BEHAVIORS AND ATTITUDES IN THE TEACHING AND LEARNING OF GEOMETRY

Helena Sousa Melo, PhD
Maria do Carmo Martins, PhD
University of Azores, Portugal

Abstract

This paper presents a case study about teaching and learning topics of Geometry that are part of the syllabus of high school courses. Internationally the teaching of Geometry has been the subject of several studies. Nevertheless, research continues to reveal that Geometry is still quite absent from the classrooms, especially in the early years; Geometry is disliked by most students, is misunderstood, and its notation is totally ignored. One of the reasons for this failure is that these issues are addressed superficially in the curriculum of the training teachers of Geometry. This paper describes the experience of teaching Geometry to a class with 90 students in academic year 2014-2015, enumerates some difficulties and behaviors faced by students and teachers, and exposes the results of the academic success.

Keywords: Geometry, teaching, learning, difficulties

Introduction

According to Isaac J. Schwatt "The average student seems to have less understanding of Geometry than of any other of the elementary mathematical disciplines. Even calculus is not excepted. This is in part due to the extreme difficulty of the subject of Geometry, the age at which the study of it is usually attempted, and the relatively small amount of time allotted to it" [4].

Given that the development of students' capabilities in geometrical reasoning continues to be an issue of considerable international concern, this paper reports on an academic experiment in a Curricular Unit involving Geometry.

The Geometry topics taught in the degree of *Educação Básica*, from the University of Azores, present a low success rate. Moreover, failed students are put together in the same class as the students that are going to attend it for the first time. During seven years, the number of students in the class increased from 20 (the number of new enrollments per year) to 90. In

the academic year of 2014-2015 we split the class in tree according to the students' knowledge. We based it on the outcome of a diagnostic test that assessed elementary notions on Geometry, for instance, the classification of triangles and quadrilaterals, the designation of angles as well as the incidence of point, line, and plane. The course Director timely informed students that a diagnostic test was going to take place. Despite that, only 70 students took the test. Based on the test results, the first class got 28 students enrolled, while the second and the third got 31 students.

We believe we set the conditions so that all students could attend classes. Week lectures are organized as follows: 2 hours for theoretical grounds, illustrated with application examples; 2 practical hours where students are motivated to solve problems and issues related to the theoretical content of that week; and 1 tutorial hour. Sadly, the number of students attending tutorial classes was only of 2 per week (on average) during the period of class. In addition, there is 1 office hour to answer to individual questions.

The goals of this subject are: develop Mathematics capabilities to be used as an instrument of interpretation and intervention in the society; develop the capacity to formulate and solve problems, as well as to train memory, to reason formally about problems, and to promote creativity; develop research, creativity, and autonomy capacity; establish a link between course contents and the curricular program of "Ensino Básico"; articulate the topics studied with the mathematical topics of the programs and curriculum guidelines for preschool, first and second cycles of basic education; and consolidate scientific knowledge in Geometry, in particular, plane and space geometry [2].

The Syllabus of the subject is: plane and space Geometry – basic concepts, definitions, and properties; polygons – general definitions, classification, and properties; polygons composition; relationship between triangle elements; congruence and similarity criteria; circumference – relative position between two circumferences and between a straight line and a circumference; relationship between polygons and the circumference; Analytical Geometry in the plane and in the space – coordinated system vectors, straight-line equations in the plane and space, plane equations; Topological Geometry in the space – relative positions between planes, relative positions of straight lines, relative positions between straight lines and planes, three-dimensional polyhedrons and curves; representation and planning of some solids; measure and base units – international system of units; measurement of base units length; plane figure area; prism volume; volume of cylinders, cones and spheres; area of a surface; time units [2].

The subject of this course is presented both in theoretical and practical lectures using computer-aided contents whenever needed. The

adopted methodology stimulates and privileges the active participation of students. The subjects are presented, discussed, and developed using problems and real situations. In order to consolidate the learning of the concepts, students are provided with exercise sheets on each course topic, as well as all additional material to support lectures [2].

The final grade is computed as the weighted average of two midterm written tests or, alternatively, a final-term written exam. The first written test assesses the skills required to Plane Geometry, while the second written test evaluates the skills required to Space Geometry.

Development

After reviewing several basic Geometry concepts, such as the notation of point, line, line segment, half-line, angle, perimeter, area and volume, and respective units of measurement, we still noticed a difficulty in the geometrical expressions of such concepts. The gaps make it difficult for students to acquire the desired knowledge as well as for teachers to devise methods to present the subjects in an understandable way. As an immediate consequence, students make a great number of mistakes and fail to acquire ground concepts that form building blocks for more advanced notions.

It was notorious the resistance to use an accurate notation. In addition to the notations presented, each student has a tendency to create his or her own notation, using various symbols for the same geometric element. The inconsistency and the notation abuse obstruct the students to understand his or her acquisition of knowledge and hinder the teacher in the correction, either in verbal form, whether in written form.

The weak or reduced concentration, the lack of motivation, the lack of language fastidiousness, the incomplete sentences, the immediate and hasty conclusions when they are not, the use of the design that does not correspond to the actual data assumed to be true, for instance, a draft of an isosceles triangle when it is scalene, the dependence on use of the calculator to perform basic operations, the lack of critical spirit, the wish of getting approved without knowledge, the use of a tutor rather than prompting the teacher, the gasp of inappropriate behavior in the classroom (cell phone, tablets, computers and side-by-side conversations unrelated to the matter under study), the entrance and exit of the classroom without proper teaching authorization restrict any kind of effort and strategy used by the teacher.

Without pretending to ridicule the students' answers, but showing the geometric immaturity of answers, we present two examples where there is a clear confusion between geometric figures in the plane and geometric figures in the space. It was observed in the 2nd written test, a true false question, that 11 of the 33 evaluated students responded, “the square is a polyhedron” (Figure 1). In another question, a student responds correctly that the sentence

"the base of a cone is a polygon" is false, correcting it and saying that "the base of a cone is a polyhedron" (Figure 2).

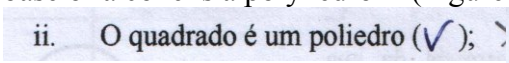

Figure 1. Example of mistake

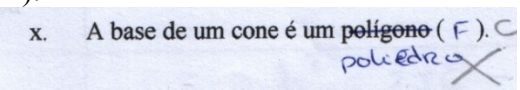

Figure 2. Misconception

A fact that makes us always surprised is the confusion between the perimeter, the area, and the volume, and, specially, the use of measurement units.

It was asked in a written test, to compute the edge of the base of a quadrangular prism given its volume and height, expressed in different measurement units. We have witnessed two failures: (1) many of the students were not able to detect that the units of measure were different, so proceeded to normal division operation, interpreting the result as the desired answer; (2) others divided the volume by the height of the prism and interpreted the results of this operation as the desired edge length of prism. In fact, in these conditions when divided the volume by the height we get the area of the base of the prism. It was evident the exchange of measurement units. The worst confusion was the use of the squared measurement unit of the area for the length measurement unit of the base edge. Notice that the Units of Measure topic is introduced in elementary school, being constantly used throughout the academic path.

The teachers must use different strategies to fill such gaps. They must insist that students need to always keep in mind the unit of measure used, in particular, the use of linear units for lengths, quadratic units for areas, and cubic units for volumes.

According to our experience in teaching in different courses, we notice that in certain classes not directed to Mathematics there is a greater difficulty in abstracting Geometry problems. We reinforce at an early stage that the appropriate path must be from the concrete to the abstract. Another difficulty we noticed in teaching and learning Geometry is that a problem can be solved in several ways. So, there is not a single solution pattern that the students “memorize" and then "reply".

Below we present the graphics with our evaluation results. Figure 3 shows the evaluation results of the first written test, where from 90 students enrolled only 62 attended. From these 62 students, 8 got a positive grade, with values ranging from 9.5 to 14.1 out of 20. Only two students achieved the highest score. The lowest grade was 0. Under these conditions, the percentage of success in the first written test was 13%.

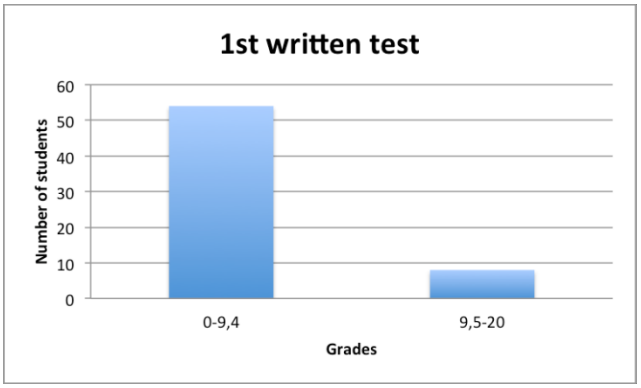

Figure 3. Results of 1st written test

In Figure 4 we observe the evaluation of the second written test. Of the 62 students who attend the first written test only 32 made the second written test. From these 32 students, 14 students got a positive grade, with values ranging from 9.5 to 19.2 out of 20. Only one student achieved the highest score. The lowest grade was 1.6. Under these conditions, the percentage of success in the second written test was 44%.

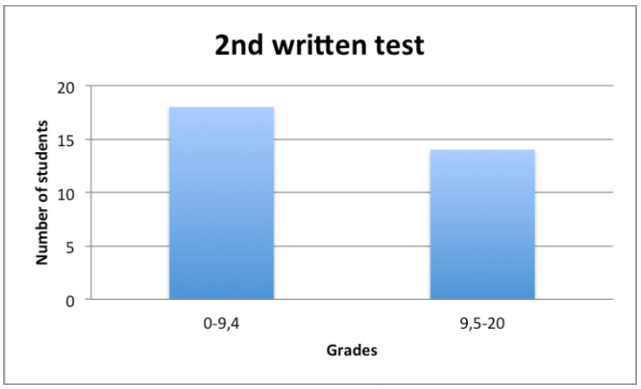

Figure 4. Results of 2nd written test

In order to give an overview of the general behavior, the Figure 5 depicts the evolution of the class. The columns represent: (a) the number of all students enrolled; (b) the average of the students who attended the geometry classes in the plane; (c) the number of students who carried out the first written test; (d) the student average that attended the geometry classes in space; (e) the number of students who made the second written test; and finally (f) the number of students that have achieved success in the Curricular Unit with the two written tests.

At the time of writing, it is impossible to inform the final performance of the class, since the exams period is still to come (from June, 15th to July, 17th of 2015).

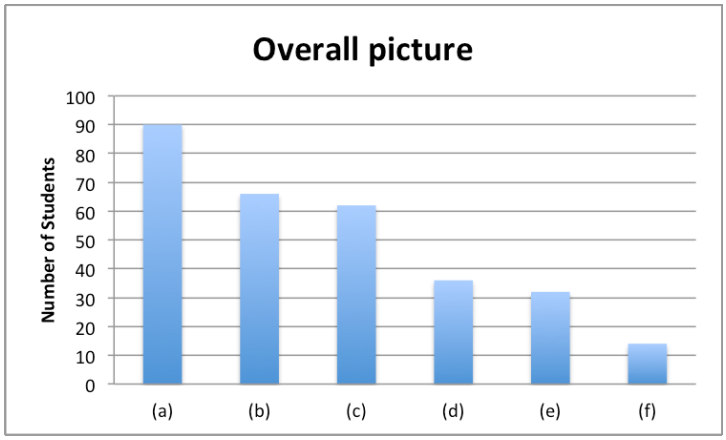

Figure 5. Evaluation of the class

With the help of the diagnostic test we were able to get a clear picture about the knowledge and skills degree of the class. Notice that only 12% of the group was in a position to be successful. Upon this background, some strategies have been applied to improve the knowledge. In the end, we achieved a success rate of 23%, which shows some improvement and a certain efficacy in the strategy we followed. Nevertheless, we are aware that there is still much work to be done.

Conclusion

Despite all efforts and exceptionally created conditions, the results were no better than in previous editions of the class. One of the reasons for the failure is the high degree of student absenteeism. From 90 students enrolled, 23 never attended any class.

We present the comments of Diana Lewis [1] on the causes for students not to like Geometry.

It goes without saying that students typically tend to dislike (or hate) Math more than other subjects.

As we know, many subjects fall under the Math umbrella, e.g., Algebra, Geometry, Trigonometry, Pre-Calculus, Calculus, and Statistics. But have you ever stopped to wonder why many students tend to struggle so much in Math?

Some answers:

1) Many students simply do not understand Geometry: while most subjects have something that students can relate to or find interesting, Geometry is conceptually difficult for some learners. Aside from not understanding it, most often they do not see why it is important for them to learn.

2) Geometry is not interactive: many subjects, like Language Arts, writing, and even Science can be very exciting for students because they get to actively take part in the learning process. Geometry may seem boring to them because they have to memorize and follow sets of rules, many of which do not have exceptions.
3) Poor relationship between student-teacher: If a student feels that she does not have a pleasant relationship with the Geometry teacher, the student may be more likely to do poorly in the class. When the class size is too big, the student can easily begin to feel lost and overlooked.
4) Negative experiences in previous Math classes: every student that has had at least one negative learning experience in her educational history, whether it was in elementary school, middle school, high school, college, or a trade school. Sometimes, all it takes is one negative experience to take the fun out of a subject forever. If this has happened to a student, it is important for them to try to reverse that process. Geometry can be very fun for students, but if they start in a state of mind that they hate Geometry, it would be very unfortunate and hinder their success.

To guarantee the success of the Geometry, either in teaching or in learning, its concepts must be explained and understood, never memorized by the use of its formulae. We cannot forget that we learn by doing!

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