SIMPLE THEORETICAL FORMULA TO ADJUST AMOUNT OF REINFORCEMENT

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Abstract

In construction industry, it is common practice that sometimes neither steel reinforcement may be available at market as required by design calculations nor poured concrete may be as strong as required. Such cases necessitate the modification of design calculations. A simple formula is required (for structural engineer as well as for site engineer) to adjust, quickly, the amount of reinforcement and to check that the adjusted materials (properties) will serve as the original ones as required in design calculations. Providing fy different than required by calculations is more safe because the adjusted calculations are done before pouring concrete. Providing fc' larger than required by calculations is acceptable because it produces good concrete quality and enhances bending capacity. Providing fc' less than required by calculations is not acceptable because it produces concrete of less quality than required. This may lead to failure. And may need some schemes to remedy. It is preferable to deal with fy being different than dealing with fc' being different than required by calculations. Because the first case occur before pouring concrete and the second case occur after pouring concrete. A formula to adjust the calculations is set to predict the required provided steel ratio if fy is different than required by calculations.

Keywords: Beam, concrete, formula, singly reinforced, steel

Introduction

Concrete is obtained by permitting a carefully proportioned mixture of cement, sand and gravel or other aggregate and water. The properties of concrete depend on the proportions of the mix. Its compressive strength is high, which makes it suitable for members primarily subjected to compression, such as columns and arches. On the other hand, it is relatively, a brittle material whose tensile strength is small compared with its compressive strength. To offset this limitation, it was found possible to use steel with its high tensile strength to reinforce concrete. While literature has been reviewed, the author did'nt find something about how to adjust the amount of reinforcement when using different construction materials (like steel reinforcement).

Purpose Of Research

The purpose of this research is to find simple formulae to predict the amount of steel reinforcement (**A_s**) for a given beam section if the steel (**f_y**) and concrete (**f'_c**) actually used on site are different from those required according to design calculations.

Research Significance

In construction industry, sometimes, materials (steel and concrete) with specific properties (**f_y, f'_c**) required by structural designer are not always available in the market. But other properties may be available instead. Such cases necessitate the modification of design calculations. A simple formula is required (for structural engineer as well as for site engineer) to adjust, quickly, the amount of reinforcement and to check that the adjusted materials (properties) will serve as the original ones as required in design calculations.

Methodology

The basis of analysis for singly reinforced concrete beam (Nawy 2005) (Wight & Macgregor 2009) (Youkhanna 2014) is shown in Fig. (1).

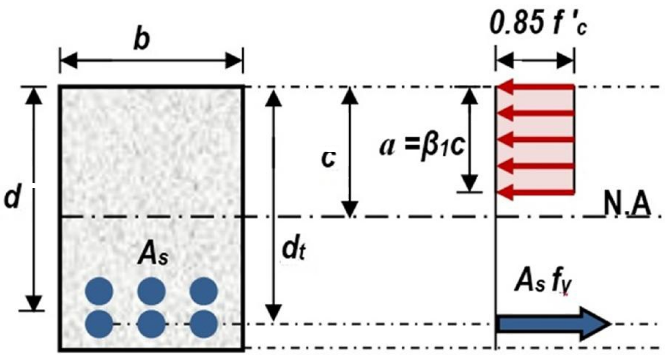

Fig. (1) Singly reinforced concrete beam.

Different approaches are available to find the amount of steel reinforcement (A_s) required for beam section. Among these approaches, the following approach is chosen.

<u>CASE:</u> Section dimensions (b, d) are known and area of reinforcement (A_s) is required

In this approach, bending moment (M_u) due to external load is to be calculated. Steel ratio may be predicted (Youkhanna 2014) (Setareh & Darvas 2007) as:

$$\rho = \frac{0.85 f'_c}{f_y}\{1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\} \tag{1}$$

$$R_n = \frac{M_u}{\phi . bd^2} \tag{2}$$

$$A_s = \rho . bd \tag{3}$$

After choosing bar sizes, the provided steel ratio should fulfill the following condition of the ACI Code (ACI 2011).

$$\rho_{\min} \le \rho \le \rho_{\max} \tag{4}$$

$$\rho_{\min} = 0.25 \frac{\sqrt{f'_c}}{f_y} \tag{5}$$

but not less than

$$\rho_{\min} = \frac{1.4}{f_y} \tag{6}$$

$$\rho_{\max} = 0.364 \beta_1 \frac{f'_c}{f_y} . \frac{d_t}{d} \qquad \text{(for } \varepsilon_t = 0.004\text{)} \tag{7}$$

$$\rho_{\max} = 0.319 \beta_1 \frac{f'_c}{f_y} . \frac{d_t}{d} \qquad \text{(for } \varepsilon_t = 0.005\text{)} \quad \text{(tension-controlled)} \tag{8}$$

Knowing that for one layer of reinforcement $d_t = d$.

$$\beta_1 = 0.85 \qquad \text{(for } f'_c \le 30 \text{ Mpa)} \tag{9}$$

$$\beta_1 = 0.85 - 0.008(f'_c - 30) \ge 0.65 \qquad \text{(for } f'_c > 30 \text{ Mpa)} \tag{10}$$

It is obvious, from Eq. (1), that steel ratio $\boldsymbol{\rho}$ depends greatly on the properties of the materials used, i.e. $\boldsymbol{f_y}$ and $\boldsymbol{f'_c}$. These two properties are considered as variables to be studied in the following paragraphs:

[I] (f_y) DIFFERENET THAN REQUIRED BY CALCULATIONS

Example:

The following data are assumed known: M_u = 100 kN.m, b = 300 mm, d = 440 mm. The following values for materials properties (f_y, f'_c) are assumed to be what is required according to design calculations: f_y = 414 MPa, f'_c = 25 MPa.

Assuming φ = 0.9 (simplifying the calculations), design calculations result the following:

Eq. (2): $R_n = \frac{100(10^6)}{(0.9)(300)(440)^2} = 1.9131$

Eq. (1): $\rho = \frac{0.85(25)}{(414)}\{1-\sqrt{1-\frac{2(1.9131)}{0.85(25)}}\} = 0.00485$

Eq. (5): $\rho_{\min} = 0.25\frac{\sqrt{25}}{414} = 0.00302$

Eq. (6): $\rho_{\min} = \frac{1.4}{414} = 0.00338 > 0.00302 \rightarrow \boldsymbol{\rho}_{\min} = 0.00338 < 0.00485$ O.K.

Eq. (9): $\beta_1 = 0.85$

Eq. (8): $\rho_{\max} = 0.319(0.85)\frac{25}{414}.\frac{440}{440} = 0.0164 > \boldsymbol{\rho} = 0.00485$ O.K.

Eq. (3): $A_s = 0.00485(300)(440) = 640.2 \cong 641\boldsymbol{mm}^2$

Now, if $\boldsymbol{f}_y$ is not the same as required by calculations ($\boldsymbol{f}_y \neq 414$MPa), Table (1) is constructed for some typical values of yield stress of steel, with all other data the same as those used in calculations. It should be noted that, in Table (1), the provided bending capacity (M_u) should be the same as that of design calculations (100 kN.m).

Table (1) Reinforcement ratio for different values of f_y [f'_c = 25 MPa].

M_u = 100 kN.m			b = 300 mm		d = 440 mm		
f_y (Mpa)	R_n	P	$\rho_{\min}$	$\rho_{\max}$	A_s (mm^2)	$\frac{f_y}{414}$	$\frac{0.00485}{\rho}$
276	1.9131	0.00728	0.00507	0.02456	961	0.667	0.667
345		0.00582	0.00406	0.01965	769	0.833	0.833
414		0.00485	0.00338	0.01637	641	1.000	1.000
490		0.00410	0.00286	0.01383	541	1.184	1.184

It is obvious that provided steel area, $A_{s,\text{provided}}$ in most practical cases will be greater than what is required in calculations. This is because of round off (integer) the number of bars to be used.

From the last two columns in Table (1), we can see that the ratio of (f_y/414) is equal to the ratio of (0.00485/ρ), where f_y represents the available steel property, 414 represent the steel property required by calculations, 0.00485 is steel ratio required by calculations, and ρ is the steel ratio available. So, there is inversely proportion between property that is required in calculations and the same property that is available. As a result, the following formula, Eq. (11) may be set as a simple guide for quick adjustment for steel ratio in design office as well as in construction site.

$$\rho_{required} = \frac{(f_y)_{Calculations}}{(f_y)_{Provided}} \times \rho_{Calculations} \tag{11}$$

Eq. (11) may be presented in other form regarding area of steel reinforcement as:

$$(A_s)_{Required} = \frac{(f_y)_{Calculations}}{(f_y)_{Provided}} \times (A_s)_{Calculations} \tag{12}$$

It is worthy to mention that for any combination of f_y and f'_c, the formula in Eq. (12) will remain applicable. Fig. (2) shows the relation between steel ratio and steel yield stress.

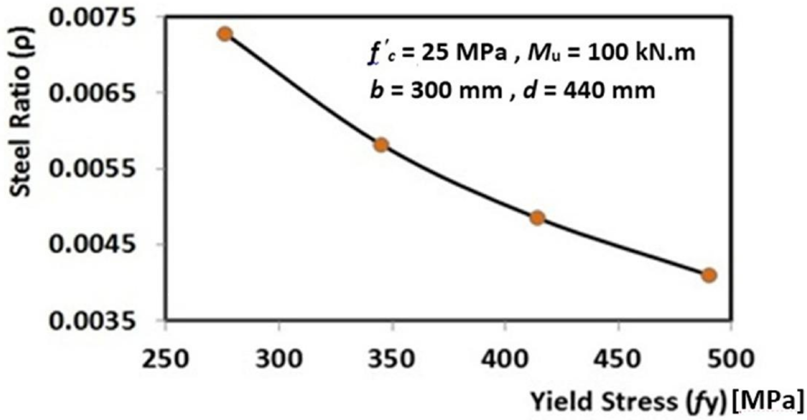

Fig (2) $\rho - f_y$ relationship.

[II] (f_c') DIFFERENET THAN REQUIRED BY CALCULATIONS

Before pouring concrete into forms of different constructional elements, it is common practice to take cylinders of same concrete mix to be tested (in most cases, after 28 days) to check that the compressive strength of poured concrete is acceptable compared to that required by calculations. If the provided compressive strength is larger than required, it is safe situation (it may increase cost a little bit), it is acceptable case. But, if the provided compressive strength is less than required, it is unsafe situation, and this may require steps of precautions which will cost more.

If all data is available, steel ratio ρ is calculated using Eq. (1). Knowing steel ratio, a bending capacity may be calculated using the following equation:

$$M_u = \phi.bd^2.\rho.f_y(1 - \frac{\rho.f_y}{1.7f'_c}) \tag{13}$$

Example:

Same data and materials properties provided in previous example are considered as required according to design calculations: f_y = 414 MPa, f'_c = 25 MPa.

If the provided compressive strength (cylinders test) is different from that required by calculations, a quick recalculation is required to judge the effect of the difference in compressive strength on the behavior (in this case bending capacity is considered) of the constructional element (in this case a beam is chosen).

In order to focus on the effect of the compressive strength values, it is assumed that provided steel ratio is the same as that required by calculations, this is logical, because after pouring of concrete, the steel provided will not be changed. As a result, adjusted calculations will use the same steel ratio of the case when f_y = 414 MPa and f'_c = 25 MPa which is considered as basis of comparison. So, steel ratio will be always taken as 0.00485 as given in Table (1). The effect of different values of compressive strength f'_c on bending capacity of the beam is represented in Table (2).

Table (2) Reinforcement ratio for different values of f'_c [f_y = 414 MPa].

M_u = 100 kN.m			b = 300 mm		d = 440 mm		
f'_c (MPa)	R_n	ρ	ρ_{min}	ρ_{max}	M_u (kN.m)	$\frac{M_u}{100}$	$\frac{f'_c}{25}$
17	1.9131	0.00485	0.00338	0.0111	97.66	0.977	0.68
21			0.00338	0.0138	99.05	0.991	0.84
25			0.00338	0.0164	100.00	1.000	1.00
30			0.00338	0.0197	100.82	1.008	1.20
35			0.00357	0.0218	101.42	1.014	1.40

Relating the ratio of compressive strength to the ratio of bending capacity from Table (2), the following formula, Eq. (14) may be set:

$$(M_u)_{Provided} = \sqrt[8]{\frac{(f'_c)_{Provided}}{(f'_c)_{Calculations}}} \times (M_u)_{Calculations} \tag{14}$$

Table (3) gives the values of the eighth root in Eq. (14) in comparison with the ratio of bending capacity. There is a slight percent of error between these two ratios, and it may be ignored. Also, in Table (3), it is given the provided bending capacity from Table (2) in comparison to calculated one (100 kN.m).

Table (3) comparison of compressive strength ratio and comparison of bending capacity (f_y = 414 MPa).

M_u = 100 kN.m			b = 300 mm		d = 440 mm
f'_c (MPa)	$\frac{f'_c}{25}$	$\frac{M_u}{100}$	$\sqrt[8]{\frac{f'_c}{25}}$	M_u (kN.m) Table (2)	M_u (kN.m) Eq.(14)
17	0.68	0.977	0.9762	97.66	97.62
21	0.84	0.991	0.9892	99.05	98.92
25	1.00	1.000	1.0000	100.00	100.00
30	1.20	1.008	1.0115	100.82	101.15
35	1.40	1.014	1.0213	101.42	102.13

From Table (3), it can be seen that Eq. (14) gives good prediction to the expected provided bending capacity of the beam. Also, there is an indication that if the concrete mix poured is not as good as required, it may produce concrete with bending capacity lower than required by design calculations. If the difference between the required bending capacity and the provided one is large, unsafe situation may arise. This may lead to weaken the beam section to resist applied moments. As a result of such case, cracks may increase as well as deflections, and eventually may produce failure.

To remedy the above unacceptable situation, one may enhance the beam using carbon fiber wrapping or strengthening using steel angles or any strengthening scheme. This will result in increasing cost. The relation between the bending capacity and concrete compressive strength is shown in Fig. (3).

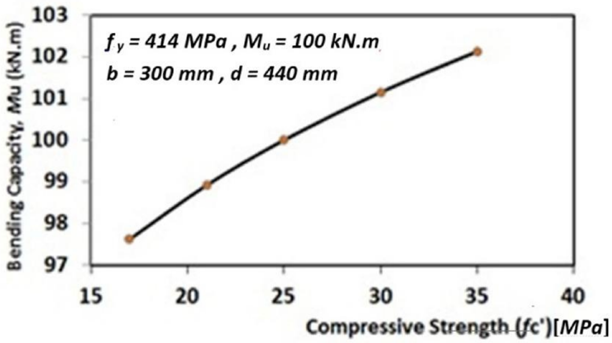

Fig. (3) $M_u - f'_c$ relationship.

Conclusion

Comparing both previous cases [I] and [II], the following may be concluded:

1. In construction industry, it is common practice that sometimes neither steel reinforcement may be available at market as required by design calculations nor poured concrete may be as strong as required.
2. Providing f_y different than required by calculations is more safe because the adjusted calculations are done before pouring concrete.
3. Providing f_c' larger than required by calculations is acceptable because it produces good concrete quality and enhances bending capacity.
4. Providing f_c' less than required by calculations is not acceptable because it produces concrete of less quality than required. This may lead to failure. And may need some schemes to remedy.
5. It is preferable to deal with f_y being different than dealing with f_c' being different than required by calculations. Because the first case occur before pouring concrete and the second case occur after pouring concrete.
6. A formula to adjust the calculations is set to predict the required provided steel ratio if f_y is different than required by calculations.

7. A formula to predict the actual provided bending capacity is set for the case when $f_c^{'}$ is less than required by calculations.

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Notations:

A_s: area of non pre-stressed tensile reinforcing steel (mm^2).
$(A_s)_{Provided}$: area of steel reinforcement different from that in design calculations (mm^2).
$(A_s)_{Required}$: area of steel reinforcement required in design calculations (mm^2).
b: width of the compression face of a flexural member (mm).
d: effective depth of a section measured from extreme compression fiber to centroid of tensile reinforcement (mm).
f'_c: specified compressive strength of concrete (MPa).
f_y : specified yield strength of non pre-stressed reinforcing (MPa) .
M_u: ultimate flexural capacity (kN.m) .
β_1 : a factor to obtain the depth of the equivalent rectangular stress block.
ρ : ratio of non pre-stressed reinforcement in tension zone.
ρ_{max}: maximum ratio of non pre-stressed reinforcement.
ρ_{min}: minimum ratio of non pre-stressed reinforcement.
φ: capacity reduction factor.