BARYON ASYMMETRY IN NEUTRINO MASS MODELS WITH AND WITHOUT θ_{13}

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Abstract

We investigate the comparative studies of cosmological baryon asymmetry in different neutrino mass models with and without θ_{13} by considering the three diagonal form of Dirac neutrino mass matrices: down quark (4,2), up-quark (8,4) and charged lepton (6,2). The predictions of any models with θ_{13} are consistent in all the three stages of leptogenesis calculations and the results are better than the predictions of any models without θ_{13} which are consistent in a piecemeal manner with the observational data. For the best model NH-IA (6,2) without θ_{13}, the predicted inflaton mass required to produce the observed baryon asymmetry is found to be $M_{\phi}\sim 3.6\times 10^{10}$ GeV corresponding to reheating temperature $T_R\sim 4.5\times 10^{6}$ GeV, while for the same model with θ_{13}: $M_{\phi}\sim 2.24\times 10^{11}$ GeV, $T_R\sim 4.865\times 10^{6}$ GeV and weak scale gravitino mass $m_{2/3}\sim 100$ GeV without causing the gravitino problem. These values apply to the recent discovery of Higgs boson of mass ~ 125 GeV. The relic abundance of gravitino is proportional to the reheating temperature of the thermal bath. One can have the right order of relic dark matter abundance only if the reheating temperature is bounded to below 10^{7}GeV.
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Introduction

Recent measurement of a moderately large value of the third mixing angle $\theta_{13}$by reactor neutrino oscillation experiments around the world particularly by Daya Bay($sin^2\theta_{13} = 0.089 \pm 0.010$ (stat) $\pm$ 0.005 (syst))[1] and RENO ($sin^2\theta_{13} = 0.113 \pm 0.013$ (stat) ± 0.019 (syst))[2], signifies an important breakthrough in establishing the standard three flavor oscillation picture of neutrinos. Thereby, will address the issues of the recent indication of non-maximal 2-3 mixing by MINOS accelerator experiment [3] leading to determining the correct octant of θ_{23}

and neutrino mass hierarchy. Furthermore, now, this has opened the door to search CP violation in the lepton sector, which in turn has profound implications for our understanding of the matter-antimatter asymmetry of the Universe. In fact, ascertaining the origin of the cosmological baryon asymmetry, $\eta_B = (6.5^{+0.4}_{-0.5}) \times 10^{-10}$ [4], is one of the burning open issues in both particle physics as well as in cosmology. The asymmetry must have been generated during the evolution of the Universe. However, it is possible to dynamically generate such asymmetry if three conditions, i) the existence of baryon number violating interactions, ii) C and CP violations and iii) the deviation from thermal equilibrium, are satisfied [5]. There are different mechanisms of baryogenesis, but leptogenesis [6] is attractive because of its simplicity and the connection to neutrino physics. Establishing a connection between the low energy neutrino mixing parameters and high energy leptogenesis parameters has received much attention in recent years in Refs. [6,7,8]. In leptogenesis, the first condition is satisfied by the Majorana nature of heavy neutrinos and the sphaleron effect in the standard model (SM) at the high temperature [8], while the second condition is provided by their CP-violating decay. The deviation from thermal equilibrium is provided by the expansion of the Universe. Needless to say the departures from thermal equilibrium have been very important-without them, the past history of the Universe would be irrelevant, as the present state would be merely that of a system at 2.75 K, very uninteresting indeed [9]! One of the key to understanding the thermal history of the Universe is the estimation of cosmological baryon asymmetry from different neutrino mass models with the inclusion of the latest non-zero θ_{13}.

Broadly the leptogenesis can be grouped into two: thermal with and without flavour effects and non-thermal. The simplest scenario, namely the standard thermal leptogenesis, requires nothing but the thermal excitation of heavy Majorana neutrinos which generate tiny neutrino masses via the seesaw mechanism [10] and provides several implications for the light neutrino mass spectrum [11]. And with heavy hierarchical right-handed neutrino spectrum, the CP-asymmetry and the mass of the lightest right-handed Majorana neutrino are correlated. In order to have the correct order of light neutrino mass-squared differences, there is a lower bound on the mass of the right-handed neutrino, $M_N \geq 10^9$ GeV [12], which in turn put constraints on reheating temperature after inflation to be $T_R \geq 10^9$ GeV. This will lead to an excessive gravitino production and conflicts with the observed data. In the post-inflation era, these gravitino are produced in a thermal bath due to annihilation or scattering processes of different standard particles. The relic abundance of gravitino is proportional to the reheating temperature of the thermal bath. One can have the right order of relic dark matter abundance only if the reheating temperature is bounded to below

10^7GeV [13].On the other hand, big-bang nucleosynthesis in SUSY theories also sets a severe constraint on the gravitino mass and the reheating temperature leading to the upper bound $T_R \leq 10^7$ GeV [14]. While thermal leptogenesis in SUSY SO(10) with high see-saw scale easily satisfies the lower bound, the tension with the gravitino constraint is manifest.

The analysis done in Ref. [15], the non-thermal leptogenesis scenario in the framework of a minimal supersymmetric *SO* (10) model with Type-I see-saw shows that the predicted inflaton mass needed to produce the observed baryon asymmetry of the universe is found to be $M_\phi \sim 5 \times 10^{11}$ GeV for the reheating temperature $T_R = 10^6$ GeV and weak scale gravitino mass $m_{3/2} \sim 100$ GeV without causing the gravitino problem. It also claims that even if these values are relaxed by one order of magnitude ($m_{3/2} \leq$ 10TeV, $T_R = 10^7$ GeV), the result is still valid. In Ref. [16] using the Closed-Time-Path approach, they performed a systematic leading order calculation of the relaxation rate of flavour correlations of left-handed Standard Model leptons; and for flavoured Leptogenesis in the Early Universe found the reheating temperature to be $T_R = 10^7$GeV to 10^{13} GeV. These values apply to the Standard Model with a Higgs-boson mass of 125 GeV [17].The recent discovery of a Standard Model (SM) like Higgs boson provides further support for leptogenesis mechanism, where the asymmetry is generated by out-of-equilibrium decays of our conjecture heavy sterile right-handed neutrinos into a Higgs boson and a lepton. Our work in this paper is consistent with the values given in Refs. [15, 16, 17].

Now, the theoretical framework supporting leptogenesis from low-energy phases has some other realistic testable predictions in view of non-zero θ_{13}. So the present paper is a modest attempt to compare the predictions of leptogenesis from low-energy CP-violating phases in different neutrino mass matrices with and without θ_{13}. The current investigation is of twofold. The first part deals with zero reactors mixing angle in different neutrino mass models within μ-τ symmetry [18], while in the second part we construct a m_{LL}matrix from fitting of $\mathrm{U_{PMNS}}$ incorporating the non-zero third reactor angle (θ_{13}) along with the observed data and subsequently predict the baryon asymmetry of the Universe (BAU).

The detailed plan of the paper is as follows. In Section 2, methodology and classification of neutrino mass models for zero θ_{13} is presented. Section 3, gives an overview of leptogenesis. The numerical and analytic results for neutrino mass models m_{LL}without and with θ_{13} are given in Sections 4 and 5 respectively. We end with summary and conclusions in Section 6.

Methodology and classification of neutrino mass models

In order to calculate the baryon asymmetry from a given neutrino mass model with zero θ_{13}, one usually starts with a suitable texture of light Majorana neutrino mass matrix, m_{LL} and then relates it with the heavy right-handed (RH) Majorana neutrinos M_{RR} and the Dirac neutrino mass matrix m_{LR} by inverting the seesaw formula in an elegant way. Since the structure of Yukawa matrix for Dirac neutrino is not known, the diagonal texture of Dirac neutrino mass matrix m_{LR} can either be of charged lepton mass matrix (6,2), up-quark type mass matrix (8,4), or down- quark type mass matrix (4,2), as allowed by SO(10) GUT models, for phenomenological analysis. The detail formalism of neutrino mass models is relegated to Appendix A. In the second part of this paper, we have constructed m_{LL} from U_{PMNS} matrix with non-zero θ_{13}.

$$m_{LL} = U_{PMNS}.m_{diag}.U_{PMNS}^T \qquad (1)$$

In the standard Particle Data Group (PDG) [19] PMNS matrix is parameterized as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{13} - s_{12}\,c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad (2)$$

$$sin^2\theta_{13} = |U_{e3}|^2, tan^2\theta_{12} = \frac{|U_{e2}|^2}{|U_{e1}|^2},\ tan^2\theta_{23} = \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}, \qquad (3)$$

$$m_{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \pm m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \qquad (4)$$

In all cases be it in NH or IH and of whatever types (A or B): $|m_2| > |m_1| \Rightarrow tan^2\theta_{12} < 1$.This condition is always true in all the calculations of leptogenesis. A global analysis [20] current best-fit data is used in the present analysis:

$$\Delta m_{21}^2 = 7.6 \times 10^{-5}\mathrm{eV}^2, \Delta m_{31}^2 = 2.4 \times 10^{-3}\mathrm{eV}^2,$$
$$sin^2\theta_{12} = 0.312, sin^2\theta_{23} = 0.42, sin^2\theta_{13} = 0.025,$$
$$\theta_{12} = 34^0 \pm 1^0,\quad \theta_{23} = 40.4^{+4.6^0}_{-1.8^0}, \theta_{13} = 9.0^0 \pm 1.3^0.$$

Oscillation data are insensitive to the lowest neutrino mass. However, it can be measured in tritium beta decay [21], neutrinoless double beta decay [22] and from the contribution of neutrinos to the energy density of the universe [23]. Very recent data from the Planck experiment have set an upper bound over the sum of all the neutrino mass eigenvalues of

$\sum_{i=1}^{3} m_i \leq$0.23 eV at 95% C.L.[24].

But, oscillations experiments are capable of measuring the two independent mass-squared differences between the three neutrino masses:

$\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{31}^2 = m_3^2 - m_1^2$. This two flavour oscillation approach has been quite successful in measuring the solar and atmospheric neutrino parameters. In future the neutrino experiments must involve probing the full three flavor effects, including the sub-leading ones proportional to $\alpha = \Delta m_{21}^2/|\Delta m_{31}^2|$.

The Δm_{21}^2 is positive as is required to be positive by the observed energy dependence of the electron neutrino survival probability in solar neutrinos but Δm_{31}^2 is allowed to be either positive or negative by the present data. Hence, two patterns of neutrino masses are possible: $m_1 < m_2 \ll m_3$, called normal hierarchy (NH) where Δm_{31}^2 is positive and $m_3 \ll m_1 < m_2$, called inverted hierarchy (IH) where $\Delta m_{31}^2$is negative. A third possibility, where the three masses are nearly quasi-degenerate with very tiny differences $m_1 \leq m_2 \sim m_3$, between them, also exists with two sub-cases of Δm_{31}^2 being positive or negative.

Leptonic CP violation (LCPV) can be established if CP violating phase δ_{CP} is shown to differ from 0 and 180^0. It was not possible to observe a signal for CP violation in the data so far. Thus, δ_{CP} can have any value in the range $[-180^0, 180^0]$. The Majorana phases $\emptyset_1$ and $\emptyset_2$ are free parameters. In the absence of constraints on the phases $\emptyset_1$and $\emptyset_2$, these have been given full variation between 0 to 2π and excluding these two extreme values.

Leptogenesis

For our estimations of lepton asymmetry [25], we list here only the required equations for computations. Interested reader can find more details in Ref. [26]. According to Type-1 Seesaw mechanism [27] the light left-handed Majorana neutrino mass matrix m_{LL}, the heavy right-handed (RH) Majorana neutrinos M_{RR} and the Dirac neutrino mass matrix m_{LR} are related in a simple way

$$m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T \tag{5}$$

Where m_{LR}^T is the transpose of Dirac neutrino mass matrix m_{LR} and M_{RR}^{-1} is the inverse of M_{RR}. In unflavoured thermal leptogenesis, the lepton asymmetry generated due to CP-violating out-of-equilibrium decay of the lightest of the heavy right-handed Majorana neutrinos, is given by

$$\epsilon_i = \frac{\Gamma(N_R \to l_L + \varphi) - \Gamma(N_R \to \bar{l}_L + \varphi^\dagger)}{\Gamma(N_R \to l_L + \varphi) + \Gamma(N_R \to \bar{l}_L + \varphi^\dagger)} \tag{6}$$

where $\bar{l}_L$ is the anti-lepton of lepton l_L, and φ is the Higgs doublets chiral supermultiplets.

$$\epsilon_i = -\frac{3}{16\pi}\left[\frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}}\frac{M_1}{M_2} + \frac{\mathrm{Im}[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}}\frac{M_1}{M_3}\right]. \tag{7}$$

where $h = m_{LR}/v$ is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of M_{RR} and v= 174 GeV is the vev of the standard model.

And finally the observed baryon asymmetry of the Universe [28] is calculated from,

$$\eta_B^{SM} = (\frac{\eta_B}{\eta_\gamma})^{SM} \approx 0.98x10^{-2} \times \kappa_1\epsilon_1 \tag{8}$$

The efficiency or dilution factor κ_1 describes the washout of the lepton asymmetry due to various lepton number violating processes, which mainly depends on the effective neutrino mass

$$\tilde{m}_1 = \frac{(h^+h)_{11}v^2}{M_1} \tag{9}$$

Where v is the electroweak vev, $v = 174$ GeV. For $10^{-2}eV < \tilde{m}_1 < 10^3 eV$, the washout factor κ_1 can be well approximated by [29]

$$\kappa_1(\tilde{m}_1) = 0.3[\frac{10^{-3}}{\tilde{m}_1}]\,[log\frac{\tilde{m}_1}{10^{-3}}]^{-0.6} \tag{10}$$

We adopt a single expression for $\kappa_1$valid only for the given range of $\tilde{m}_1$[29,30].

In the flavoured thermal leptogenesis[31], we look for enhancement in baryon asymmetry over the single flavour approximation and the equation for lepton asymmetry in $N_1 \rightarrow l_\alpha \varphi$ decay where $\alpha = (e, \mu, \tau)$, becomes

$$\varepsilon_{\alpha\alpha} = \frac{1}{8\pi}\frac{1}{(h^+h)_{11}}[\sum_{j=2,3}\text{Im}[h^*{}_{\alpha1}(h^+h)_{1j}h_{\alpha j}]g(x_j) + \sum_j \text{Im}[h^*{}_{\alpha1}(h^+h)_{j1}h_{\alpha j}]\frac{1}{(1-x_j)}].$$

(11)

where $x_j = \frac{M_j^2}{M_i^2}$ and $g(x_j) \sim \frac{3}{2}\frac{1}{\sqrt{x_j}}$. The efficiency factor is given by $\kappa_\alpha = \frac{m_*}{\tilde{m}_{\alpha\alpha}}$. Here $m_* = 8\pi H v^2/M_1^2 \sim 1.1x10^{-3}$ eV, and $\tilde{m}_{\alpha\alpha} = \frac{h_{\alpha1}^\dagger h_{\alpha1}}{M_1}v^2$.

This leads to the BAU,

$$\eta_{3B} = \frac{n_B}{n_\gamma} \sim 10^{-2}\sum_\alpha \epsilon_{\alpha\alpha}\kappa_\alpha \sim 10^{-2} m_* \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}}.$$

(12)

For single flavour case, the second term in $\epsilon_{\alpha\alpha}$ vanishes when summed over all flavours.

Thus

$$\epsilon_1 \equiv \sum_\alpha \epsilon_{\alpha\alpha} = \frac{1}{8\pi}\frac{1}{(h^\dagger h)_{11}}\sum_j Im[(h^\dagger h)_{ij}^2]g(x_j),$$

(13)

this leads to baryon symmetry,

$$\eta_{1B} \approx 10^{-2} m_* \frac{\epsilon_1}{\widetilde{m}} = 10^{-2}\kappa_1\epsilon_1, \quad (14)$$

where $\epsilon_1 = \sum_\alpha \epsilon_{\alpha\alpha}$ and $\widetilde{m} = \sum_\alpha \widetilde{m}_{\alpha\alpha}$.

However, in non-thermal leptogenesis scenario [32], the mechanism of generation of lepton asymmetry is different from the standard thermal leptogenesis. Here the right-handed neutrinos are produced through the direct non-thermal decay of the inflaton ϕ. And the inflation decay rate is given by

$$\Gamma_\phi = \Gamma(\phi \to N_i N_i) \approx \frac{|\lambda_i|^2}{4\pi} M_\phi \quad (15)$$

The reheating temperature after inflation is given by the expression,

$$T_R = \left(\frac{45}{2\pi^2 g_*}\right)^{1/4} \left(\Gamma_\phi M_p\right)^{1/2} \quad (16)$$

and the produced baryon asymmetry of the universe can be calculated by the following relation [33],

$$Y_B = \frac{n_B}{s} = CY_L = C\frac{3}{2}\frac{T_R}{M_\phi}\epsilon \quad (17)$$

where $s = 7.0n_\gamma$, is related to $Y_B = n_B/s = 8.7x10^{-11}$ in eq. (17). From eq. (17) the connection between T_R and M_ϕ is expressed as,

$$T_R = \left(\frac{2Y_B}{3C\epsilon}\right) M_\phi. \quad (18)$$

Two more boundary conditions are: $M_\phi > 2M_1$ and $T_R \leq 0.01\, M_1$. M_1 and ϵ_1 for all neutrino mass models are used in the calculation of theoretical bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi < M_\phi^{max}$. Only those models which satisfy the constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$ simultaneously, can survive in the non-thermal leptogenesis.

Numerical analysis and results without θ_{13}

Classifications of different neutrino mass matrices with zero θ_{13} employed for numerical analysis are given in Appendix A. The predictions of mass-squared differences and mixing

Type	Δm^2_{21} ($10^{-5} eV^2$)	Δm^2_{23} ($10^{-3} eV^2$)	$tan^2\theta_{12}$	$tan^2\theta_{23}$	$sin\theta_{13}$
(IA)	7.82	2.20	0.45	1.0	0.0
(IB)	7.62	2.49	0.45	1.0	0.0
(IC)	7.62	2.49	0.45	1.0	0.0
(IIA)	7.91	2.35	0.45	1.0	0.0
(IIB)	8.40	2.03	0.45	1.0	0.0
(IIC)	7.53	2.45	0.45	1.0	0.0
(III)	7.61	2.42	0.45	1.0	0.0

Table 1 Predicted values of the solar and atmospheric neutrino mass-squared difference and mixing angles.

angles are given in Table-1. These values are consistent with the observed neutrino oscillation data at global best fit value at 1σ level. For the calculation of baryon asymmetry, we then translate these mass matrices to M_{RR} via the inversion of the seesaw formula, $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis U_R where $M_{RR}^{diag} = U_R^T M_{RR} U_R = diag(M_1, M_2, M_3)$ with real and positive eigenvalues [34, 35]. We then transform diagonal form of Dirac mass matrix, $m_{LR} = diag(\lambda^m, \lambda^n, 1)v$ to the basis $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$, where Q= $diag(1, e^{i\emptyset_1}, e^{i\emptyset_2})$ is the complex matrix containing CP-violating Majorana phases derived from M_{RR}. Here λ is the Wolfenstein parameter and the choice *(m, n)* in

m_{LR} gives the type of Dirac neutrino mass matrix. At the moment we consider phenomenologically three possible forms of Dirac neutrino mass matrix such as (i) *(m,n)*= (4,2) for the down-quark mass matrix, (ii) (6,2) for the charged-lepton type mass matrix, and (iii) (8,4) for up-quark type mass matrix. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = \frac{m'_{LR}}{v}$ which enters in the expression of CP-asymmetry ϵ_i in Eq. (7). The new Yukawa coupling matrix h also becomes complex, and hence the term $Im(h^\dagger h)_{1j}$ appearing in lepton asymmetry ϵ_1 gives a non-zero contribution. In our numerical estimation of lepton asymmetry, we choose some arbitrary values of ϕ_1 and ϕ_2 other than $\pi/2$ and 0. The corresponding baryon asymmetries η_B are estimated for both unflavoured η_{1B} and flavoured η_{3B} leptogenesis respectively in Table-2. As expected there is enhancement in baryon asymmetry in case of flavoured leptogenesis η_B as shown in Table 2. We also observe the sensitivity of baryon asymmetry predictions on the choice of models with zero θ_{13} all but the five models are favourable with good predictions.

Type	(m,n)	M_1	ϵ_1	η_{1B}	η_{3B}	status
(IA)	(4,2)	5.43×10^{10}	1.49×10^{-5}	7.03×10^{-9}	2.16×10^{-8}	✓
(IA)	(6,2)	4.51×10^{8}	1.31×10^{-7}	5.76×10^{-11}	1.34×10^{-10}	✓
(IA)	(8,4)	3.65×10^{6}	1.16×10^{-9}	5.72×10^{-13}	1.19×10^{-12}	×
(IB)	(4,2)	5.01×10^{9}	2.56×10^{-14}	7.15×10^{-15}	1.09×10^{-9}	×
(IB)	(6,2)	4.05×10^{7}	2.06×10^{-16}	5.76×10^{-20}	8.84×10^{-12}	×
(IB)	(8,4)	3.28×10^{5}	1.68×10^{-18}	4.67×10^{-22}	7.16×10^{-14}	×
(IC)	(4,2)	5.01×10^{9}	1.85×10^{-13}	5.12×10^{-17}	7.16×10^{-9}	×
(IC)	(6,2)	4.05×10^{7}	1.47×10^{-15}	3.77×10^{-19}	5.80×10^{-11}	×
(IC)	(8,4)	3.28×10^{5}	1.02×10^{-16}	2.82×10^{-20}	4.34×10^{-12}	×
(IIA)	(4,2)	4.02×10^{10}	1.12×10^{-12}	2.49×10^{-15}	7.90×10^{-11}	×
(IIA)	(6,2)	3.25×10^{8}	9.00×10^{-15}	2.00×10^{-17}	6.34×10^{-13}	×
(IIA)	(8,4)	2.63×10^{6}	7.53×10^{-17}	1.67×10^{-19}	5.35×10^{-15}	×
(IIB)	(4,2)	9.76×10^{10}	4.02×10^{-6}	3.25×10^{-9}	7.53×10^{-9}	×
(IIB)	(6,2)	8.10×10^{8}	3.33×10^{-8}	2.57×10^{-11}	5.96×10^{-11}	✓
(IIB)	(8,4)	$6..56\times10^{6}$	2.71×10^{-10}	2.09×10^{-13}	4.86×10^{-13}	×
III	(4,2)	3.73×10^{12}	3.09×10^{-5}	8.13×10^{-8}	1.85×10^{-6}	×
III	(6,2)	4.08×10^{11}	3.74×10^{-5}	7.37×10^{-10}	1.62×10^{-9}	✓
III	(8,4)	3.31×10^{9}	3.09×10^{-7}	6.06×10^{-11}	1.13×10^{-10}	✓

Table 2 For zero θ_{13}, lightest RH Majorana neutrino mass M_1 and values of CP asymmetry and baryon asymmetry for QDN models (IA, IB, IC), IH models (IIA, IIB) and NH models (III), with $\tan^2\theta_{12} = 0.45$, using neutrino mass matrices given in the Appendix A. The entry (m, n) in m_{LR} indicates the type of Dirac neutrino mass matrix taken as charged lepton mass matrix (6, 2) or up quark mass matrix (8,4), or down quark mass matrix (4,2) as explained in the text. IA (6,2) and III (8,4) appears to be the best models.

Streaming lining further, by taking the various constraints into consideration, quasi-degenerate type-1A, QD-1A (6, 2) and NH-III (8, 4) are competing with each other, which can be tested for discrimination in the next level-the non-thermal leptogenesis.

In case of non-thermal leptogenesis, the lightest right-handed Majorana neutrino mass M_1 and the CP asymmetry ϵ_1 from Table-2, for all the neutrino mass models, are used in the calculation of the bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $\mathrm{M}_{\emptyset}^{\min} < M_{\emptyset} \leq \mathrm{M}_{\emptyset}^{\max}$ which are given in Table-3. The baryon asymmetry $Y_B = \frac{\eta_B}{s}$ is taken as input value from

WMAP observational data. Only those neutrino mass models which simultaneously satisfy the two constraints, $T_R^{max} > T_R^{min}$and $M_\phi^{max} > M_\phi^{min}$, could survive in the non-thermal

leptogenesis scenario. Certain inflationary models such as chaotic or natural inflation predict the inflaton mass $M_\phi \sim 10^{13}$ GeV and from Table-3, the neutrino mass models with

Type	m,n	$T_R^{min} < T_R \leq T_R^{max}$	$M_\emptyset^{min} < M_\emptyset \leq M_\emptyset^{max}$	status
(IA)	(4,2)	$1.2 \times 10^6 < T_R \leq 5.4 \times 10^8$	$1.1 \times 10^{11} < M_\emptyset \leq 4.9 \times 10^{13}$	✓
(IA)	(6,2)	$1.1 \times 10^6 < T_R \leq 4.5 \times 10^6$	$9.0 \times 10^8 < M_\emptyset \leq 3.6 \times 10^{10}$	✓
(IA)	(8,4)	$5.1 \times 10^5 < T_R \leq 3.6 \times 10^4$	$7.3 \times 10^6 < M_\emptyset \leq 9.6 \times 10^6$	×
(IB)	(4,2)	$6.0 \times 10^{13} < T_R \leq 5.0 \times 10^7$	$1.0 \times 10^{10} < M_\emptyset \leq 7.4 \times 10^3$	×
(IB)	(6,2)	$6.4 \times 10^{13} < T_R \leq 4.1 \times 10^5$	$8.10 \times 10^7 < M_\emptyset \leq 0.51 \times 1$	×
(IB)	(8,4)	$6.4 \times 10^{13} < T_R \leq 3.3 \times 10^3$	$6.6 \times 10^5 < M_\emptyset \leq 3.4 \times 10^{-5}$	×
(IC)	(4,2)	$8.9 \times 10^{12} < T_R \leq 5.0 \times 10^7$	$1.0 \times 10^{10} < M_\emptyset \leq 5.7 \times 10^4$	×
(IC)	(6,2)	$9.0 \times 10^{12} < T_R \leq 4.1 \times 10^6$	$8.10 \times 10^7 < M_\emptyset \leq 0.36 \times 1$	×
(IC)	(8,4)	$1.1 \times 10^{12} < T_R \leq 3.3 \times 10^3$	$6.6 \times 10^6 < M_\emptyset \leq 1.8 \times 10^{-2}$	×
(IIA)	(4,2)	$1.3 \times 10^{13} < T_R \leq 5.0 \times 10^8$	$8.0 \times 10^{10} < M_\emptyset \leq 2.8 \times 10^6$	×
(IIA)	(6,2)	$1.2 \times 10^{13} < T_R \leq 4.1 \times 10^6$	$6.5 \times 10^8 < M_\emptyset \leq 1.8 \times 10^2$	×
(IIA)	(8,4)	$1.1 \times 10^{14} < T_R \leq 3.3 \times 10^4$	$5.3 \times 10^6 < M_\emptyset \leq 1.8 \times 10^{-2}$	×
(IIB)	(4,2)	$8.9 \times 10^6 < T_R \leq 5.0 \times 10^8$	$2.0 \times 10^{12} < M_\emptyset \leq 7.0 \times 10^{13}$	✓
(IIB)	(6,2)	$8.0 \times 10^6 < T_R \leq 4.1 \times 10^6$	$1.6 \times 10^{11} < M_\emptyset \leq 9.3 \times 10^9$	×
(IIB)	(8,4)	$7.9 \times 10^6 < T_R \leq 7.3 \times 10^4$	$1.3 \times 10^9 < M_\emptyset \leq 6.3 \times 10^5$	×
(III)	(4,2)	$4.0 \times 10^7 < T_R \leq 3.7 \times 10^{10}$	$7.5 \times 10^{11} < M_\emptyset \leq 7.0 \times 10^{15}$	✓
(III)	(6,2)	$3.6 \times 10^6 < T_R \leq 4.1 \times 10^9$	$8.2 \times 10^{11} < M_\emptyset \leq 9.3 \times 10^{14}$	✓
(III)	(8,4)	$3.5 \times 10^6 < T_R \leq 3.3 \times 10^7$	$6.3 \times 10^9 < M_\emptyset \leq 6.3 \times 10^{10}$	✓

Table 3 Theoretical bound on reheating temperature T_R and inflaton masses M_ϕ in non-thermal leptogenesis, for all neutrino mass models with $\tan^2\theta_{12} = 0.45$. Models which are consistent with observations are marked in the status column.

(m, n) which are compatible with $M_\phi \sim 10^{13}$GeV, are listed as IA-(4, 2), IIB-(4, 2), III-(4, 2) and III-(6, 2) respectively. The neutrino mass models with (m, n) should be compatible with $M_\phi \sim (10^{10} - 10^{13})$ GeV and $T_R \approx (10^4 - 10^8)$ GeV. Again in order to avoid gravitino problem [36] in supersymmetric models, one has the bound on reheating temperature, $T_R \approx (10^6 - 10^7)$ GeV.

This streamlines to allow models as IA-(4,2), IIB-(4,2) and III-(6,2) respectively. If we prefer the Dirac neutrino mass matrix as either charged lepton or up-quark mass matrix, then we have only one allowed model III-(6, 2) in the list. The predictions of thermal leptogenesis [table-2] and non-thermal leptogenesis [table-3] are not consistent for the given model [say QD-1A(6,2) or NH-III(6,2)]; therefore, there is a problem with neutrino mass models with zero θ_{13}. In the next section, we study neutrino mass models with non-zero θ_{13} and check the consistency of above predictions.

Numerical analysis and results with Θ_{13}

In this Section, we investigate the effects of inclusion of non-zero θ_{13} [cf. 1, 2] on the cosmological baryon asymmetry in neutrino mass models. Unlike in Section IV analysis, we don't use the particular form of matrices, but we have constructed the lightest neutrino mass matrix m_{LL} using Eq. (1) through Eqns. (2) and (4). Observational [37] inputs used in U_{PMNS} are:

$\theta_{12} = 34^0, \theta_{23} = 45^0,\ \theta_{13} = 9^0.\ c_{12} = 0.82904,\ c_{23} = 0.707106, c_{13} = 0.98769,\ s_{12} = 0.55919,\ s_{23} = 0.707106, c_{13} = 0.156434.$ We obtained

$$U_{PMNS} = \begin{pmatrix} 0.81883 & 0.55230 & 0.156434 \\ -0.48711 & 0.52436 & 0.69840 \\ 0.30370 & -0.64807 & 0.69840 \end{pmatrix}. \quad (19)$$

Using Eq. (3) this leads to $sin^2\theta_{13} = 0.0244716, tan^2\theta_{12} = 0.45495, tan^2\theta_{23} = 1$.

Then the m_{diag} of Eq. (4) are obtained from the observation data [cf. 20] $(\Delta m_{12}^2 = m_2^2 - m_1^2 = 7.6 \times 10^{-5} \mathrm{eV}^2, |\Delta m_{23}^2| = |m_2^2 - m_3^2| = 2.4 \times 10^{-3} \mathrm{eV}^2)$, and calculated out for normal and inverted hierarchy patterns. The mass eigenvalues m_i (i=1, 2, 3) can also be taken from Ref. [cf. 25]. The positive and negative value of m_2 corresponds to Type-IA and Type-IB respectively. Once the matrix m_{LL} is determined the procedure for subsequent calculations are same as in Section IV.

Here, we give the result of only the best model due to inclusion of reactor mixing angle θ_{13} in prediction of baryon asymmetry, reheating temperature (T_R) and Inflaton mass (M_ϕ). Undoubtedly, for $\tan^2 \theta_{12} = 0.45$, the best model is NH-IA (6, 2) with: baryon asymmetry in unflavoured thermal leptogenesis $B_{uf} = 3.313\times 10^{-12}$; single flavoured approximation $B_{1f} = 8.844\times 10^{-12}$; and full flavoured $B_{3f} = 2.093\times 10^{-11}$. If we examine these values, we found that, expectedly, there is an enhancement is baryon asymmetry due to flavour effects. Similarly in non-thermal leptogenesis, we found that NH-IA is the best model and the predicted results are:

$$T_R^{min} < T_R \leq T_R^{max}(\mathrm{GeV}) = 7.97x10^3 < T_R \leq 4.486\mathrm{x}10^6,$$
$$M_\phi^{min} < M_\phi \leq M_\phi^{max}(\mathrm{GeV}) = 8.97x10^8 < M_\phi \leq 2.24x10^{11}.$$

Summary and Conclusion

We now summarise the main points. We have investigated the comparative studies of baryon asymmetry in different neutrino mass models (viz QDN, IH and NH) with and without θ_{13} for $tan^2\theta_{12} = 0.45$, and found models with θ_{13} are better than models without θ_{13}. We found that the predictions of any models with zero θ_{13} are erratic or haphazard in spite of the fact that their predictions are consistent in a piecemeal manner with the observational data (see Tables 2 & 3) whereas the predictions of any models with non-zero θ_{13} are consistent throughout the calculations. And among them, only the values of NH-IA (6,2) satisfied Davidson-Ibarra upper bound on the lightest RH neutrino CP asymmetry $|\epsilon_1| \leq 3.4 \times 10^{-7}$ [38] and M_1 lies within the famous Ibarra-Davidson bound [38], i.e., $M_1 > 4 \times 10^8$ GeV. Neutrino mass models either with or without θ_{13}, Type-IA for charged lepton matrix (6,2) in normal hierarchy appears to be the best if $Y_B^{WMAP} =$

8.7×10^{-11} is taken as the standard reference value, on the other hand if $Y_B^{CMB} = 6.1 \times 10^{-10}$, then charged lepton matrix (5,2) is not ruled out. We observed that unlike neutrino mass models with zero θ_{13}, where $\mu -$ predominate over e and τ contributions, for neutrino mass models with non-zero θ_{13}, $\tau -$ predominate over e and μ contributions. This implies the predominate factor changes for neutrino mass models with and without θ_{13}. When flavour dynamics is included the lower bound on the reheated temperature is relaxed by a factor ~ 3 to 10 as in Ref. [39]. We also observe enhancement effects in flavoured leptogenesis [40] compared to non-flavoured leptogenesis by one order of magnitude as in Ref. [41]. Such predictions may also help in determining the unknown Dirac Phase δ in lepton sector, which we have not studied in the present paper. The overall analysis shows that normal hierarchical model appears to be the most favourable choice in nature. Further enhancement from brane world cosmology [42] may marginally modify the present findings, which we have kept for future work.

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Appendix A: Classification of Neutrino Mass Models with zero $\boldsymbol{\theta_{13}}$

We list here the zeroth order left-handed Majorana neutrino mass matrices (m_{LL}^0) [43,44] with texture zeros left-handed Majorana neutrino mass matrices, $m_{LL} = m_{LL}^0 + \Delta m_{LL}$, corresponding to three models of neutrinos, viz., Quasi-degenerate (QD1A, QD1B, QD1C), inverted hierarchical (IH2A, IH2B) and normal hierarchical (NH3) along with the inputs parameters used in each model. m_{LL} which obey μ-τ symmetry are constructed from their zeroth-order (completely degenerate) mass models m_{LL}^0 by adding a suitable perturbative term Δm_{LL}, having two additional free parameters. All the neutrino mass matrices given below predict $tan^2\theta_{12} = 0.45$. The values of three input parameters are fixed by the predictions on neutrino masses and mixings in Table 1.

Type	m_{LL}^{diag}	m_{LL}^0	$m_{LL} = m_{LL}^0 + \Delta m_{LL}$
QD1A	$diag(1,-1,1)m_0$	$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\begin{pmatrix} \epsilon-2\eta & -a\epsilon & -a\epsilon \\ -a\epsilon & \frac{1}{2}-b\eta & \frac{1}{2}-\eta \\ -a\epsilon & \frac{1}{2}-\eta & \frac{1}{2}-b\eta \end{pmatrix} m_0$
Inputs $\epsilon = 0.66115, \eta = 0.16535, m_0 = 0.4$ (for $tan^2\theta_{12} = 0.45$, a =0.868, b = 1.025).			
QD1B	$diag(1,1,1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} \epsilon-2\eta & -a\epsilon & -a\epsilon \\ -a\epsilon & \frac{1}{2}-b\eta & \frac{1}{2}-\eta \\ -a\epsilon & \frac{1}{2}-\eta & \frac{1}{2}-b\eta \end{pmatrix} m_0$
Inputs are: $\epsilon = 8.314 \times 10^{-5}, \eta = 0.00395, m_0 = 0.4$ eV, (a =0.945 and b = 0.998).			
QD1C	$diag(1,1,-1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} \epsilon-2\eta & -a\epsilon & -a\epsilon \\ -a\epsilon & -b\eta & 1-\eta \\ -a\epsilon & 1-\eta & -b\eta \end{pmatrix} m_0$
Inputs are: $\epsilon = 8.314 \times 10^{-5}$, $\eta = 0.00395, m_0 = 0.4$ eV (for a =0.945 and b = 0.998).			
IH2A	$diag(1,1,0)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0$	$\begin{pmatrix} \epsilon-2\eta & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} & \frac{1}{2}-\eta \\ -\epsilon & \frac{1}{2}-\eta & \frac{1}{2} \end{pmatrix} m_0$
Inverted Hierarchy with even CP parity in the first two eigenvalues (IIA), ($m_i = m_1, m_2, m_3$): $\frac{\eta}{\epsilon} = 1.0, \eta = 0.0048, m_0 = 0.05$ eV.			
IH2B	$diag(1,-1,0)m_0$	$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0$	$\begin{pmatrix} \epsilon-2\eta & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} & \frac{1}{2}-\eta \\ -\epsilon & \frac{1}{2}-\eta & \frac{1}{2} \end{pmatrix} m_0$
Inverted Hierarchy with odd CP parity in the first two eigenvalues (IIB), ($m_i = m_1, -m_2, m_3$): $\frac{\eta}{\epsilon} = 1.0, \eta = 0.6607, m_0 = 0.05$ eV.			
NH3	$diag(0,0,1)m_0$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix} m_0$	$\begin{pmatrix} 0 & -\epsilon & -\epsilon \\ -\epsilon & 1-\epsilon & 1+\eta \\ -\epsilon & 1+\eta & 1-\epsilon \end{pmatrix} m_0$
Inputs are: $\frac{\eta}{\epsilon} = 0.0, \epsilon = 0.146, m_0 = 0.028$ eV.			

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