved puzzles for microscopic objects. Hence, even for a simple quantum problem, any new approach should be valuable for promoting the final resolution of all conceptual perplexities.

Particle in one-dimensional potential well is the simplest model that demonstrates main quantum mechanical behaviors among which quantization of the dynamic quantities is the most significant. Previously, several renowned physicists dealt with the problem through solving the corresponding Schrodinger equation and gave different probability distributions for the particle's momentum. Einstein [1] and Pauli [2] presented a discrete distribution, but Landau [3] obtained a continuous distribution.

In this paper, the problem is solved by using a particle model that qualifies for quantum wave description. The particle model[4] is briefly introduced and then applied, yielding results that cover majority of previous conclusions. Our results reveal that, quantization can be realized from the condition of closure of the particle's trajectory, and the Schrodinger wave functions for different energies can be generated from lateral projections of the three dimensional trajectories, which represent various modes of particle's motion. The probabilistic implication of the Schrodinger wave function is explored.

The Quasi-classical Particle Model

It is known that appearance of the Planck's constant marks the beginning of investigating the behaviors of microscopic objects. That this constant has a dimension of angular momentum should be a cue to what the motion of microscopic object is different from macroscopic object. Indeed, a mass point that is conjectured to possess intrinsic circular motion with its angular momentums being measured by the Planck's constant was proved to be describable with quantum wave. In this quasi-classical model, the free particle is assumed to have a spiral trajectory defined by the following parameter equations [4]

$$\left.\begin{aligned} x(t) &= ut \\ y(t) &= r\cos(\omega t) \\ z(t) &= r\sin(\omega t) \end{aligned}\right\} \qquad (1)$$

and its angular momentums are constrained by

$$m\omega r^2 = s\hbar \qquad (2)$$

$$mur = \sqrt{s}\hbar \qquad (3)$$

where m is the mass, t the time, u the velocity component along the x-axis (rectilinear velocity), r the radius of trajectory, ω the circular frequency, s the spin constant, and $\hbar$ the Planck's constant.

The above assumptions lead to the following relations

$$\hbar\omega = mu^2 \qquad (4)$$

$$L = \sqrt{s(s+1)}\,\hbar \qquad (5)$$

$$E_k = \hbar\omega(s+1)/2 \qquad (6)$$

$$\lambda = 2\pi\hbar/(mu) \qquad (7)$$

where L is the total angular momentum, E_k the total kinetic energy, and λ the pitch of the spiral trajectory.

If the motion of the hypothesized particle is described with quantum wave, □□n (7) is exactly the wavelength. For convenience, we assign the wave number as

$$k_x = 2\pi / \lambda = mu / \hbar \qquad (8)$$

Application for One-dimensional Potential Well

The one-dimensional potential well with the selected coordinate is shown as in Fig.1. For the potential in the well is zero, the particle will behave like a free particle there.

To tackle the problem with the new particle model, we define the stationary state of the particle as that the particle's trajectory is closed. Gratifyingly, this definition is almost a copy from the stationary state of a classical phenomenon such as the periodical motion of a planet in its orbit. In addition, the two walls of the well are supposed to be ideal. That is, when the particle collides with either of the walls, the magnitudes of its two velocity components keep invariant but the direction of the velocity of *x*-component (the rectilinear velocity) is reversed.

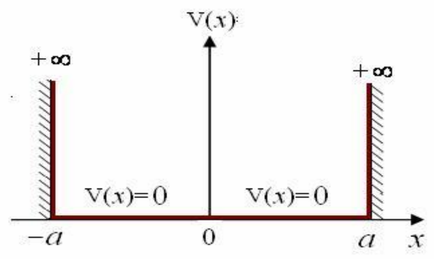

Fig.1. One-dimensional potential well

In order to form a closed trajectory with the wave number given by (8), the particle must have a phase shift of integral multiples of 2π for a round trip over the well. This condition is given by

$$4k_x a = 2n\pi \qquad n = 1, 2, \cdots \qquad (9)$$

Substitution the wave number of (8) into (7) gives the quantized momentum as

$$p_n = \frac{n\pi\hbar}{2a} \qquad n = 1, 2, \cdots \qquad (10)$$

Then, putting (10) into (6) making use of (4) yields

$$E_{k,n} = (s+1)\frac{n^2\pi^2\hbar^2}{8ma^2} \qquad n = 1, 2, \cdots \qquad (11)$$

And from (3) and (10), the radius of trajectory is found to be

$$r_n = \frac{2\sqrt{s}a}{n\pi} \qquad n = 1,2,\cdots \qquad (12)$$

Other quantities like circular frequency, circular velocity component et al. are also quantized but omitted for conciseness.

To illustrate the results intuitively, we take the particle's mass as of an electron, *s* as 1, and the well's size *a* as $10\times10^{-10}\,m$. The particle's trajectories are drawn for n=1, 5 and 20, as shown in Fig.2 and Fig.3 in a different view angle. Particularly, the projections of these trajectories on the *x*-*z* plane are presented as in Fig. 4, which are sine curves for *n* even and cosine curves for *n* odd, exactly the un-normalized wave functions for the corresponding energies obtained from solution of the Schrodinger equation.

	n=1	n=5	n=20
E(eV)	0.19	4.7	57.2
v(m/s)	2.57×10^5	1.29×10^6	5.14×10^6
ω(rad/s)	2.86×10^{14}	7.14×10^{15}	1.14×10^{17}

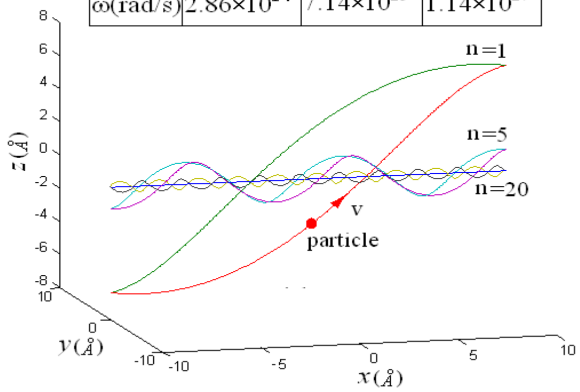

Fig.2. Particle's trajectories for energy levels as n=1, 5 and 20 respectively

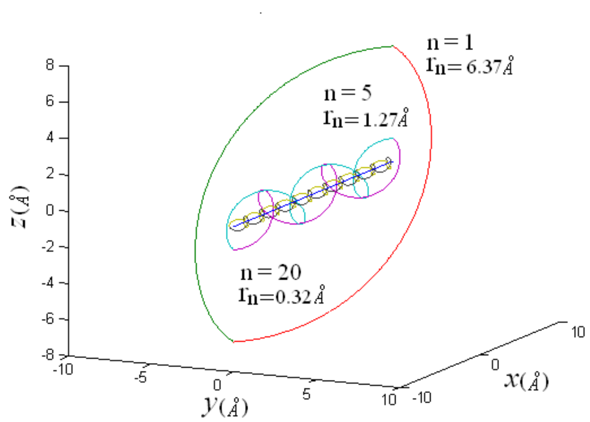

Fig.3. Radii of particle's trajectories for energy levels as n=1, 5 and 20 respectively

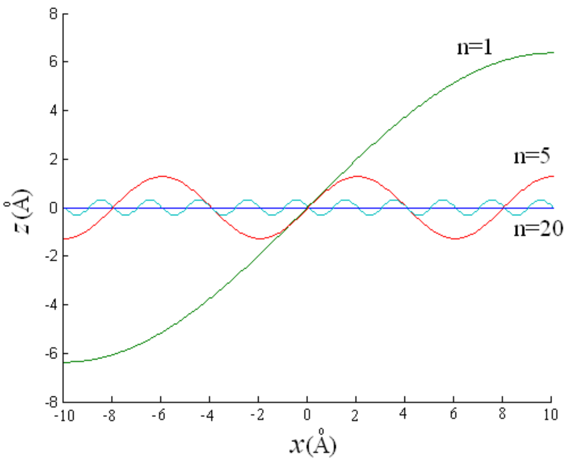

Fig.4. Projections of the one-way trajectories on *x-z* plane, which are identical to the unnormalized wave functions from the solution of the corresponding Schrodinger equation [7].

Discussions

The total kinetic energy given by (11) consists of two terms. One proportional to spin constant *s* is resulted from the velocity component of circular motion, and the other term from the rectilinear velocity is identical to that given by the solution of the corresponding Schrodinger equation [3]. It is particularly noted that lateral projections of the particle's trajectories on *x-z* plane correspond to the unnormalized wave functions of the solution of the Schrodinger equation, as shown in Fig.4. This explains that the wave functions associated with different energies represent various modes of the particle's motion.

By orthodox interpretation of wave function, square of these cosine or sine functions should give standing wave like distributions of probability density to find the particle. But one serious problem exits for this interpretation. Since, the probability in the vicinities containing zero points of the wave function are approximately zero, which means the particle is almost not allowed to pass through these narrow regions. This problem is formally bypassed by the complementarity principle, which forbids the idea of real existence of a particle before it is actually observed. Apparently, the complementarity principle is more like a philosophical argument rather than a scientific statement, for it does not give any reason how the wave collapses to make the particle appear at a given spatial point. Nothing is more absurd as the unreality property implied by the orthodox interpretation as considering the situation that, when one applies the mass and the electric charge for an electron in the Schrodinger equation, the electron even cannot be considered as an object really existing. Could wave be massive and charged? Unbelievably, as the strongest and simplest evidence for the probability wave the standing-wave-like probability distribution for the particle in the well has never been confirmed experimentally.

As shown in Fig.3, the particle with lower energy has bigger radius of trajectory and thereby relative greater amplitude of wave function due to the projection relation. On the other hand, by the equations (10) and (12), the radius of the trajectory is inversely proportional to the rectilinear velocity, so is the amplitude of wave function. Therefore, greater wave amplitude means that the particle stays longer in a given interval of its one-dimensional path along *x-axis*. This relation may explain the probability implication of the wave amplitude. Consider potential in the well be a function of *x-coordinate*. In this case, the particle in definite motion state will have all its dynamic quantities varying with the *x-coordinate*. As the varying potential field can be approached approximately by a step function that the potential is constant in each sector, the particle's trajectory will be about the concatenation of a series of spiral curves of different radii. Accordingly, in the sector where the radius is greater, the particle will have relative bigger amplitude of wave function, and thereby lower rectilinear velocity or longer staying time, implying a relative bigger probability to be observed. This explains that the wave amplitude, not the absolute square of wave function, might measure the probability density. By this interpretation, interference of Schrodinger waves invalids the probabilistic implication and causes the zero probability puzzle in a standing wave like probability distribution.

As the outcome of observation is concerned, dynamic quantities of an object regardless macroscopic or microscopic, in deterministic motion or in random motion, are qualified for probability descriptions provided the object is in a stationary state of motion. For the particle under our consideration, obviously, the probability density of finding the particle is a uniform distribution over the well; and the probability density for the momentum is a discrete distribution that gives probability 1/2 for each of the two opposite momentums, which is exactly what Einstein and Pauli gave. The continuous momentum distribution obtained by Landau is apparently resulted from the effect of truncation of the triangular wave functions by the well's width. So, this difference makes no significance.

It is noted from the tablet in Fig.2, even an electron has the energy as low as 0.2eV, its rotation frequency gets about 10^{14} Hz, in orders higher than the highest working frequency of contemporary electronic devices; and its trajectory with a radius of several angstroms is generated with an extreme high velocity. To observe the detailed motion of an electron must be extremely difficult. Therefore, a single electron or other microscopic object in similarly quick motion state will continuously be mysterious for quite long period before the measuring technique is equal to the task. Hence, those claims that microscopic object was observed appearing simultaneously at two different spatial places are unconvincing, for no one could make a strictly instantaneous measurement with an instrument of finite temporal resolution.

Conclusion

Quantization as the typical characteristics of microscopic object can be brought by a quasi-classical particle description provided the particle's trajectory is required to be closed. The eigen Schrodinger wave functions represent various modes of particle's motion and their amplitudes might measure the probability density that the particle is to be observed, but their interference makes nonsense.

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